Lecture 3: 12 January, 2023

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Slides by Madhavan Mukund

Data Mining and Machine Learning
January–April 2023

Supervised learning

- A set of items
  - Each item is characterized by attributes $(a_1, a_2, \ldots, a_k)$
  - Each item is assigned a class or category $c \leftarrow$ label
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- Classification problem
  - Usually assumed to binary — two classes
- Supervised learning as each training example has a category, i.e. it is *labeled*.

Example: Loan application data set

ID	Age	Has_job	Own_house	Credit_rating	Class
1	young	false	false	fair	**No**
2	young	false	false	good	**No**
3	young	true	false	good	**Yes**
4	young	true	true	fair	**Yes**
5	young	false	false	fair	**No**
6	middle	false	false	fair	**No**
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10	middle	false	true	excellent	**Yes**
11	old	false	true	excellent	**Yes**
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How do we evaluate the performance of a model?

- Model is optimized for the training data. How well does it work for unseen data?
- Don't know the correct answers in advance to compare — different from normal software verification

The road ahead

Many different models

- Decision trees
- Probabilistic models — naïve Bayes classifiers
- Models based on geometric separators
  - Support vector machines (SVM)
  - Neural networks

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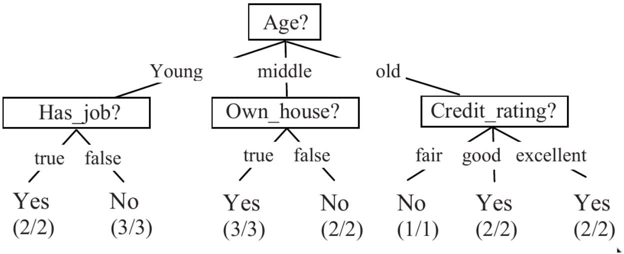
Important issues related to supervised learning

- Evaluating models
- Ensuring that models generalize well to unseen data
  - A theoretical framework to provide some guarantees
- Strategies to deal with the training data bottleneck

Decision trees

- Play "20 Questions" with the training data



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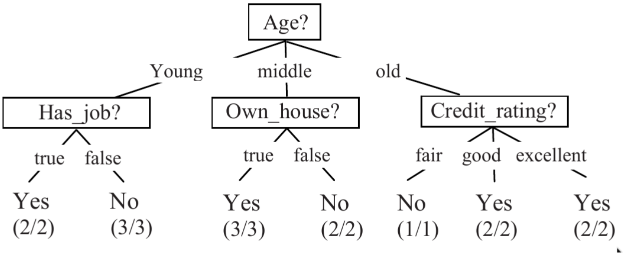

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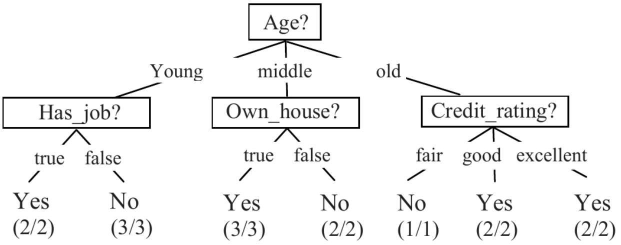

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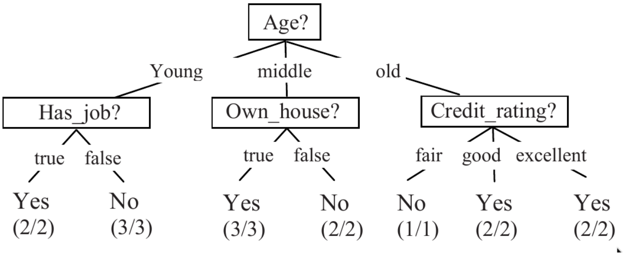
- Queries are adaptive
  - Different along each path, depends on history



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Decision tree algorithm

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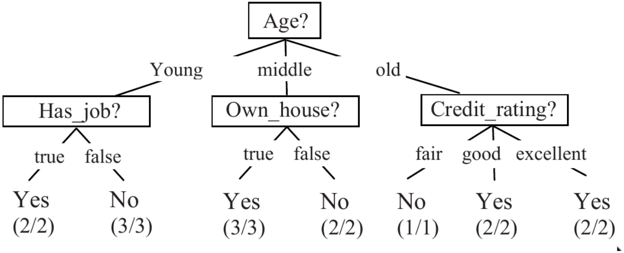

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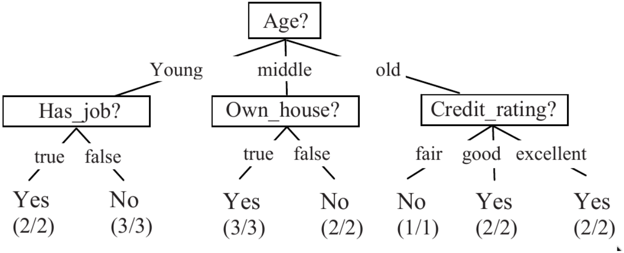

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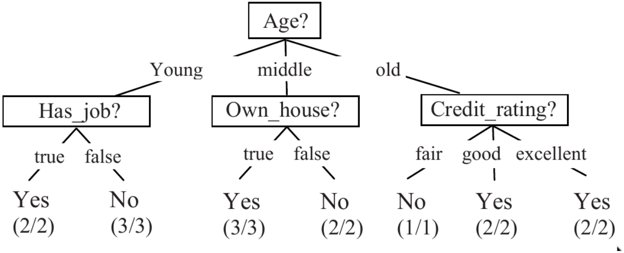

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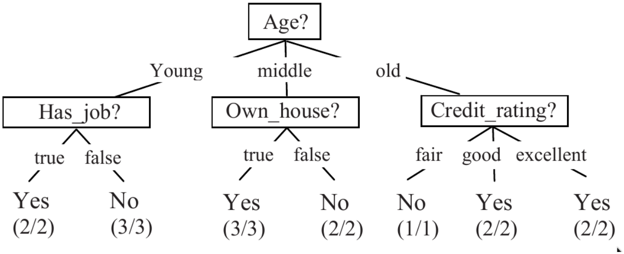

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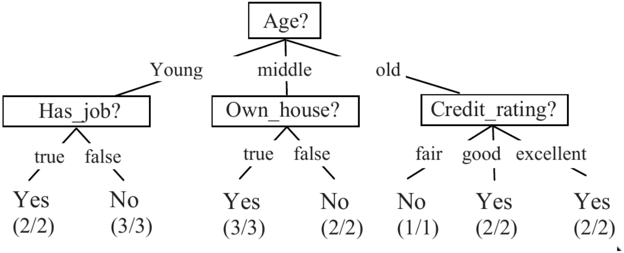
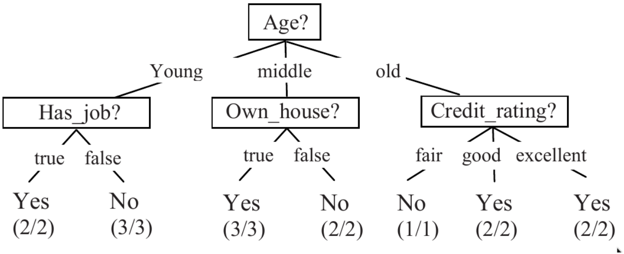
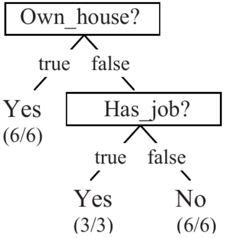


- Non-uniform node — identical combination of attributes, but different classes
- Given attributes may not capture all the criteria for classification. So two identical rows in the training data may have different labels!

Decision trees

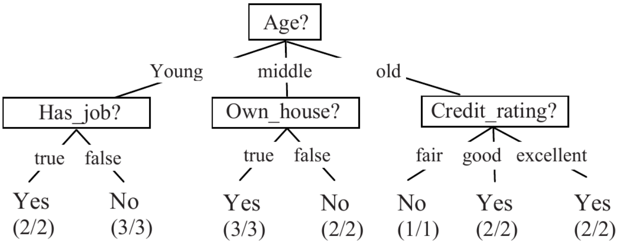
- Tree is not unique

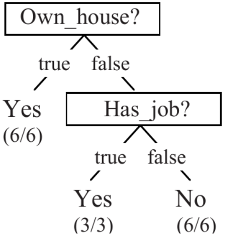




Decision trees

- Tree is not unique
- Which tree is better?





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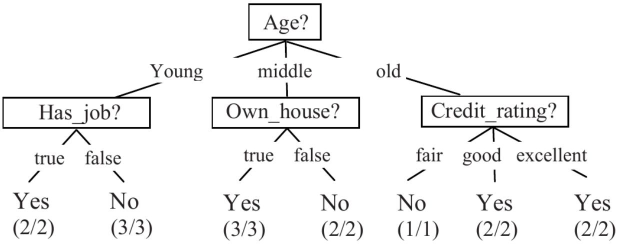

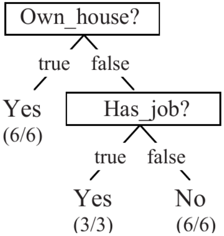

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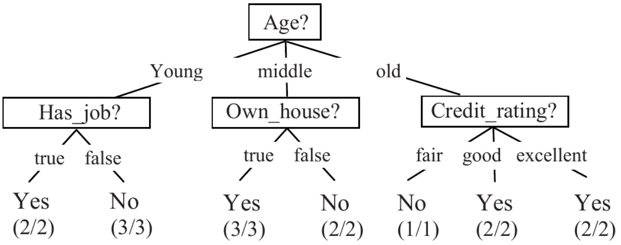

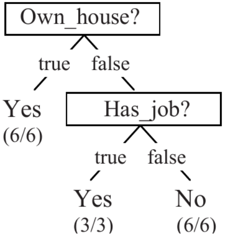

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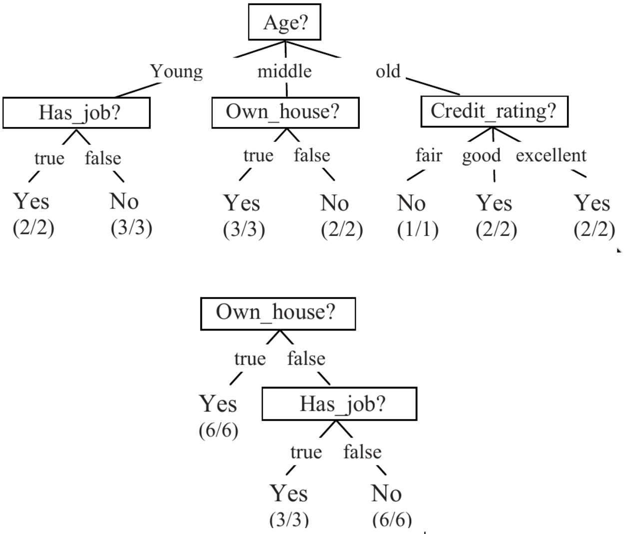

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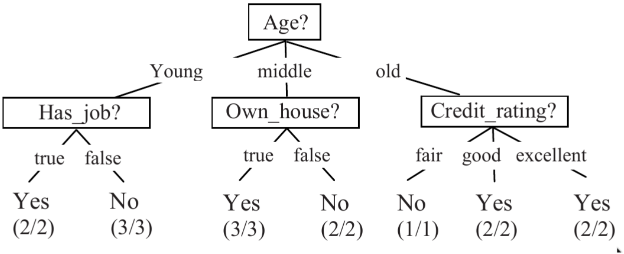

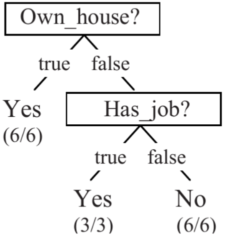

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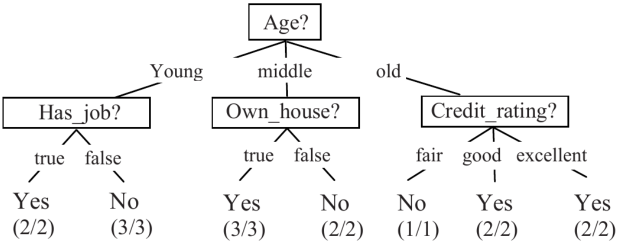
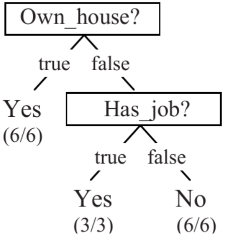
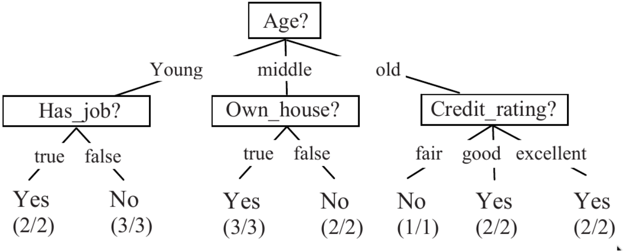
- Instead, greedy heuristic

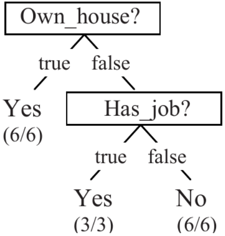




Greedy heuristic

- Goal: partition with uniform category — pure leaf





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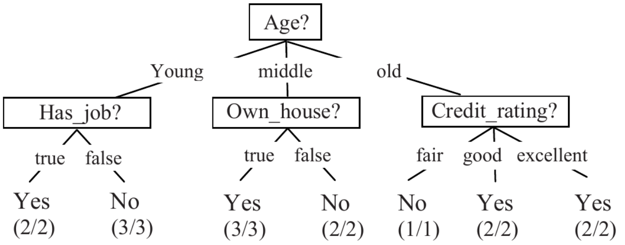

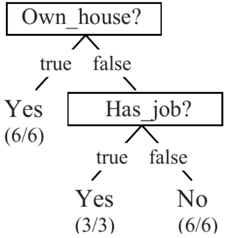

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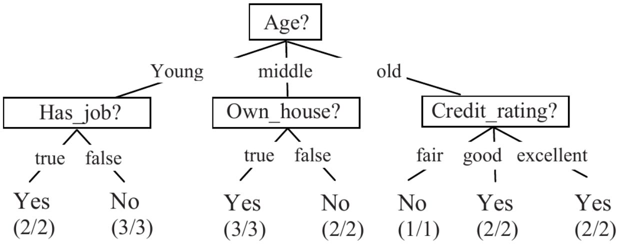

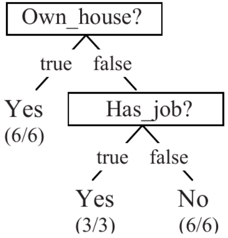

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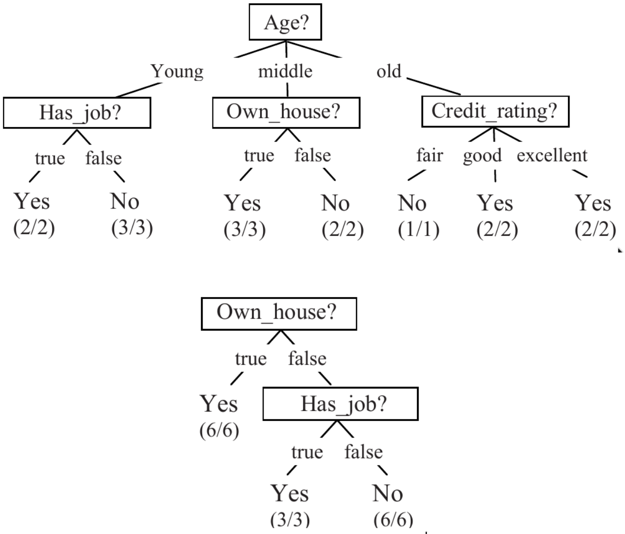

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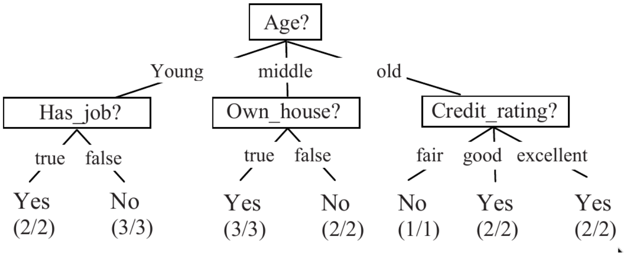

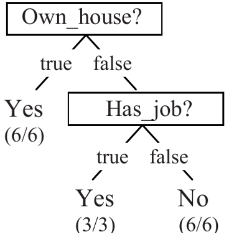

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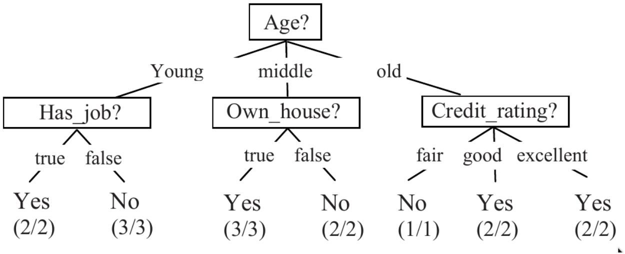
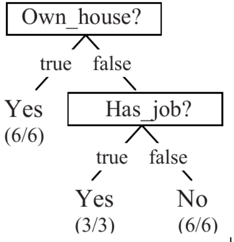
- Heuristic: Choose the attribute yielding minimum impurity

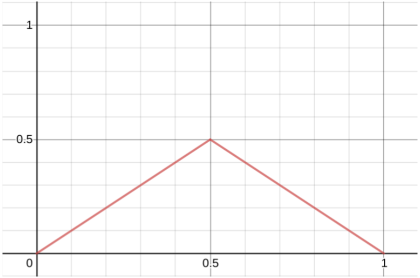




A better impurity function

What is wrong with this impurity function?

- Misclassification rate is linear



X-axis: fraction of data-rows at the node with label $c = 1$

Y-axis: impurity of the node

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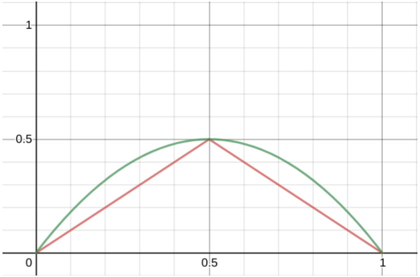
  - Intuitively, the green curve increases the urgency of moving towards a pure state.



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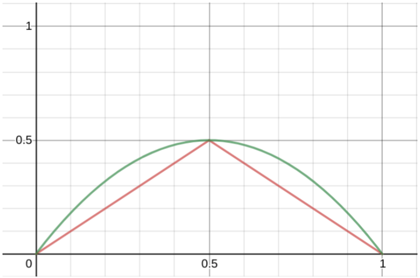

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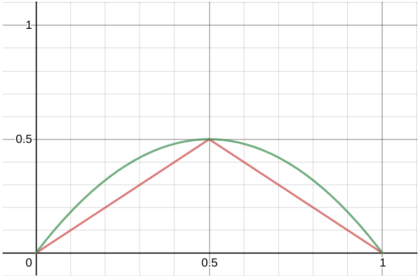
- Entropy — [Quinlan]
- Gini index — [Breiman]



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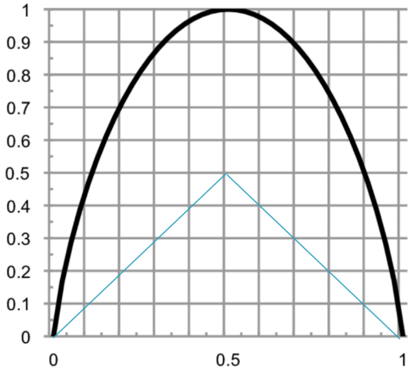
- Entropy
  $E = -(p_0 \log_2 p_0 + p_1 \log_2 p_1)$
- Minimum when $p_0 = 1, p_1 = 0$ or vice versa — note, declare $0 \log_2 0$ to be 0
- Maximum when $p_0 = p_1 = 0.5$



$c = 1$

Gini Index

- Measure of unequal distribution of wealth
- Economics — [Corrado Gini]
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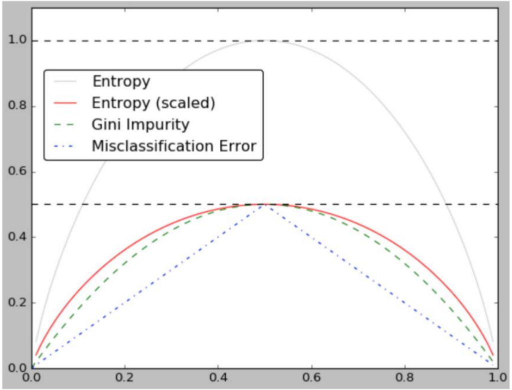
- Gini Index $G = 1 - (p_0^2 + p_1^2)$

Gini Index

- Measure of unequal distribution of wealth
- Economics — [Corrado Gini]
- As before, n data items
  - n_0 with $c = 0$, $p_0 = n_0/n$
  - n_1 with $c = 1$, $p_1 = n_1/n$
- Gini Index $G = 1 - (p_0^2 + p_1^2)$
- $G = 0$ when $p_0 = 0$, $p_1 = 0$ or v.v.
  $G = 0.5$ when $p_0 = p_1 = 0.5$



$c = 1$

Gini Index

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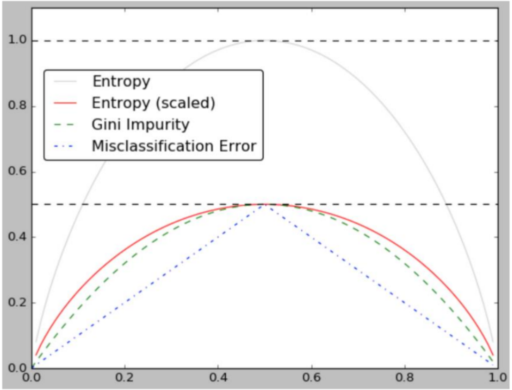
- Entropy curve is slightly steeper, but Gini index is easier to compute



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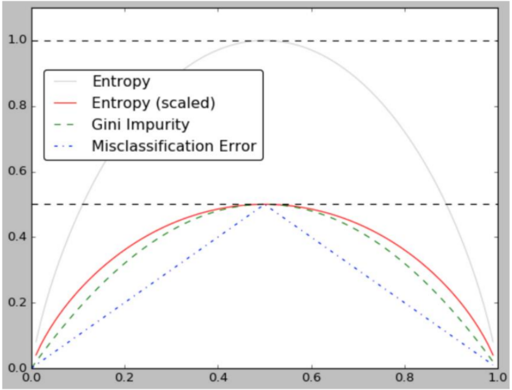
  $G = 0.5$ when $p_0 = p_1 = 0.5$
- Entropy curve is slightly steeper, but Gini index is easier to compute
- Decision tree libraries usually use Gini index



$c = 1$