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Data Mining and Machine Learning
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Market-basket analysis

- Set of items $I = \{i_1, i_2, \ldots, i_N\}$
- A transaction is a set $t \subseteq I$ of items, set of transactions $T = \{t_1, t_2, \ldots, t_M\}$

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  - $X, Y \subseteq I$, $X \cap Y = \emptyset$
  - If $X \subseteq t_j$ then it is likely that $Y \subseteq t_j$

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  - Want $\dfrac{(X \cup Y).count}{X.count} \geq \chi$ (Confidence)
- How significant is this pattern overall?
  - Want $\dfrac{(X \cup Y).count}{M} \geq \sigma$ (support)

$X \cup Y$

$\dfrac{Z.count}{M} \geq \sigma$

$Z.count \geq \sigma \cdot M$

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- Given sets of items I and transactions T, with confidence χ and support σ, find all valid association rules $X \rightarrow Y$

Apriori

Apriori observation

If Z is not a frequent itemset, no superset $Y \supseteq Z$ can be frequent

- For any frequent pair $\{x, y\}$, both $\{x\}$ and $\{y\}$ must be frequent
- Build frequent itemsets bottom up, size 1,2,...

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- For any frequent pair $\{x, y\}$, both $\{x\}$ and $\{y\}$ must be frequent
- Build frequent itemsets bottom up, size 1,2,...
- F_i : frequent itemsets of size i — Level i
- F_1: Scan T, maintain a counter for each $x \in I$
- C_k = subsets of size k, every $(k-1)$-subset is in F_{k-1}
- F_k: Scan T, maintain a counter for each $X \in C_k$

Association rules

- Given sets of items I and transactions T, with confidence χ and support σ, find all valid association rules $X \rightarrow Y$
  - $X, Y \subseteq I$, $X \cap Y = \emptyset$
  - $\frac{(X \cup Y).count}{X.count} \geq \chi$
  - $\frac{(X \cup Y).count}{M} \geq \sigma$

Association rules

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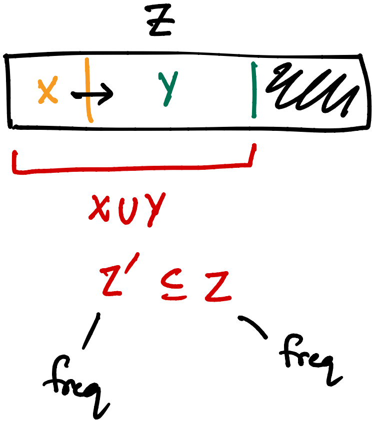
  - $X, Y \subseteq I$, $X \cap Y = \emptyset$
  - $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
  - $\dfrac{(X \cup Y).count}{M} \geq \sigma$
- For a rule $X \to Y$ to be valid, $X \cup Y$ should be a frequent itemset
- Apriori algorithm finds all $Z \subseteq I$ such that $Z.\text{count} \geq \sigma \cdot M$

Naïve strategy

- For every frequent itemset Z
  - Enumerate all pairs $X, Y \subseteq Z$, $X \cap Y = \emptyset$
  - Check $\dfrac{(X \cup Y).count}{X.count} \geq \chi$



Association rules

Naïve strategy

- For every frequent itemset Z
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- Can we do better?

Association rules

Naïve strategy

- For every frequent itemset Z
  - Enumerate all pairs $X, Y \subseteq Z$, $X \cap Y = \emptyset$
  - Check $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
- Can we do better?
- Sufficient to check all partitions of Z
  - If $X, Y \subseteq Z$, $X \cup Y$ is also a frequent itemset

Association rules

- Sufficient to check all partitions of Z
- Suppose $Z = X \uplus Y$, $X \to Y$ is a valid rule and $y \in Y$
- What about $(X \cup \{y\}) \to Y \setminus \{y\}$?

$$\frac{(X \cup Y).\text{count}}{X.\text{count}} > \chi$$

$X.\text{count} > X \cup \{y\}.\text{count}$

$X \to Y$ is a rule

$X \cup \{y\} \to Y \setminus \{y\}$

$X' \to Y'$

✓ bigger

$\frac{A}{b} > \chi \quad \frac{A}{c} > \chi \quad b \geq c$

Association rules

- Sufficient to check all partitions of Z
- Suppose $Z = X \uplus Y$, $X \rightarrow Y$ is a valid rule and $y \in Y$
- What about $(X \cup \{y\}) \rightarrow Y \setminus \{y\}$?
  - Know $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
  - Check $\dfrac{(X \cup Y).count}{(X \cup \{y\}).count} \geq \chi$

Association rules

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- What about $(X \cup \{y\}) \to Y \setminus \{y\}$?
  - Know $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
  - Check $\dfrac{(X \cup Y).count}{(X \cup \{y\}).count} \geq \chi$
  - $X.\text{count} \geq (X \cup \{y\}).\text{count}$, always
  - Second fraction has smaller denominator, so $(X \cup \{y\}) \to Y \setminus \{y\}$ is also a valid rule

Association rules

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  - X.count $\geq (X \cup \{y\})$.count, always
  - Second fraction has smaller denominator, so $(X \cup \{y\}) \to Y \setminus \{y\}$ is also a valid rule

Observation: Can use apriori principle again!

$X \to Y$ is not valid

$X \setminus \{x\} \to Y \cup \{y\}$ cannot be valid

Apriori for association rules

- If $X \to Y$ is a valid rule, and $y \in Y$, $(X \cup \{y\}) \to Y \setminus \{y\}$ must also be a valid rule
- If $X \to Y$ is not a valid rule, and $x \in X$, $(X \setminus \{x\}) \to Y \cup \{x\}$ cannot be a valid rule

Apriori for association rules

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- If $X \to Y$ is not a valid rule, and $x \in X$, $(X \setminus \{x\}) \to Y \cup \{x\}$ cannot be a valid rule
- Start by checking rules with single element on the right
  - $Z \setminus z \to \{z\}$
- For $X \to \{x, y\}$ to be a valid rule, both $(X \cup \{x\}) \to \{y\}$ and $(X \cup \{y\}) \to \{x\}$ must be valid
- Explore partitions of each frequent itemset "level by level"

Complications

Variable thresholds

How to fix χ, σ?

Mixed baskets

Association rules for classification

- Classify documents by topic
- Consider the table on the right

Words in document	Topic
student, teach, school	Education
student, school	Education
teach, school, city, game	Education
cricket, football	Sports
football, player, spectator	Sports
cricket, coach, game, team	Sports
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Association rules for classification

- Classify documents by topic
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Association rules for classification

- Classify documents by topic
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- Look for association rules of a special form
  - {student, school} → {Education}
  - {game, team} → {Sports}

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$X \rightarrow \{y\}$

Cannot make predictions for combinations we have never seen!

Association rules for classification

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- Look for association rules of a special form
  - $\{$student, school$\} \rightarrow \{$Education$\}$
  - $\{$game, team$\} \rightarrow \{$Sports$\}$
- Right hand side always a single topic
- Class Association Rules

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football, player, spectator	Sports
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Summary

- Market-basket analysis searches for correlated items across transactions
- Formalized as association rules
- Apriori principle helps us to efficiently
  - identify frequent itemsets, and
  - split these itemsets into valid rules
- Class association rules — simple supervised learning model

Supervised learning

- A set of items
  - Each item is characterized by attributes $(a_1, a_2, \ldots, a_k)$
  - Each item is assigned a class or category c
- Given a set of examples, predict c for a new item with attributes $(a'_1, a'_2, \ldots, a'_k)$

A_1	A_2	...	A_k	C
a_1	a_2	...	a_k	c_1
b_1	b_2	...	b_k	c_2

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- Aim is to learn a mathematical model that generalizes the training data
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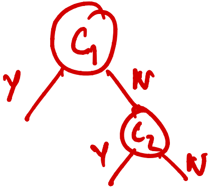
- Classification problem
  - Usually assumed to binary — two classes

$\{c_1, c_2, c_3\}$



Example: Loan application data set

ID	Age	Has_job	Own_house	Credit_rating	Class
1	young	false	false	fair	No
2	young	false	false	good	No
3	young	true	false	good	Yes
4	young	true	true	fair	Yes
5	young	false	false	fair	No
6	middle	false	false	fair	No
7	middle	false	false	good	No
8	middle	true	true	good	Yes
9	middle	false	true	excellent	Yes
10	middle	false	true	excellent	Yes
11	old	false	true	excellent	Yes
12	old	false	true	good	Yes
13	old	true	false	good	Yes
14	old	true	false	excellent	Yes
15	old	false	false	fair	No

Number

Answer

6N
9Y

Basic assumptions

Fundamental assumption of machine learning

- Distribution of training examples is identical to distribution of unseen data

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Fundamental assumption of machine learning

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What does it mean to learn from the data?

- Build a model that does better than random guessing
  - In the loan data set, always saying Yes would be correct about $9/15$ of the time
- Performance should ideally improve with more training data

Basic assumptions

Fundamental assumption of machine learning

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- Build a model that does better than random guessing
  - In the loan data set, always saying Yes would be correct about $9/15$ of the time
- Performance should ideally improve with more training data

How do we evaluate the performance of a model?

- Model is optimized for the training data. How well does it work for unseen data?
- Don't know the correct answers in advance to compare — different from normal software verification

The road ahead

Many different models

- Decision trees
- Probabilistic models — naïve Bayes classifiers
- Models based on geometric separators
  - Support vector machines (SVM)
  - Neural networks

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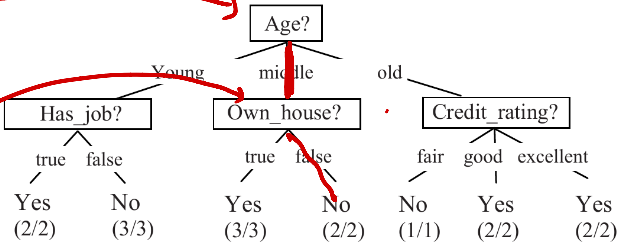
Important issues related to supervised learning

- Evaluating models
- Ensuring that models generalize well to unseen data
  - A theoretical framework to provide some guarantees
- Strategies to deal with the training data bottleneck

Decision trees

New item

(Middle, Y, N, Fair)



Questions are "adaptive"

20 Questions

What is the "strategy"? ✓

ID	Age	Has_job	Own_house	Credit_rating	Class
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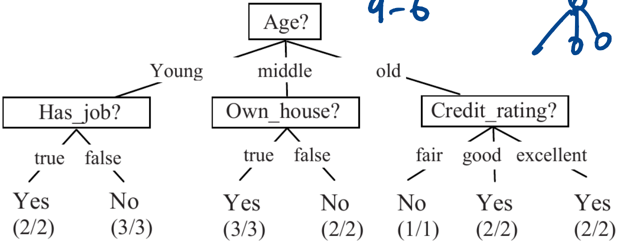
15	old	false	false	fair	**No**

How to choose questions?

Level 1 ✓

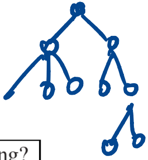
Level 2

⋮



9-6

2-0 0-3 3-0 0-2



How long do I continue?

- Till I run out of questions (i.e no. of colums)
- If we reach a "uniform" answer

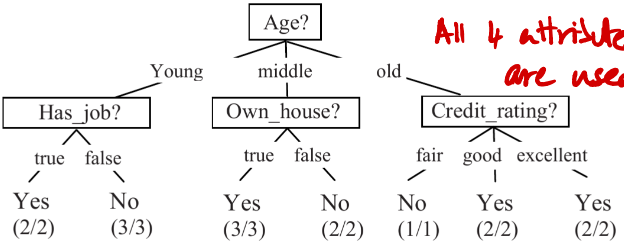
Comparing decision trees

Prefer "less complex" model

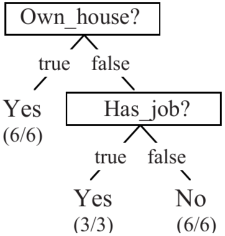
"Simplest explanation is best"

Occam's Razor

William of Ockham



All 4 attributes are used



Age, Credit Rating are redundant

Simplest is best

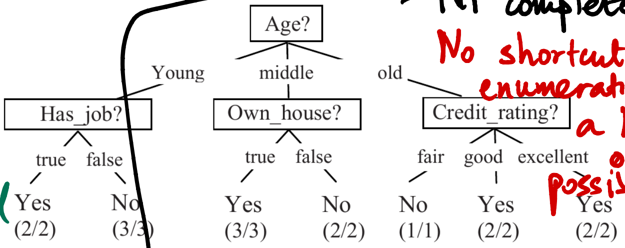
Simplest?

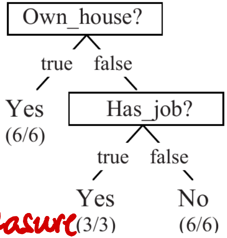
- No of questions overall
- No of levels in tree

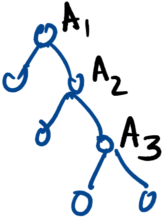
Bad news Building "simplest" model is computationally intractable, for any reasonable measure

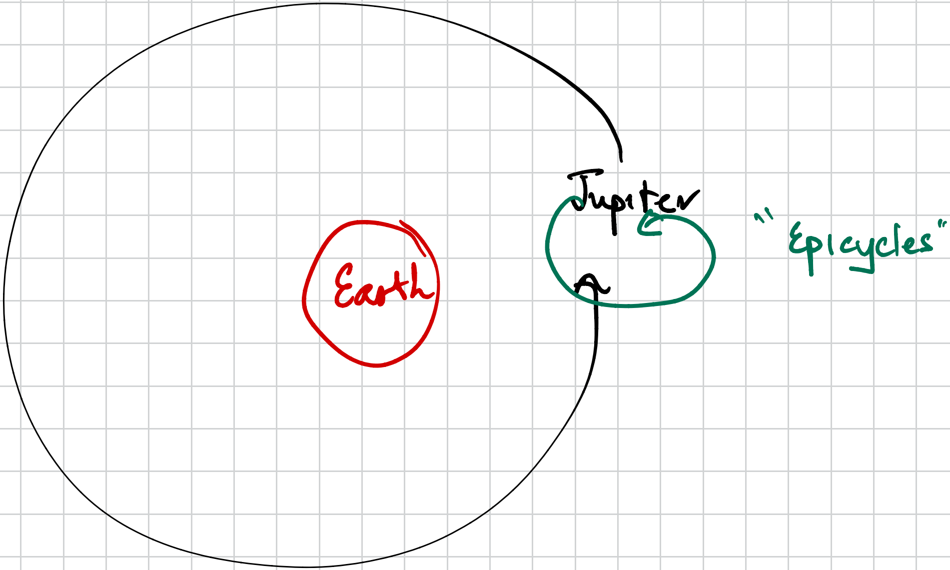
→ NP complete

No shortcut to enumerating a lot of possibilities








Jupiter
Earth
"Epicycles"

Greedy heuristic — impurity

Goal?

Reach "uniform" blocks

Reduce "randomness"