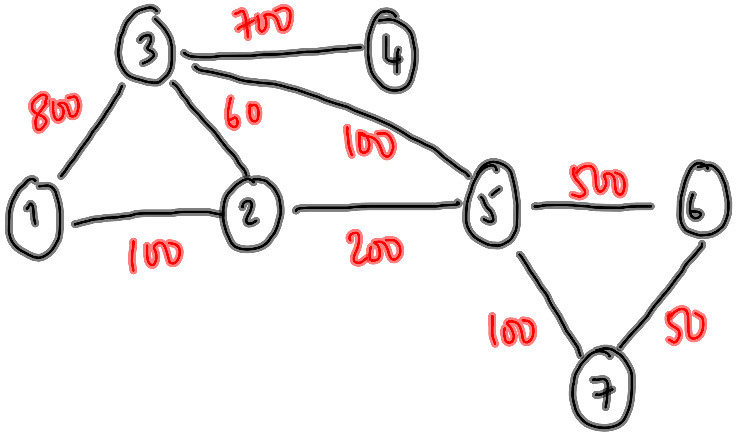
Shortest paths in weighted graphs



Want shortest path from 1 to every other vertex

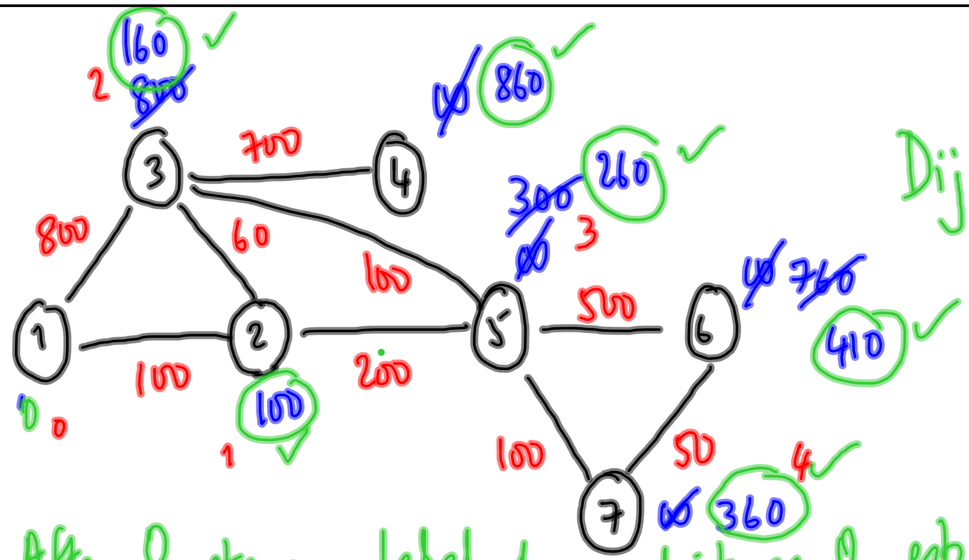
Edge weights $w : E \to \mathbb{N} = \{1, 2, ..\}$

↳ represent "costs" – money, distance, time

replace 1 by w in adj matrix

in an adj list – pairs $i \to \{(j,w), (k,w') ..\}$

Dijkstra's algorithm



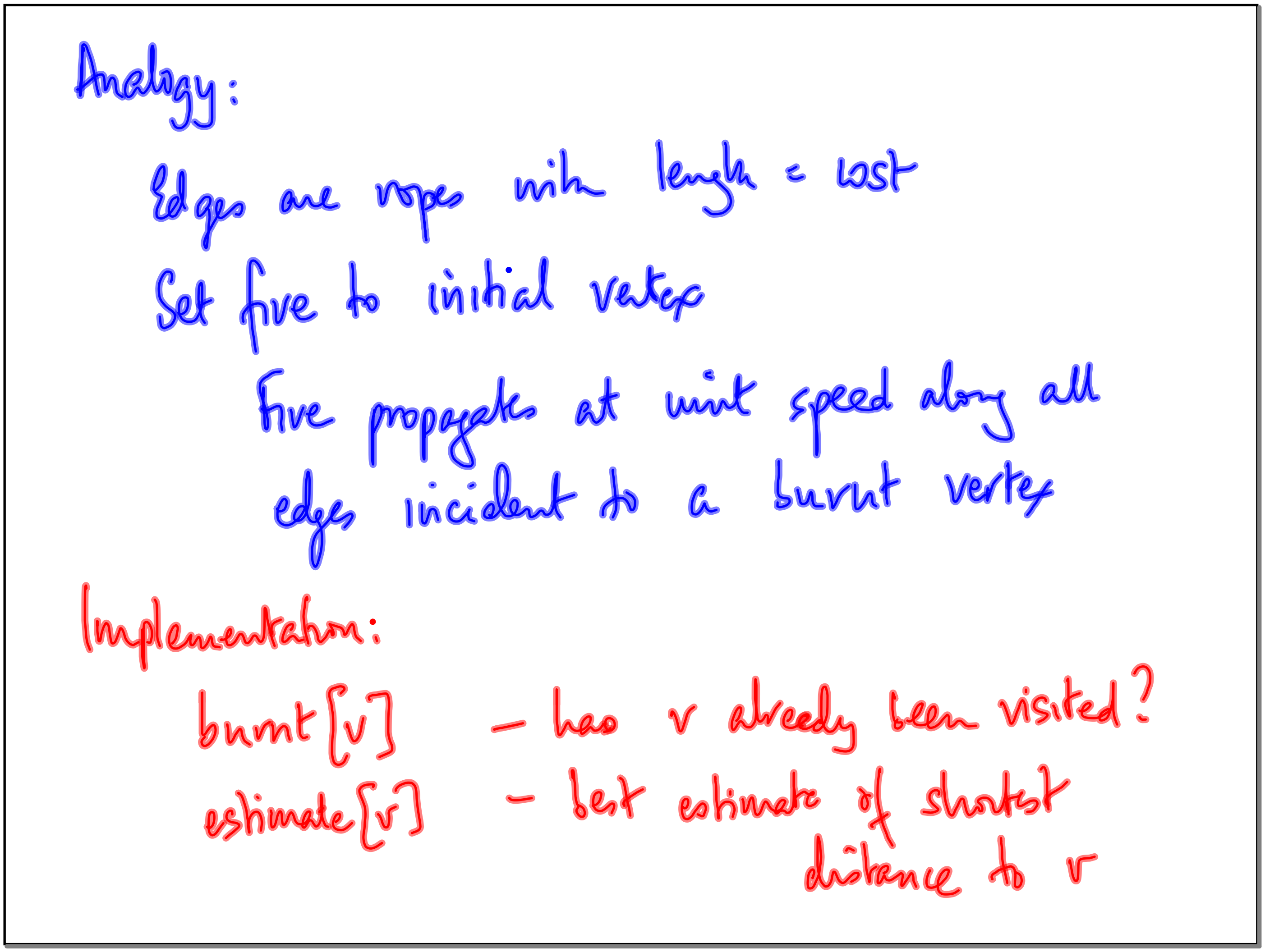
After 0 steps – label 1 as distance 0, estimates to other nodes

After 1 step – label shortest distance to 2 as 100

recompute all estimates

2nd step: Have explored {1,2} Possible extension are 3, 5 ✓ ✗

recompute estimates



Analogy:

Edges are ropes with length = cost

Set fire to initial vertex

Fire propagates at unit speed along all edges incident to a burnt vertex

Implementation:

burnt[v] – has v already been visited?

estimate[v] – best estimate of shortest distance to v

Initially: (assume we start at 1)

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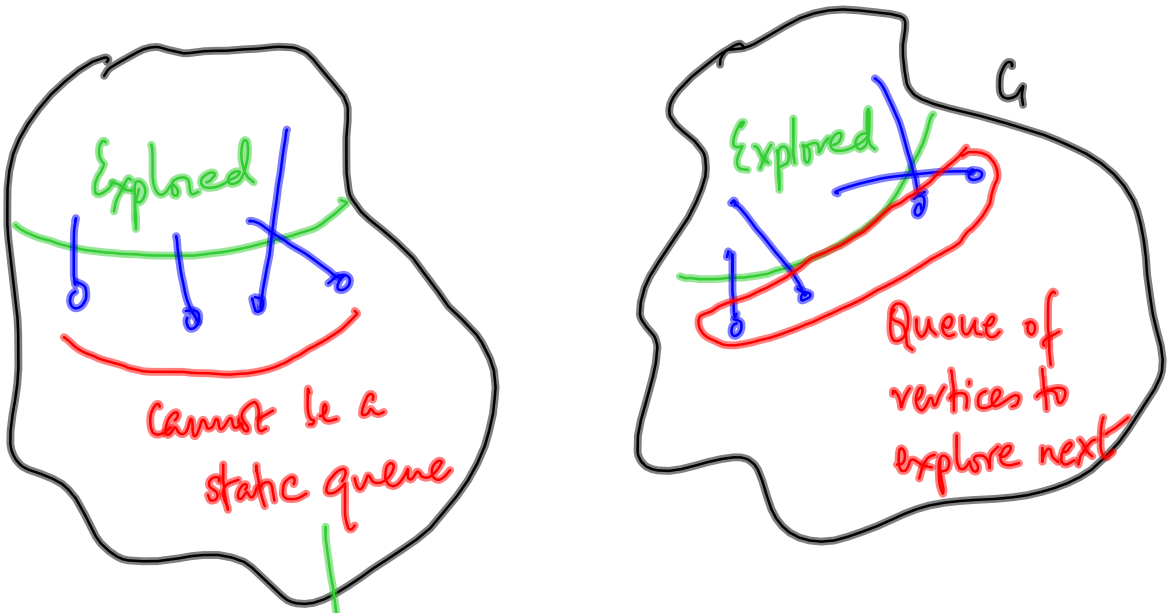
$\forall v.$ burnt[v] = 0                 $O(n)$              $O(n^2)$ as written
estimate[1] = 0                       } $O(n)$
$\forall v \neq 1$ estimate[v] = $\infty$

while there are unburnt vertices
    pick v s.t. burnt[v]==0 & estimate[v] is min
                                  $O(n)$
    burnt[v] = 1
    $\forall w, (v,w) \in E$ update estimate[w] as
        min(estimate[w], estimate[v] + weight(v,w))
```

n iterations

Dijkstra's algorithm vs BFS



extract smallest estimate in this "queue"

priority queue

Priority queue is an evolving/dynamic set of elements

Add an element with a given priority

Remove the element with "best" priority

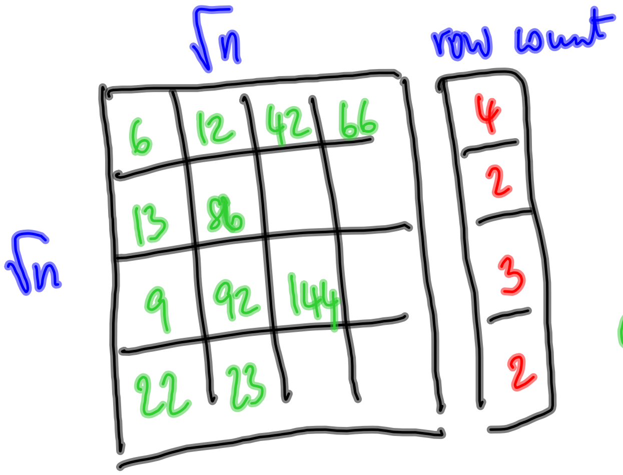
Update priority of elements already in queue (extension to standard priority queue)

Implementing priority queue

	Insert	Delete-best
Sorted list	$O(n)$	$O(1)$
Unsorted list	$O(1)$	$O(n)$

Way to go is to leave 1-demensional data structures behind

Trivial 2D structure (assumes we have an upper bound n on size of P.Q.)



				row count
6	12	42	66	4
13	86			2
9	92	144		3
22	23			2

Each row is sorted

delete-best : find min of col 1 + shift row $O(\sqrt{n})$

Insert : find first non-full row
Insert $O(\sqrt{n})$