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Data Mining and Machine Learning
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Limitations of classification models

Recall

- Bias : Expressiveness of model limits classification
- Variance: Variation in model based on sample of training data

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Overcoming limitations

- Bagging is an effective way to overcome high variance
  - Ensemble models
    - Sequence of models based on independent bootstrap samples
    - Use voting to get an overall classifier
- How can we cope with high bias?

Dealing with bias

- A biased model always makes mistakes
    - Build an ensemble of models to average out mistakes

Dealing with bias

- A biased model always makes mistakes
  - Build an ensemble of models to average out mistakes
- Mistakes should be compensated across models in the ensemble
  - How to build a sequence of models, each biased a different way?
  - Again, we assume we have only one set of training data

Boosting

- Build a sequence of weak classifiers $M_1, M_2, \ldots, M_n$ on inputs $D_1, D_2, \ldots, D_n$
  - A weak classifier is any classifier that has error rate strictly below 50%

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  - Initially all weights equal, D_1
  - Going from D_i to D_{i+1} : increase weights where M_i makes mistakes on D_i
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- Ensemble output
  - Individual classification outcomes are $\{-1, +1\}$
  - Unknown input x: ensemble outcome is weighted sum $\sum_{i=1}^{n} \alpha_i M_i(x)$
  - Check if weighted sum is negative/positive

The boosting algorithm — Adaboost

- Initially, all data items have equal weight

AdaBoost(D, Y, BaseLeaner, k)

$\boxed{\text{Initialize } D_1(w_i) \leftarrow 1/n \text{ for all } i;}$

for $t = 1$ to k **do**

  $f_t \leftarrow$ BaseLearner(D_t);

  $e_t \leftarrow \sum_{i: f_t(D_t(\mathbf{x}_i)) \neq y_i} D_t(w_i)$;

  if $e_t > ½$ **then**

    $k \leftarrow k - 1$;

    exit-loop

  else

    $\beta_t \leftarrow e_t / (1 - e_t)$;

    $D_{t+1}(w_i) \leftarrow D_t(w_i) \times \begin{cases} \beta_t & \text{if } f_t(D_t(\mathbf{x}_i)) = y_i \\ 1 & \text{otherwise} \end{cases}$;

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- Damping factor — reduce weight of correct inputs
- Reweight data items and normalize
- Final classifier

$$f_{\text{final}}(x) = \arg\max_{y \in Y} \sum_{t : f_t(x) = y} \log \frac{1}{\beta_t}$$

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  - Pick model with lowest error rate on D_{j+1} as M_{j+1}
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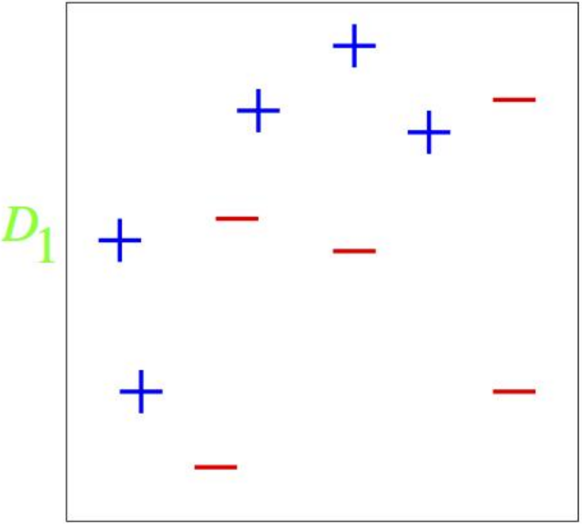
  - Reweight all training data based on error rate of M_{j+1}
- Note that same model M may be picked in multiple iterations, assigned different weights α

Boosting: An example

- Weak classifiers are horizontal and vertical lines
- Initial training data has equal weights



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- Weak classifiers are horizontal and vertical lines
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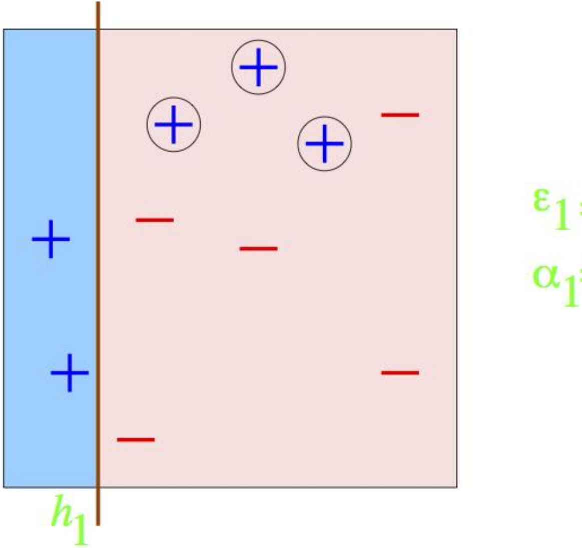

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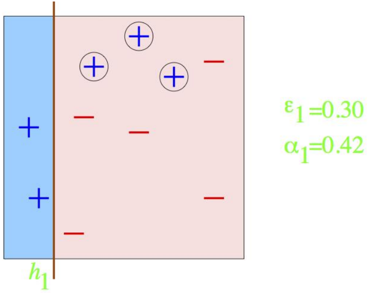

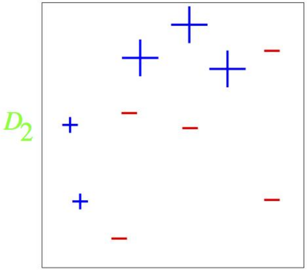

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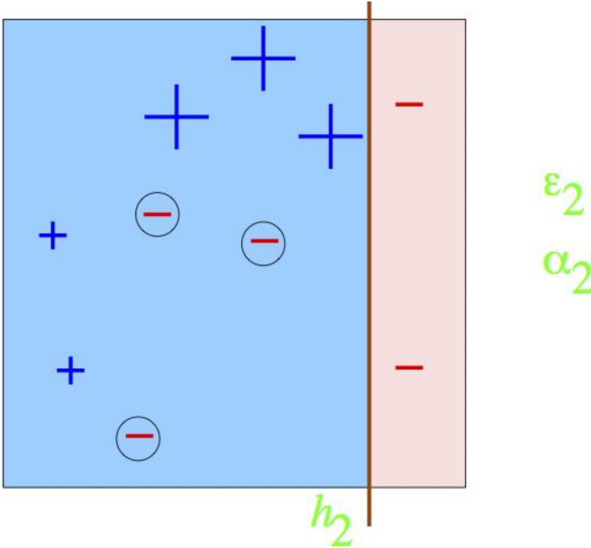

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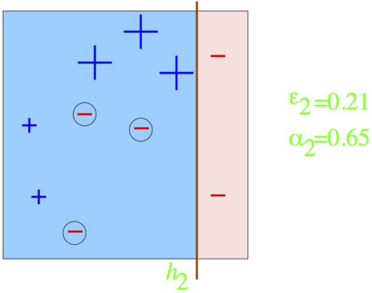

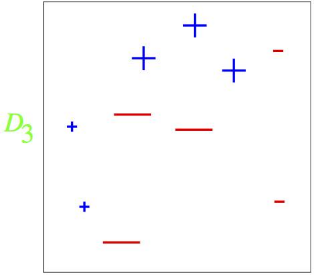

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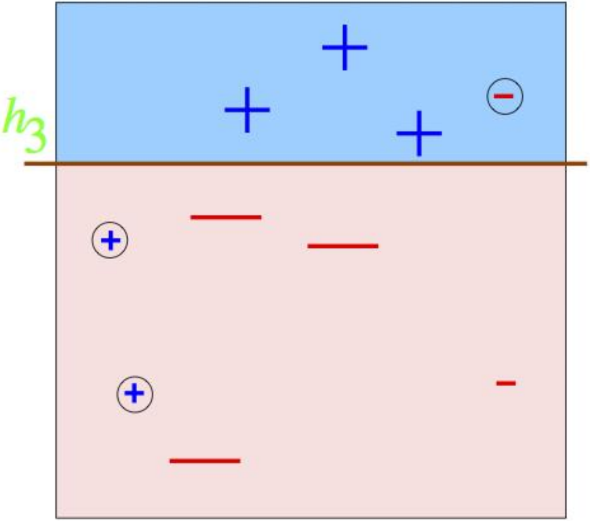
- Second separator: vertical line
  - Increase weight of misclassified inputs
- Third separator: horizontal line



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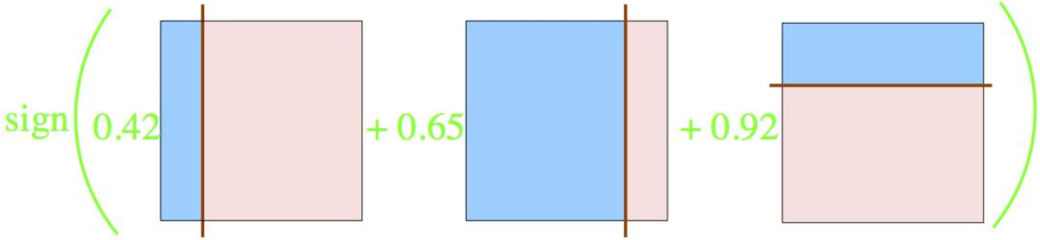
- Final classifier is weighted sum of three weak classifiers



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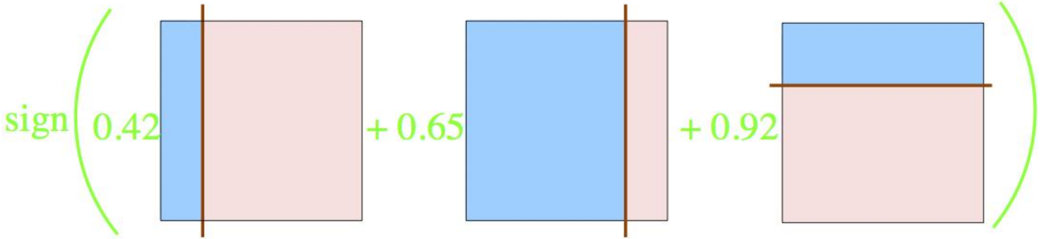

- Pictorially