
Guyana
Venezuela
Suriname
Columbia
French Guiana
(France)
Amazon R.
Amazon
Basin
Brazil
Brazilian
Highlands
Peru
Bolivia
Paraguay
Uruguay
Chile
Argentina
ATLANT
Falkland Is.
(U.K.)


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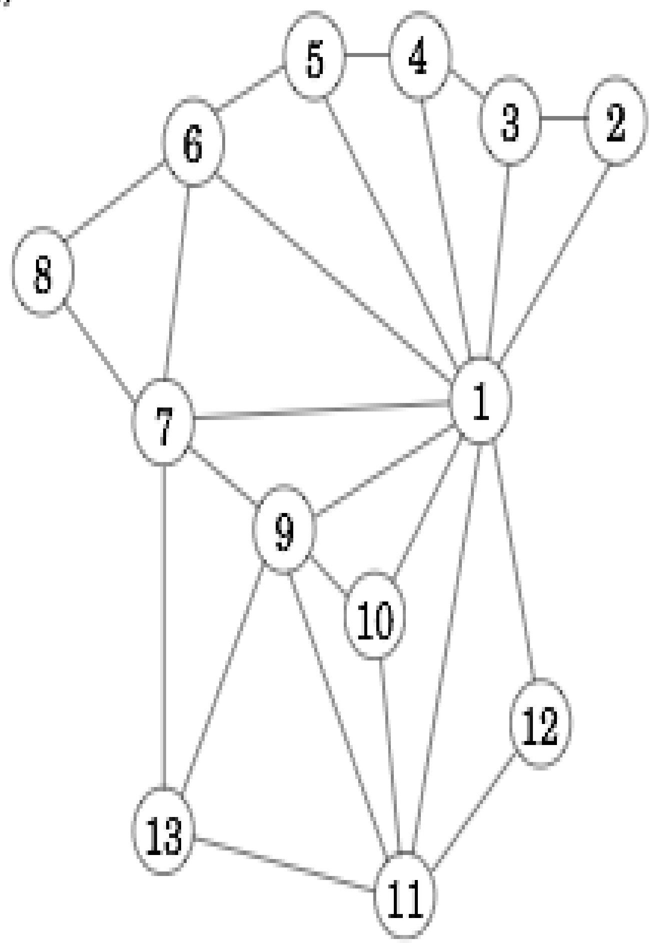
(b)


1
2
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13

Graph $G = (V, E)$ Vertices

Edges $E \subseteq V \times V$

Map colouring : Countries are vertices

$(c_1, c_2) \in E$ if c_1 & c_2 share a border

Legal Map Colouring : $f : V \to C$ (C a set of colours)

s.t $(v_1, v_2) \in E \Rightarrow f(v_1) \neq f(v_2)$

Airline

Cities served

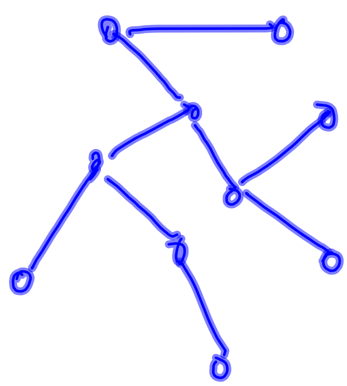
Connections the airline has between cities

How to go from Amritsar to Coimbatore?

Cities = nodes

vertices

Edges : direct flights



A & B share a border ⇒ B & A share a border

KF flies Delhi → Amritsar $\overset{?}{\Rightarrow}$ KF flies Amritsar → Delhi?

$E \subseteq V \times V$ may not be symmetric

Directed vs undirected graphs

asymmetric symmetric edges

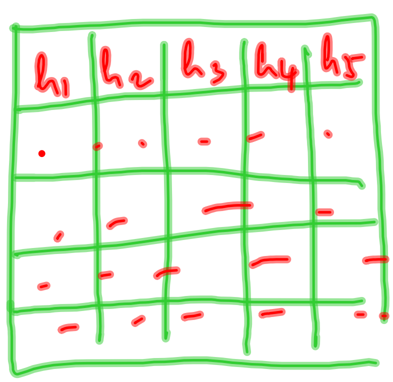
Usually assume irreflexive

Explicit vs implicit graphs

Airline route graph is explicit

Country graph is explicit

Field

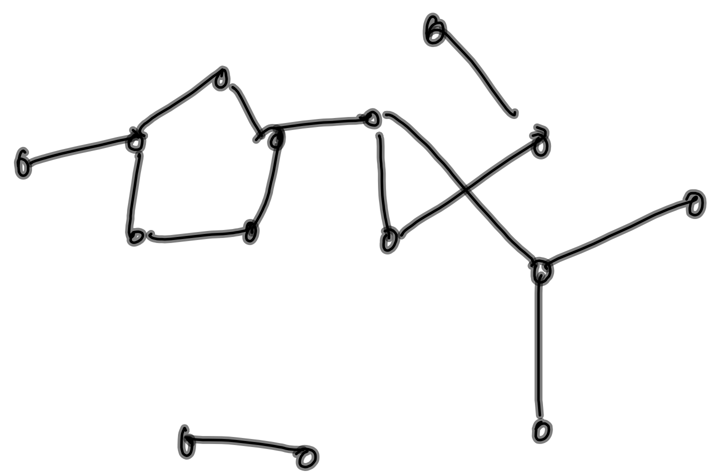


Go from one square patch to an adjacent square patch (N/S/E/W) if height diff $\leq$ threshold

Questions about graphs:

Colouring

Reachability – can I go from x to y?



Visually

Check connectivity

Program cannot do this

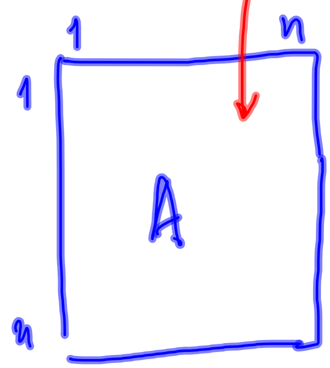
$A(i,i) = 0$, generally (irreflexive)

symmetric if G is undirected

Need to represent the graph

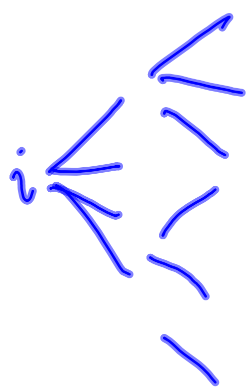
Adjacency Matrix

$V = \{1, 2, \ldots, n\}$



$$A(i,j) = \begin{cases} 1, & \text{if } (i,j) \in E \\ 0, & \text{otherwise} \end{cases}$$

Write a program to check if j is reachable from i

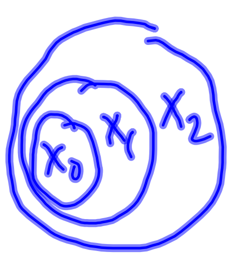


$X_0 = \{i\}$

$X_1 = \{j \mid (i,j) \in E\} \cup X_0$

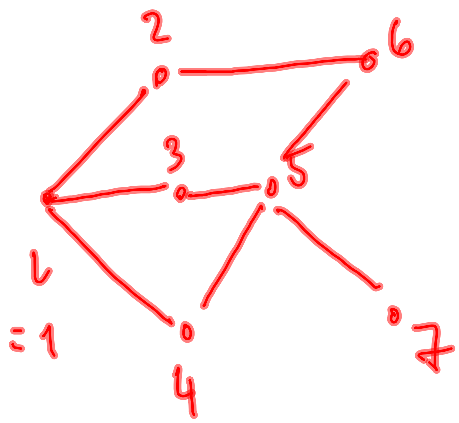
$X_2 = \{j \mid \exists k \in X_1 \ (k,j) \in E\} \cup X_1$

$\vdots$



$\exists m .\ X_{m+1} = X_m$ (V is finite!)

Check if $j \in X_m$



$X_0 = \{1\}$

$X_1 = X_0 \cup \{2,3,4\} = \{1,2,3,4\}$

$X_2 = X_1 \cup \{6, 5, \cancel{5}\}$

$X_3 = X_2 \cup \{7\}$

Two attributes associated with each vertex:

Has it been seen before?

Has it been explored (i.e. have its neighbours been computed)?

Level by level exploration

- "Mark" each vertex when we see it first

mark[v] = 1

When I explore an edge (j,k)

if mark[k] == 0, set mark[k] = 1
& add it to the list of vertices to be explored

Process all outgoing edges from j at one shot

List of unexplored vertices (i.e. reached, but yet to explore neighbours)

Natural to use queue

explore a vertex v:

for each $(v,w) \in E$ Scan row v in A

if mark(w) == 0

mark(w) = 1

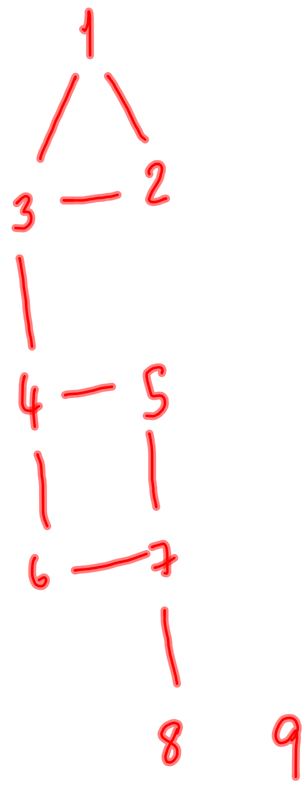
add w to queue

Breadth First Search (starting at v)

```
BFS(v):
    mark[v] = 1
    add v to queue Q

    while Q is not empty
        w = extract_head(Q)
        for each u s.t. (w,u) ∈ E
            if mark[u] == 0:
                mark[u] = 1
                add u to Q
```

BFS(5)



Mark:

1	2	3	4	5	6	7	8	9
~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1	0

Queue = [~~5~~, ~~4~~, ~~7~~, ~~3~~, ~~6~~, ~~8~~, ~~1~~, ~~2~~

How much time does this take?

Each vertex enters Q at most once

Processing nbrs of v takes $O(n)$

Naf

$O(n^2)$ assuming adj. matrix

$|V| = n$ $|E| = m$ Clearly, undirected graph $\Rightarrow m \leq \binom{n}{2}$

directed $\Rightarrow m \leq n(n-1)$

Sparse graphs m is $O(n)$

Can we improve on $O(n^2)$ BFS for sparse graphs?

Maintain edges as a list

Adjacency list

$1 \rightarrow \{2, 7\}$ $(1,2), (1,7) \in E$

$2 \rightarrow \{1, 8\}$ $(2,1), (2,8) \in E$

$\vdots$ $\vdots$

$8 \rightarrow \{2, \ldots\}$

What is the complexity of BFS now?

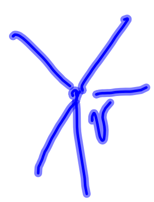
Outer loop is still $O(n)$ — each vertex enters Q at most once

Processing neighbours(v)

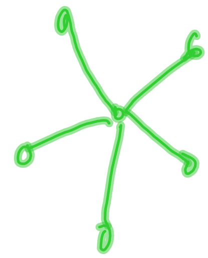
Proportional to no. of neighbours

$\Downarrow$

$$\text{degree}(v) = \left|\{w \mid (v,w) \in E\}\right|$$



Accounting costs more carefully



$$\deg(v_1) + \deg(v_2) + \dots + \deg(v_n)$$

n outer loops

Assuming entire graph is connected

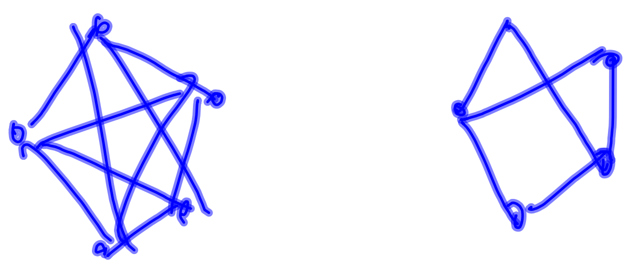
BFS takes $\sum_{v \in V} \deg(v) = 2m$

$(i,j) \in E$ contributes 1 each to $\deg(i), \deg(j)$

Graphs have 2 size parameters $|V| = n$

$|E| = m$

An algorithm is said to be linear if it works in time $O(n+m)$



To follow

(1) BFS identifies reachable vertices level by level. Can we recover the length of the shortest path (in terms of no. of edges) to each reachable vertex?

(2) Extract an actual path from v to w for each reachable vertex w.