

Lecture 22: 4 April, 2023

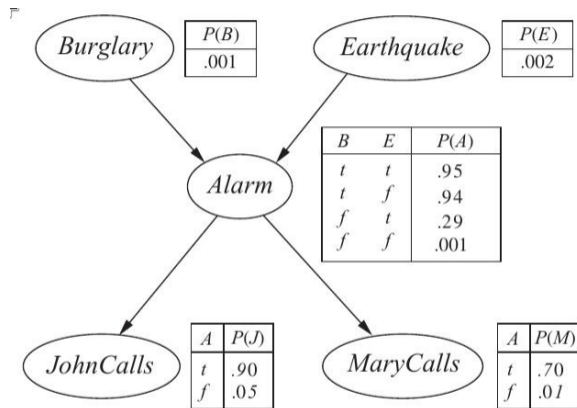
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Data Mining and Machine Learning
January–April 2023

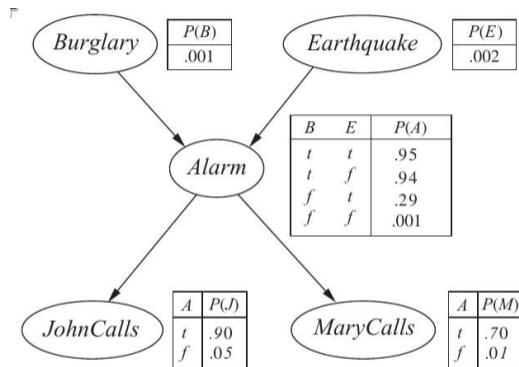
Probabilistic graphical models

- Underlying DAG, no cyclic dependencies
- Each node has a local (conditional) probability table



Conditional independence

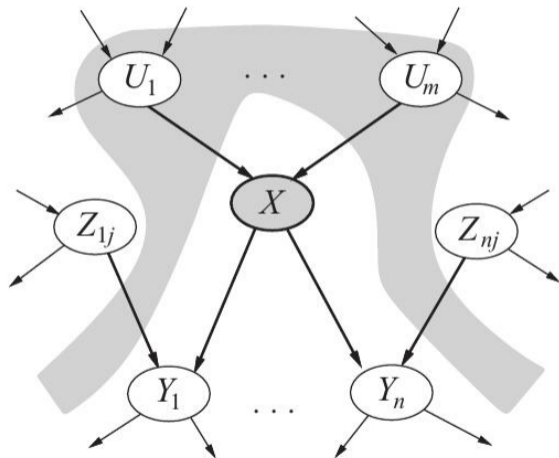
- $x \perp y$ — x and y are independent
 - $P(x \wedge y) = P(x) \cdot P(y)$
- $x \perp y \mid z$
 - x and y are independent given z
 - $P(x \wedge y \mid z) = P(x \mid z) \cdot P(y \mid z)$
- Is *JohnCalls* independent of *MaryCalls* ($j \perp m$)?
 - No — value of j tells us something about value of m and vice versa
- Is *JohnCalls* independent of *MaryCalls* given *Alarm* ($j \perp m \mid a$)?
 - Yes — by semantics of network, local independence



Probabilistic graphical models

- **Fundamental assumption**

A node is conditionally independent of non-descendants, given its parents



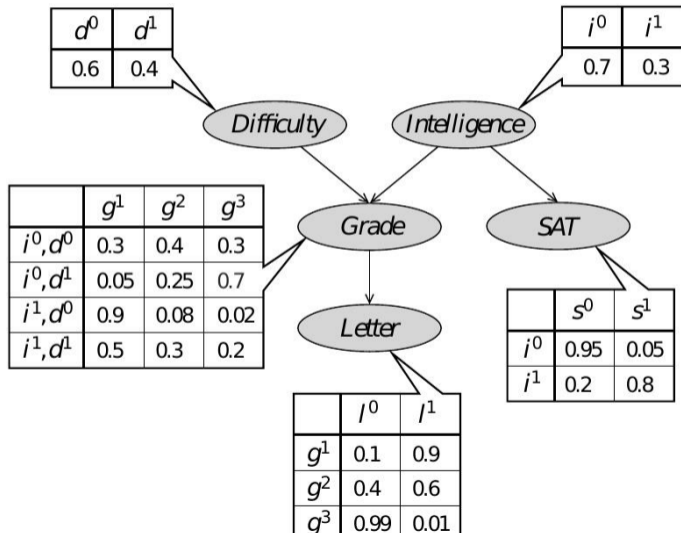
Student example

- $SAT \perp Grade \mid Difficulty$?

- No

- Can we calculate conditional independence from the graph?

- In general, check if $X \perp Y \mid Z$ for sets of variables X, Y, Z



Conditional independence

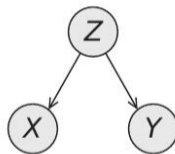
- How does dependence “flow” through a network?
- For neighbouring nodes, dependence flows both ways
 - If $x \rightarrow y$, knowing x tells us about y and vice versa
- Examine **trails** between nodes
 - Paths in the underlying undirected graph
- **Basic trails** — (undirected) paths of length 2
 - Four basic trails



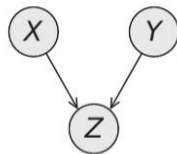
(a)



(b)



(c)



(d)

Basic trails

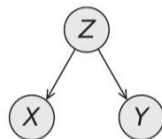
- (a), (b) and (c): Z blocks flow between X and Y , by semantics of Bayesian networks
- In (d), knowing Z allows influence to flow
 - Z : Car does not start
 X : Low Battery, Y : No Fuel
 - Z : Grass is wet
 X : Overnight rain, Y : Sprinkler ran
 - Simplest form of **V-structure**



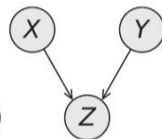
(a)



(b)



(c)



(d)

D-Separation

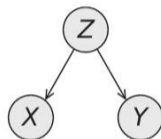
- Check if $X \perp Y \mid Z$
- Dependence should be blocked on every trail from X to Y
 - Each undirected path from X to Y is a sequence of basic trails
 - For (a), (b), (c), need Z present
 - For (d), need Z absent
 - In general, V-structure includes descendants of the bottom node



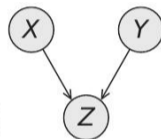
(a)



(b)



(c)



(d)

- x and y are **D-separated** given z if all trails are blocked
- Variation of **breadth first search (BFS)** to check if y is reachable from x through some trail
- Extends to sets — each $x \in X$ is D-separated from each $y \in Y$

Conditional independence, example

- Is **SAT** independent of **Difficulty** given **Intelligence**?
 - Yes, **Difficulty** – **Grade** – **Intelligence** – **SAT** trail is blocked at **Grade** (V-structure) and **Intelligence**
- Is **SAT** independent of **Difficulty** given **Letter**?
 - No, **Difficulty** – **Grade** – **Intelligence** – **SAT** trail is open
 - **Letter** is known, hence something about **Grade** is known (V-structure)
 - **Intelligence** is not known

