Mathematical Logic, August–December 2015

Assignment 3: MSO and Temporal Logic

November 19, 2015

Due: November 27, 2015

Note: *Only electronic submissions accepted, via Moodle.*

1. Write an MSO formula that captures the language of finite words over $\Sigma = \{a, b, c\}$ in which a's and b's alternate. For instance, *ccc*, *cacc*, *bcacbccac*, *ababa* are all in the language, while *caacb*, *bcbb*, *aa* are not.

2. Suppose we do *not* reduce our MSO language to have only set variables. Construct an automaton whose language is equivalent to the formula $x \in X$, where x is an individual variable and X is a set variable.

3. The dual of the until operator U in LTL is written W and called *unless* or *weak until.* This operator is defined as follows:

$$\varphi \mathsf{W} \psi \equiv (\varphi \mathsf{U} \psi) \vee G\varphi$$

   (a) Write a first-order formula $\alpha_{\varphi \mathsf{W} \psi}(x)$ that captures the semantics of $\sigma, x \models \varphi \mathsf{W} \psi$, where $\sigma = s_0 s_1 \ldots s_n$ is a run and $x \in \{0, 1, \ldots, n\}$. As usual, you can assume that corresponding formulas $\alpha_\varphi(x)$ and $\alpha_\psi(x)$ have already been defined, inductively.

   (b) Show that U can be expressed in terms of W. (First show that $G\varphi$ can be expressed in terms of W and then show how to express U using W and $F\varphi$.)

4. Construct an example to show that the CTL formula $AFAG\varphi$ is not equivalent to the LTL formula $FG\varphi$, assuming that both CTL and LTL are interpreted over infinite runs.