Knapsack

Items i_1 i_2 . . . i_n

Value v_1 v_2 . . . v_n

Weight w_1 w_2 . . . w_n

Bag of fixed weight capacity W

Choose a set of items that fits in your bag to maximize value

→ unlimited copies of each item

→ one copy of each item

Unlimited copies

Suggestion

Order items, say by $\frac{v_j}{w_j}$

Optimal solution has n_j copies of item i_j

⋮

OR

Just pick items one at a time

Picking i_j reduces capacity by w_j, adds value v_j

Start with capacity W

pick i_1 | i_2 | . . . | i_n

v_1 + soln to $W - w_1$

v_2 + soln to $W - w_2$

v_n + soln to $W - w_n$

i_1 | i_2 | . . .

$$\text{MaxValue}(w) = \max_{j \text{ s.t. } w_j \le w} \left(v_j + \text{MaxValue}(w - w_j) \right)$$

current capacity

Final solution is MaxValue(W)

— need MaxValue(0), MaxValue(1), ... , MaxValue$(W-1)$

DP of order $O(n \cdot W)$

Depends on W, and the "fractions" of W generated by the w_j's

Input size of value W is $\log W$ (no. of bits to write W)

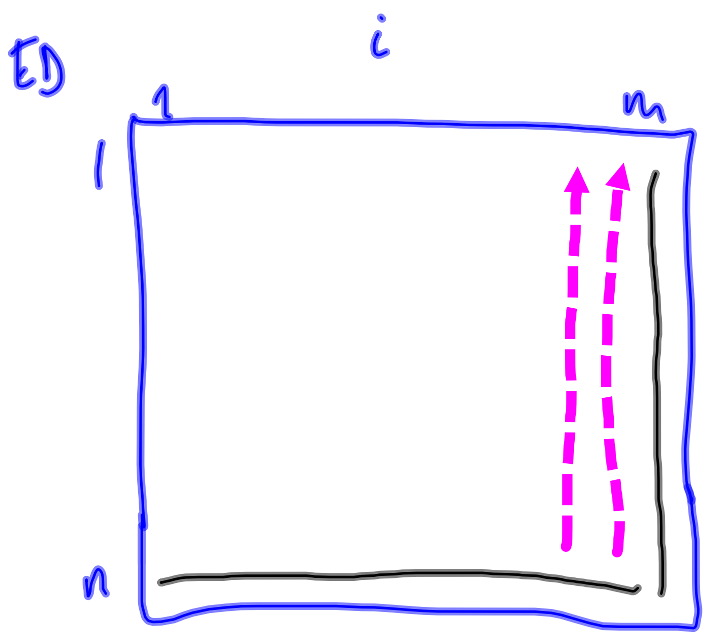
Not poly time

Space: Do we need space proportional to W to compute MaxValue(W)?

Naively we need such an array to store subproblem values

Subproblems reachable from MaxValue(w) are bounded by max w_j

What about edit distance



$O(\min(m,n))$ space suffices — row wise or columnwise

only need current & previous row/column

Knapsack, limited (=1) copy of each item

MaxValue(w) $\xrightarrow{\text{choose } i_j}$ $\underline{\text{MaxValue}(w - w_j)}$

cannot choose i_j again

Need to incorporate list of available items as a parameter

Fix some order of items $i_1, i_2, \ldots, i_n$

MaxValue(w, k) Optimum solution for capacity w and items available are $i_1, i_2 \ldots, i_k$

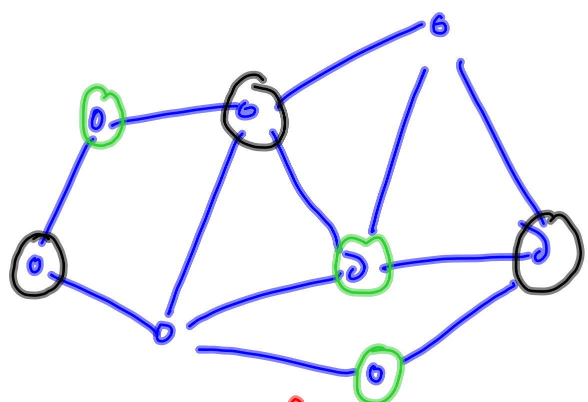
(or $i_k, i_{k+1} \ldots i_n$)

$\text{MaxValue}(w, k)$

- choose i_k: $\text{MaxValue}(w - w_k, k-1) + v_k$
- skip i_k: $\text{MaxValue}(w, k-1)$

$i_1, i_2, \ldots, i_k$

GRAPHS



Independent set of vertices – no edges between them

Aim: Find an independent set of max size

MIS

Interval scheduling problem

requests $r_1, \dots, r_n$ each $r_i = (s_i, f_i)$

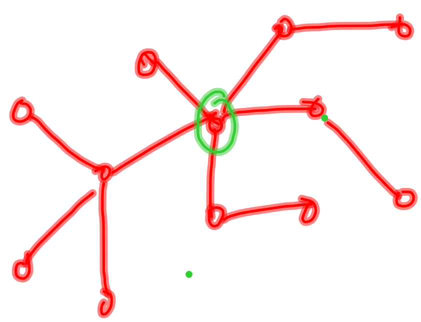
graph $V = \{r_1, \dots, r_n\}$

$E = \{(r_i, r_j) \mid (s_i, f_i) \text{ \& } (s_j, f_j) \text{ overlap}\}$

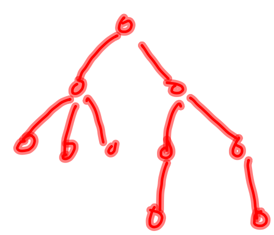
Solution is an MIS in this graph

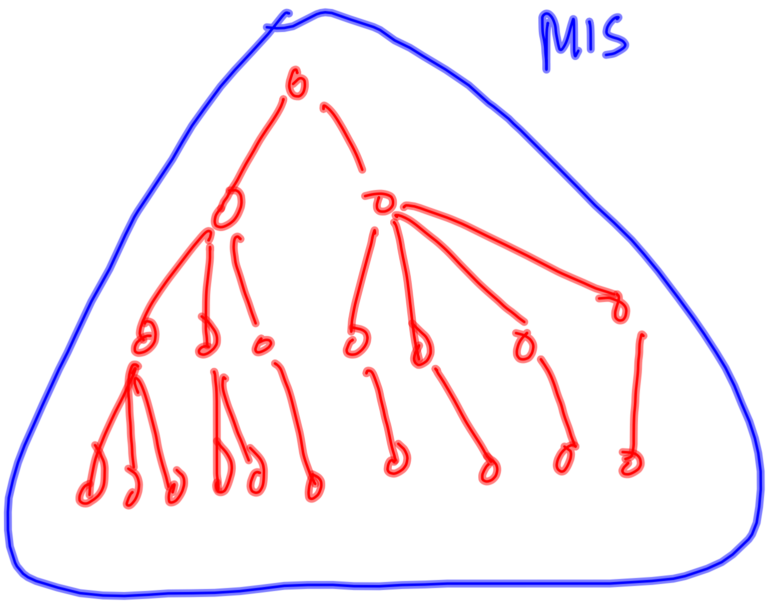
In general MIS is hard

But, it has a nice solution over trees



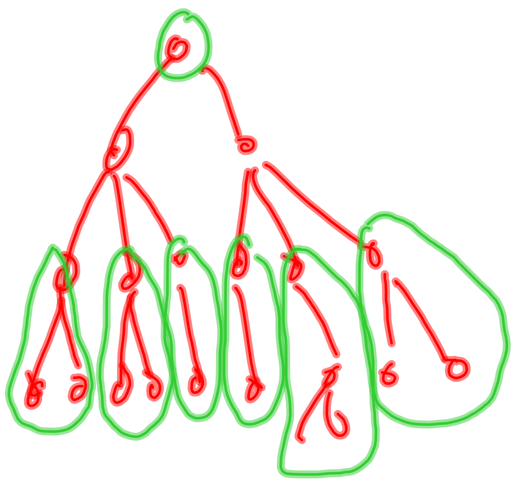
Pick a root and "hang" up the tree



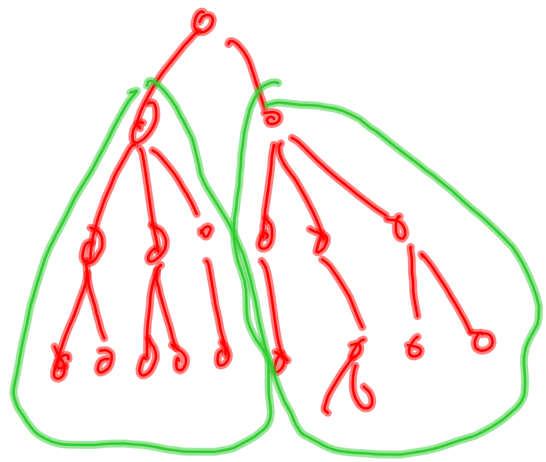


choose root — add MIS for grandchildren
exclude root — add MIS for children

Choose root



Exclude root



Also bottom up?

Tree structure controls/limits no. of subproblems