

Graphs: $G = (V, E)$ $E \subseteq V \times V$ $\begin{cases} \text{undirected} \\ \text{directed} \end{cases}$

Represent as adjacency matrix A $n \times n$ 0-1 matrix
 $|V| = n$

adjacency list $i \mapsto$ nbrs of i

Exploring a graph via breadth first search (BFS)

BFS (s) $\leftarrow \forall v. \text{mark}[v]=0 \leftarrow O(n)$ when $|V|=n$

$\text{mark}[s]=1$

$\text{insertqueue}(Q, s)$

while Q is not empty

$v = \text{extract-head}(Q)$

for each nbr u of v

if u is not marked

$\text{mark}[u]=1$

$\text{insertqueue}(Q, u)$

With adjacency list

$O(m)$

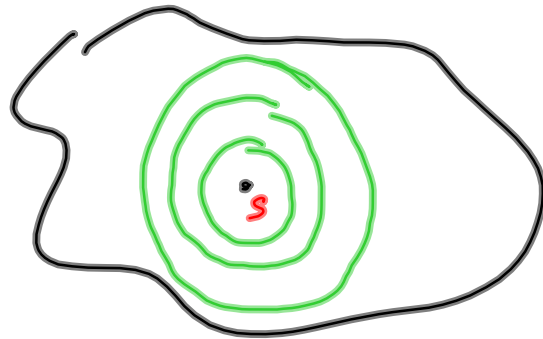
where $|E|=m$

$O(n+m)$

by defn, linear time

BFS visits vertices in order of distance (no. of hops) from source

Can record this distance in BFS



Initialize $\text{distance}[s] = 0$

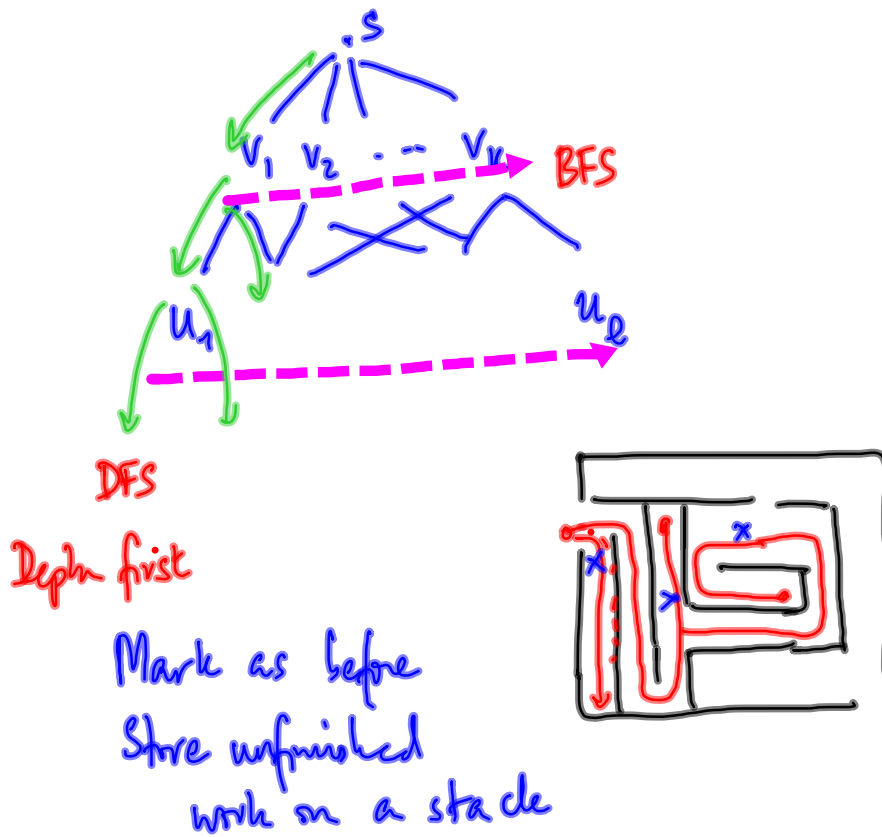
Each time v adds neighbour u to the queue

$$\text{distance}[u] = \text{distance}[v] + 1$$

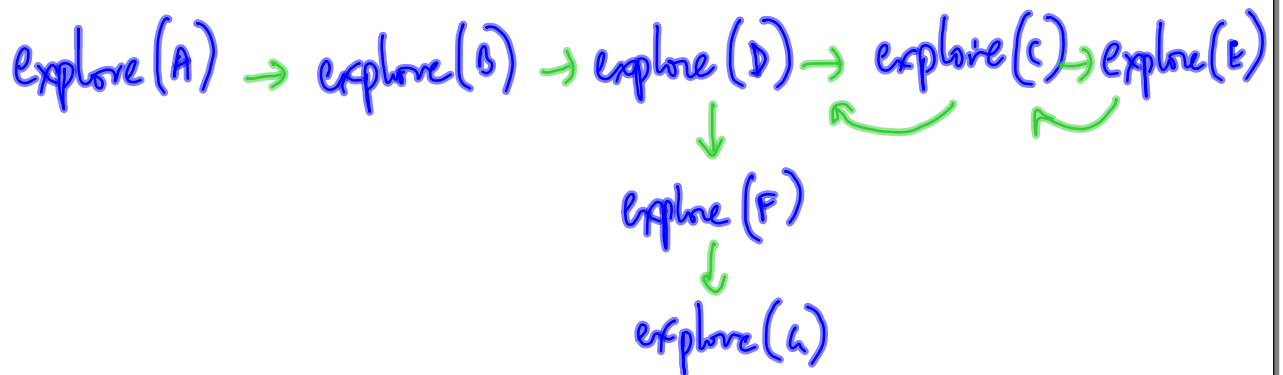
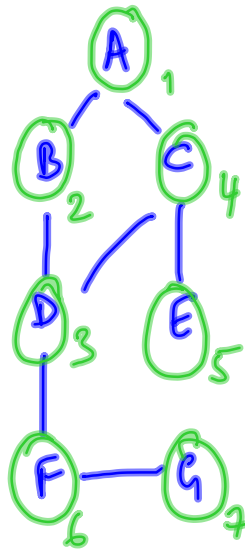
Record path to each reachable vertex

$$\text{parent}[u] = v \quad \text{when } v \text{ adds } u \text{ to queue}$$

An alternative strategy for exploration



explore(v)
 mark[v]=1
 for each nbr u of v
 if u is unmarked
 explore(u)



dfs(\hat{a})

for all v $mark[v] = 0$

for each u in V

if u is unmarked
 explore(u)

e.g. previsit(v)

sequence [v] = count

count = count + 1 // count set to 0 initially

explore(v)

mark [v] = 1

previsit(v)

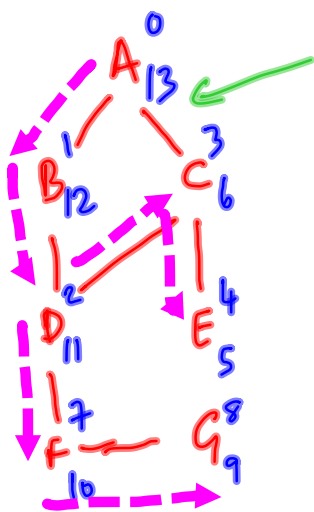
for each nbr u of v

if u unmarked
 explore(u)

postvisit(v)

Using $previsit(v)$ & $postvisit(v)$ can maintain two values $pre[v]$, $post[v]$ that refer to a common counter

Initially $count = 0$



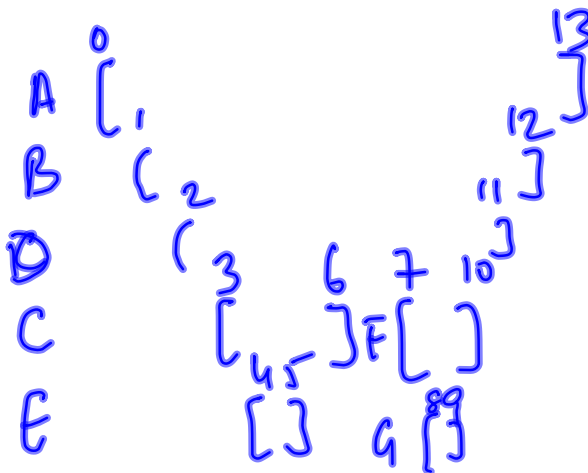
---> DFS Tree
Connected, no loops

$previsit(v)$

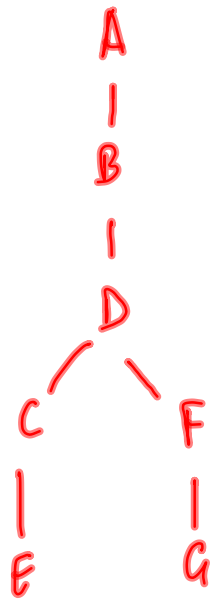
$pre[v] = count$
 $count = count + 1$

$postvisit(v)$

$post[v] = count$
 $count = count + 1$



DFS tree



$Pre(v), Post(v)$ intervals are
well behaved

Must be nested or disjoint
Cannot overlap in some
nontrivial way

$Interval(u) \subseteq Interval(v)$

u was explored via v

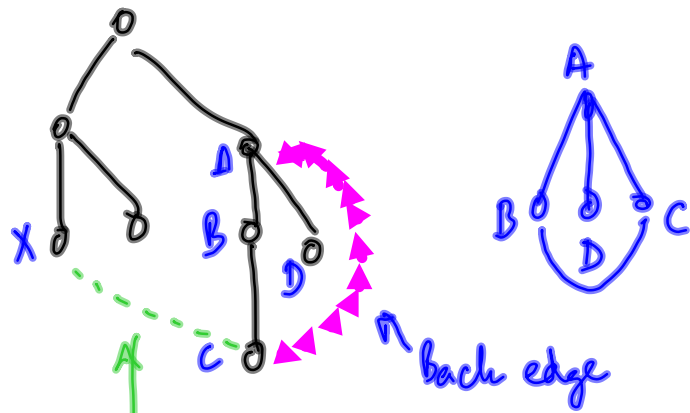
$Interval(u) \cap Interval(v) = \emptyset$

unconnected in DFS tree

G has DFS tree edges & back edges

undirected

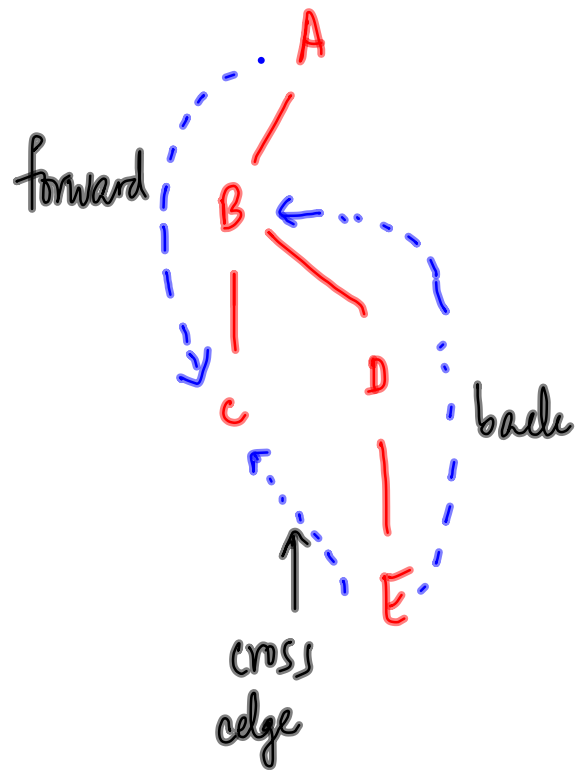
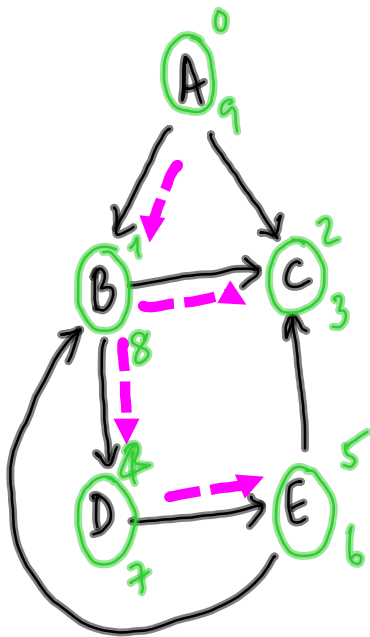
G has a cycle
iff DFS tree
has a back
edge



not possible!

Should have visited C from X

What if G is directed?



Supplementary Reading

Algorithm Design

Kleinberg & Tardos

Algorithms

Dasgupta, Papadimitriou
& Vazirani