Mathematical Logic, August–December 2015

Assignment 2: First-Order Logic

October 12, 2015

Due: October 20, 2015

Note: *Only electronic submissions accepted, via Moodle.*

1. Let $L = (R, F, C)$ be a finite first-order relational language with $F = C = \emptyset$ and let $\mathcal{M} = (S, \iota)$ be a finite L-structure. Show that there is an L-sentence $\phi_{\mathcal{M}}$ whose models are precisely the L-structures isomorphic to $\mathcal{M}$.

2. (a) Let $L = \{0, +, \times\}$ where $+$ and $\times$ are binary function symbols and 0 is a constant symbol. Consider an L-structure $(\mathbb{R}, +, \times, 0)$, where $\mathbb{R}$ is the set of real numbers with the conventional interpretation of and $+$, $\times$ and 0 as addition, multiplication and zero.

   Show that the relation $<$ ("less-than") is *elementary definable* in $(\mathbb{R}, +, \times, 0)$ – i.e, there is a formula $\phi(x, y)$ over L such that for all $a, b \in \mathbb{R}$,

   $((\mathbb{R}, +, \times, 0), [x \mapsto a, y \mapsto b]) \models \phi(x, y)$ iff $a < b$.

   (b) Let $L = \{+, 0\}$. Show that $<$ is not elementary definable in $(\mathbb{R}, +, 0)$. [Hint: Work with a suitable isomorphism of $(\mathbb{R}, +, 0)$ onto itself.]

3. Using Ehrenfeucht-Fraïssé games show that acyclicity of finite graphs is not first-order definable in the language $L = (\{E\}, \emptyset, \emptyset)$, where E is interpreted as the edge relation.