Dijkstra, Prim, Kruskal

Greedy

Make locally optimal choices, never backtrack

Prove global optimality

Job scheduling

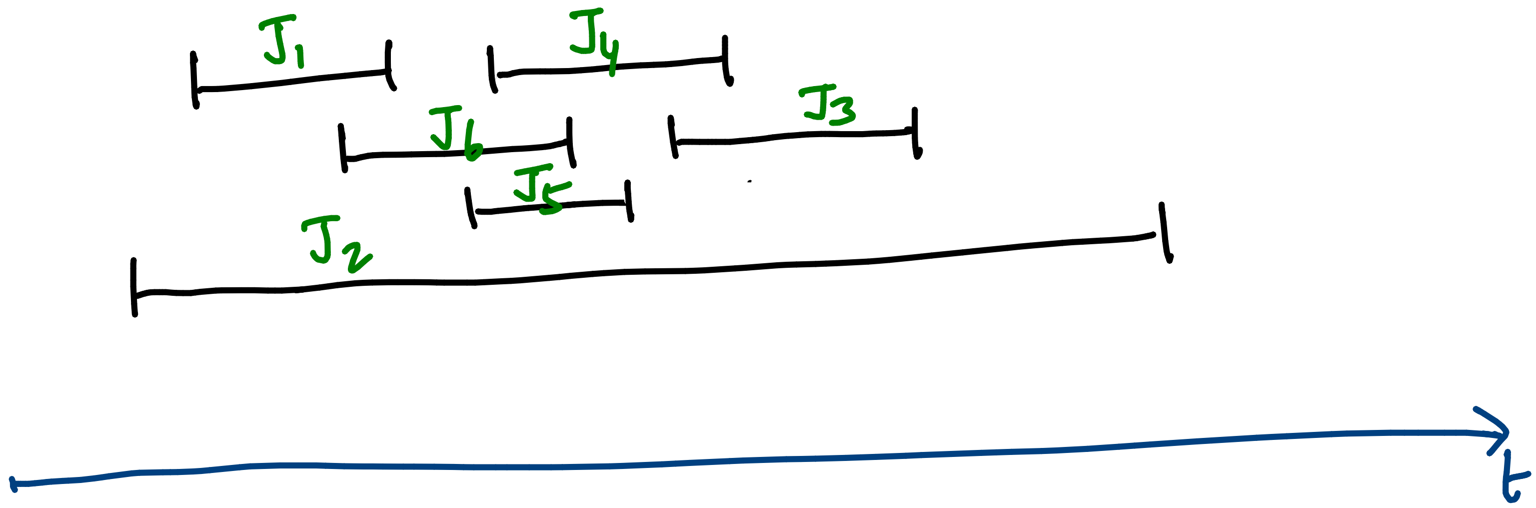
Collection of tasks with constraints

Find an optimal schedule

Interval scheduling

Each job has a time interval $(start_i, finish_i)$

Cannot undertake jobs with overlapping intervals



Compute max no. of compatible jobs that can be executed

Greedy strategy?

Local criterion to choose a job

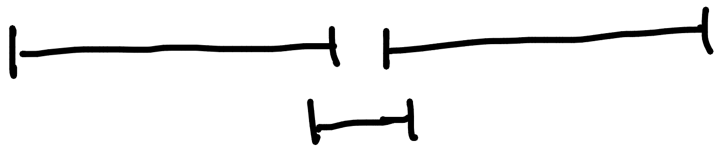
e.g. length of job - choose shortest

start time - earliest / latest ?

finish time - earliest / latest ?

overlaps - choose min

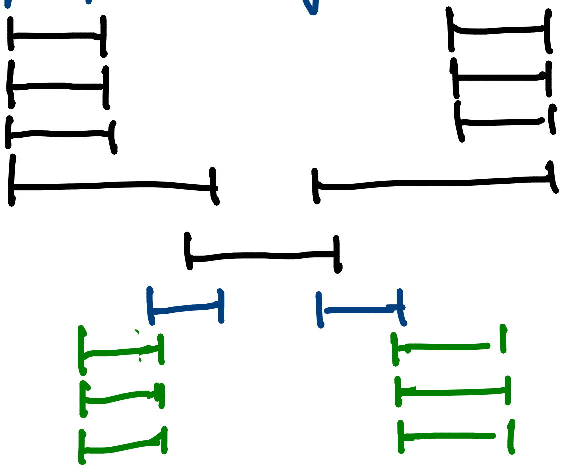
length does not work



overlaps ?

min overlaps?

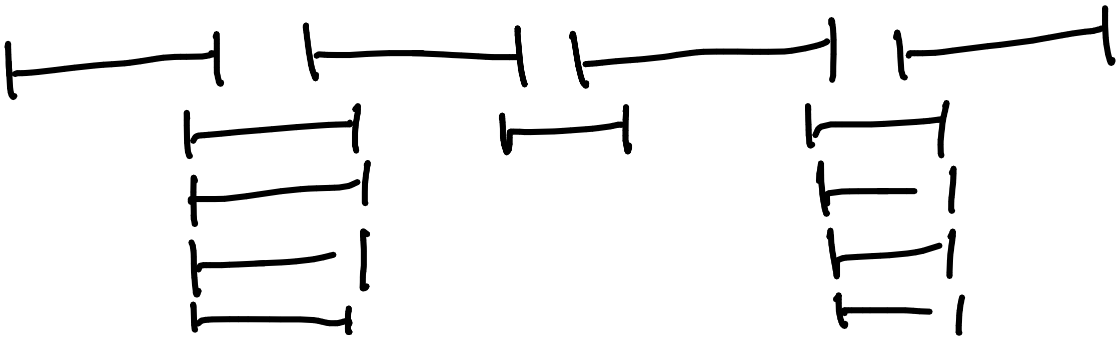
At each step pick a job with min overlaps



ALMOST

CLOSER

SIMPLER



start/finish times

earliest finish (= latest start)

Prove correctness?

earliest start ~~x~~

Assume there is an optimum soln

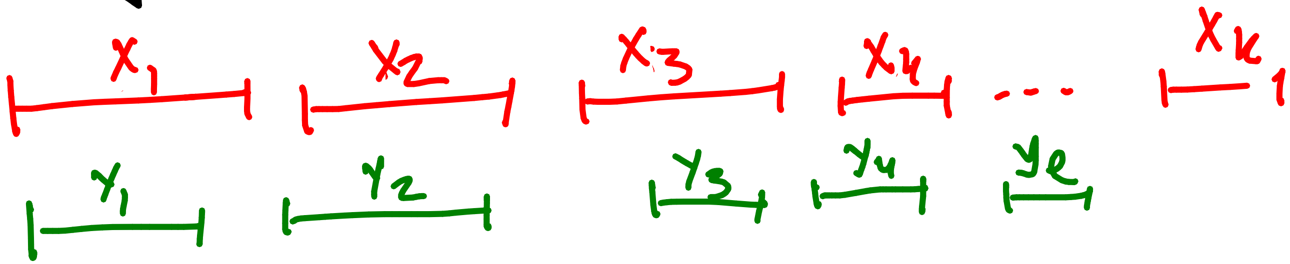
$x_1\ x_2 \ldots x_k$ s.t. $s_1 < f_1 \le s_2 < f_2 \le \ldots$

Claim: Greedy can produce a soln that is equally good

Greedy produces $y_1\ y_2 \ldots y_\ell$ $\ell \le k$

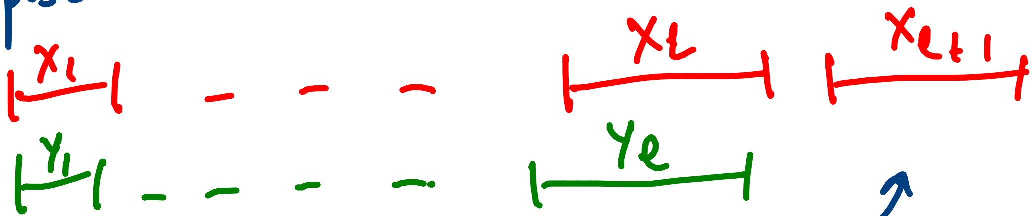
By induction, show $\text{finish}(y_i) \le \text{finish}(x_i)$

Pictorially



Inductive proof is clear

Suppose $\ell < k$



greedy can be extended

$\therefore \ \ell = k$

Alternatively, "transform" $x_1 \ldots x_k$ to $y_1 \ldots y_k$ "EXCHANGE ARGUMENT"

Another variant

Jobs have durations & deadlines

All jobs must be eventually completed

Penalty for not meeting deadlines

Penalty is proportional to delay

Minimize penalty of schedule

Minimize maximum delay across all jobs

Jobs are indivisible, cannot overlap

Job i: deadline $d(i)$

time $t(i)$

Greedy criteria

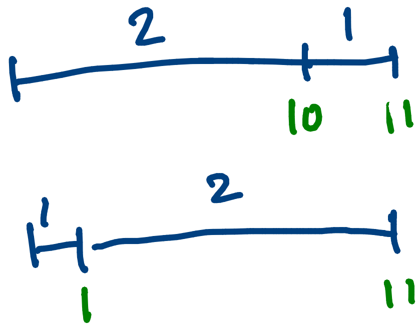
Duration min $t(i)$ — $d(1)=11$ $t(1)=2$

$d(2)=10$ $t(2)=9$

Slack $d(i)-t(i)$

$d(1)=2$ $t(1)=1$

$d(2)=10$ $t(2)=10$



Penalty 9

Penalty 1

Deadline min $d(i)$ ✓ Why?