Lecture 9: 7 February, 2023

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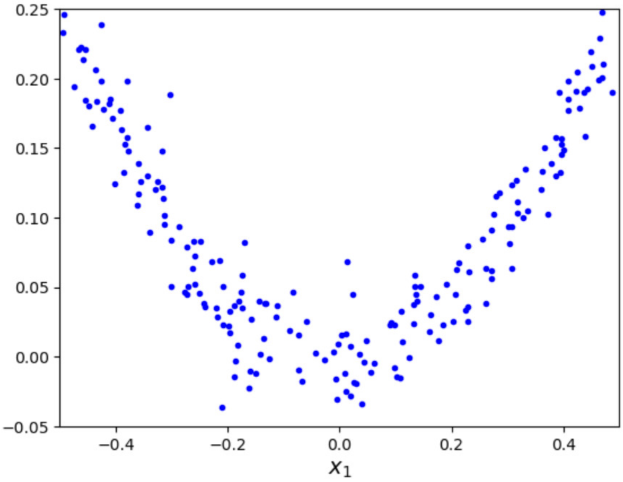
Data Mining and Machine Learning
January–April 2023

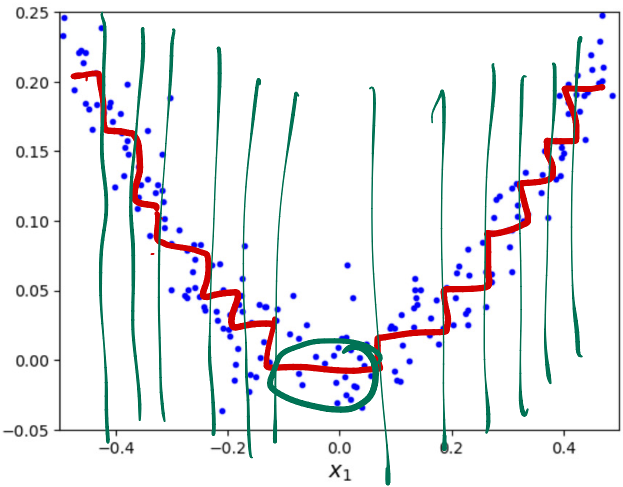
Decision trees for regression

- How do we use decision trees for regression?



Decision trees for regression

- How do we use decision trees for regression?
- Partition the input into intervals



Decision trees for regression

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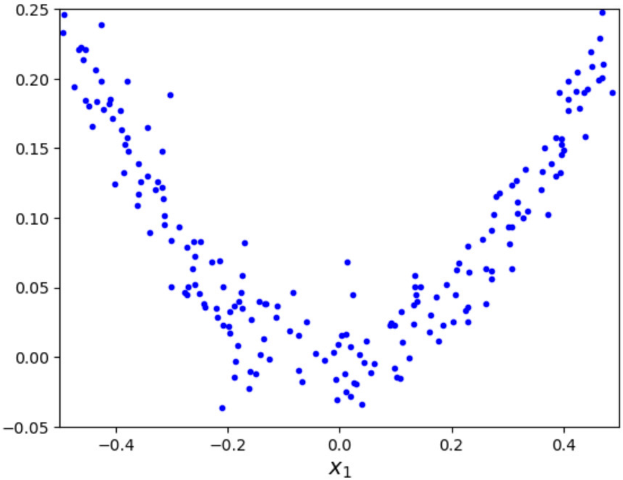
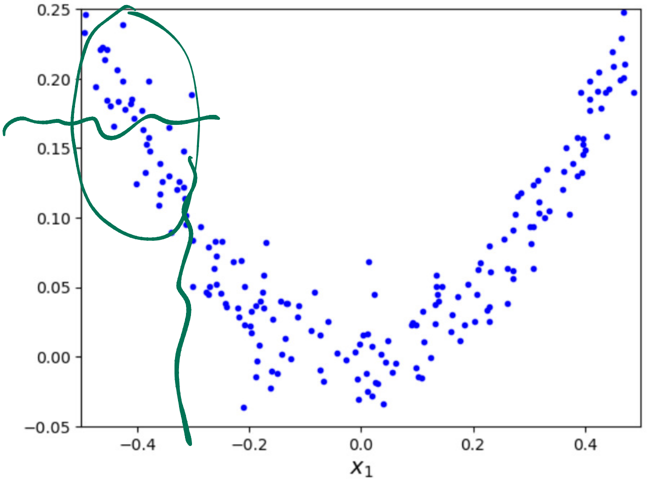
- Partition the input into intervals
- For each interval, predict mean value of output, instead of majority class

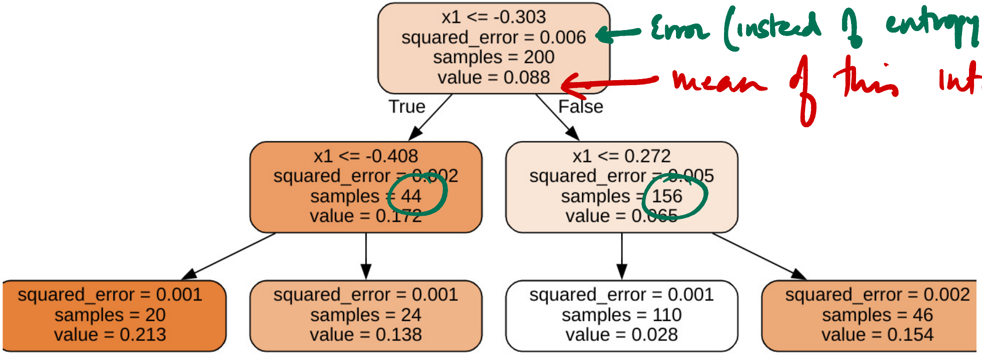


- How do we use decision trees for regression?
- Partition the input into intervals
- For each interval, predict mean value of output, instead of majority class
- Regression tree



Decision trees for regression

- Regression tree for noisy quadratic

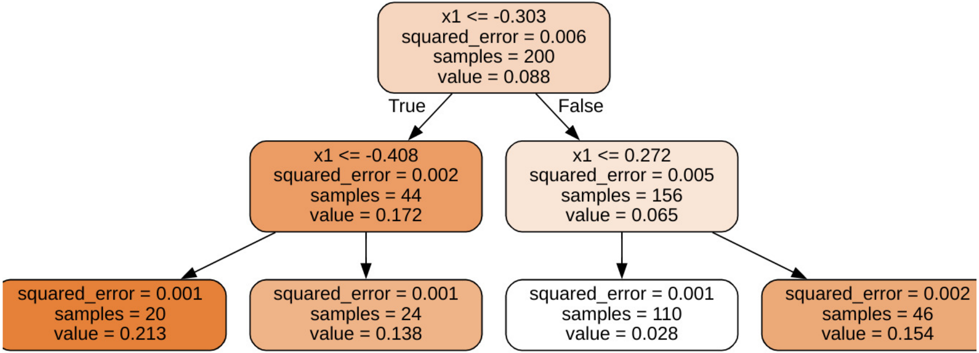


Error (instead of entropy etc)

mean of this interval

Decision trees for regression

- Regression tree for noisy quadratic
- For each node, the output is the mean y value for the current set of points



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- Regression tree for noisy quadratic
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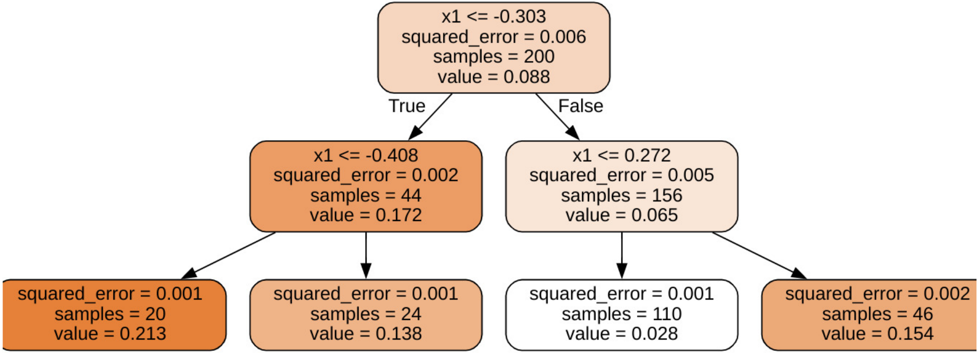
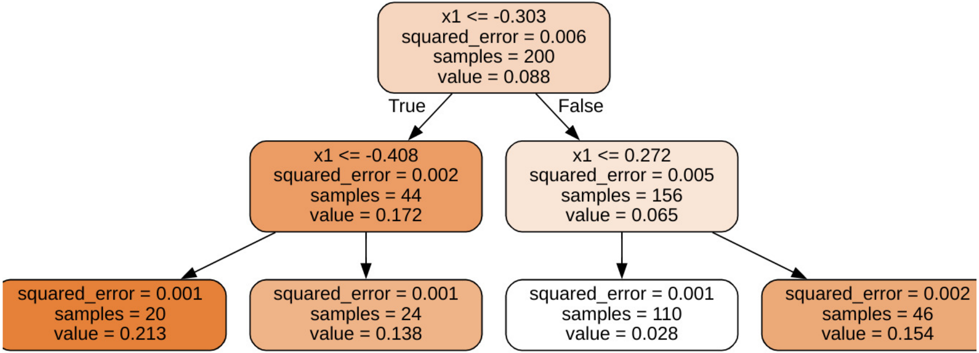
- Instead of impurity, use mean squared error (MSE) as cost function



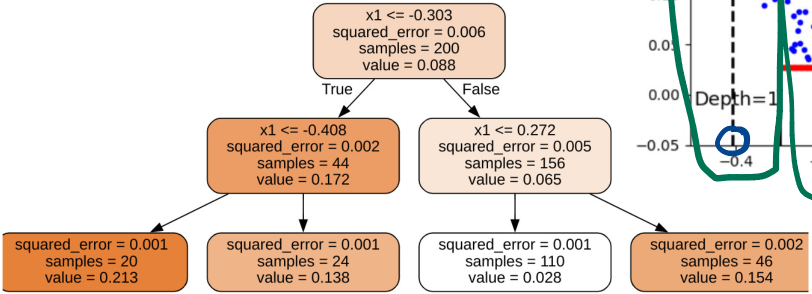
Decision trees for regression

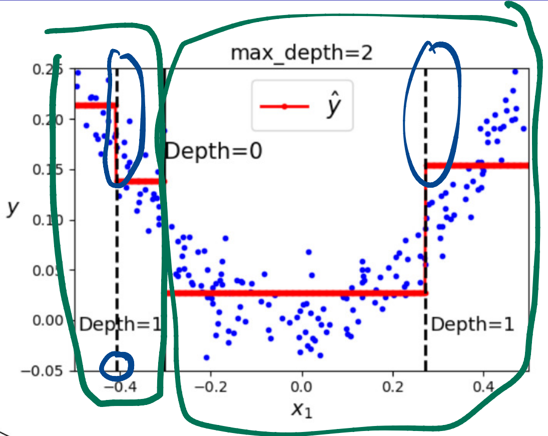
- Regression tree for noisy quadratic
- For each node, the output is the mean y value for the current set of points
- Instead of impurity, use mean squared error (MSE) as cost function
- Choose a split that minimizes MSE



Regression trees

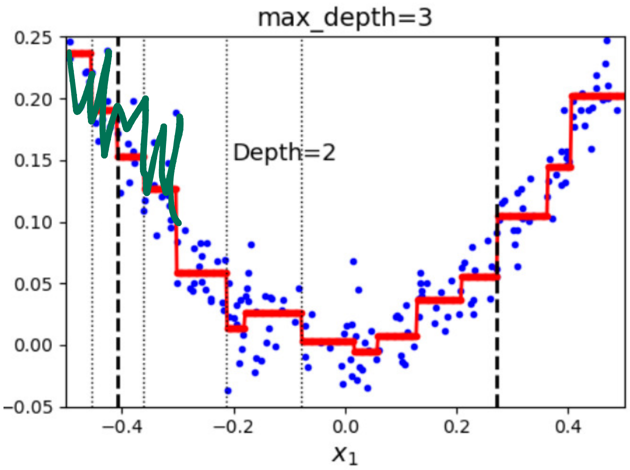
- Approximation using regression tree





Regression trees

- Extend the regression tree one more level to get a finer approximation



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- Extend the regression tree one more level to get a finer approximation
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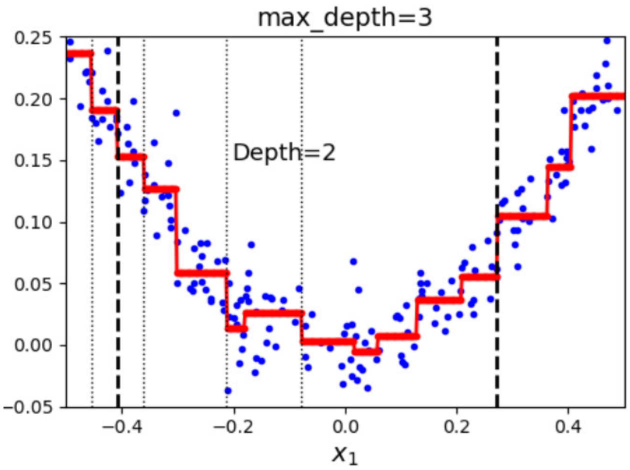

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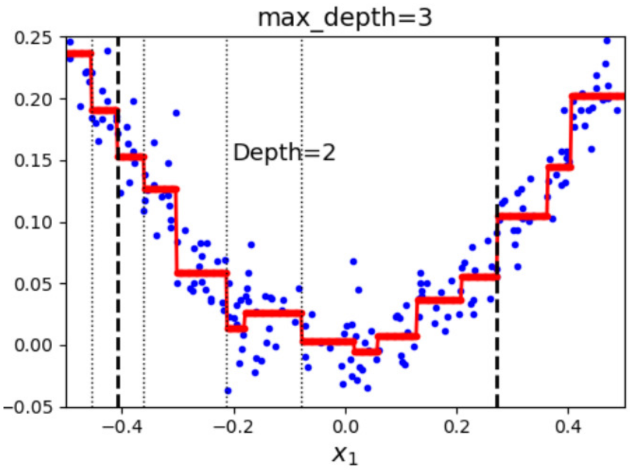
- Classification and Regression Trees (CART)

Leo Breiman



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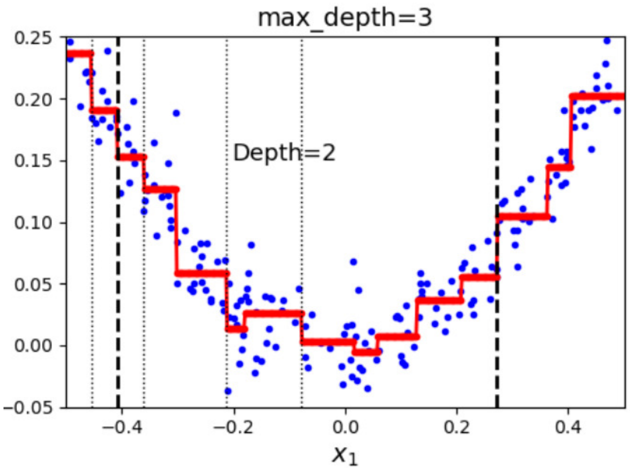
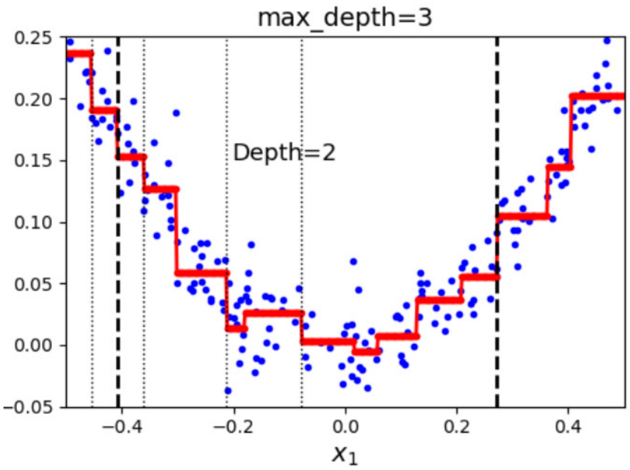


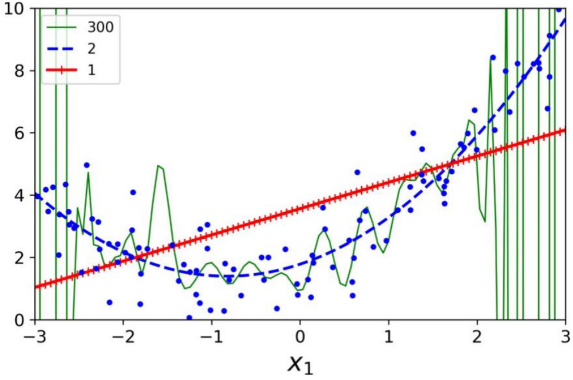
Regression trees

- Extend the regression tree one more level to get a finer approximation
- Set a threshold on MSE to decide when to stop
- Classification and Regression Trees (CART)
  - Combined algorithm for both use cases
- Programming libraries typically provide CART implementation



Overfitting

- Overfitting: model too specific to training data, does not generalize well

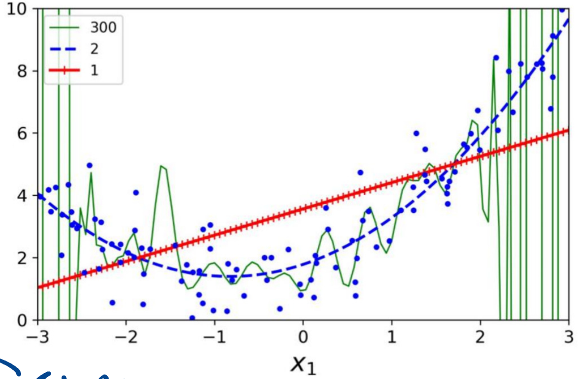


Overfitting

- Overfitting: model too specific to training data, does not generalize well
- Regression — use regularization to penalize model complexity

MSE + $\alpha \cdot$ REG

Ridge Lasso Elastic Net



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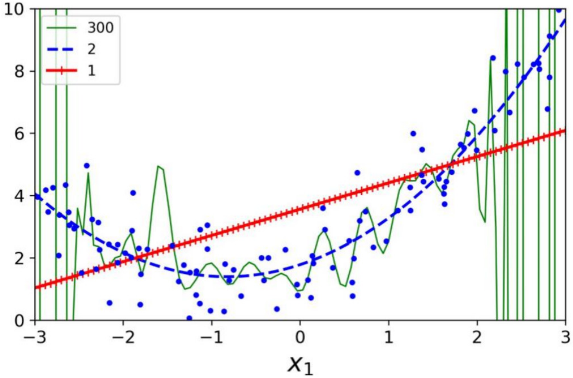

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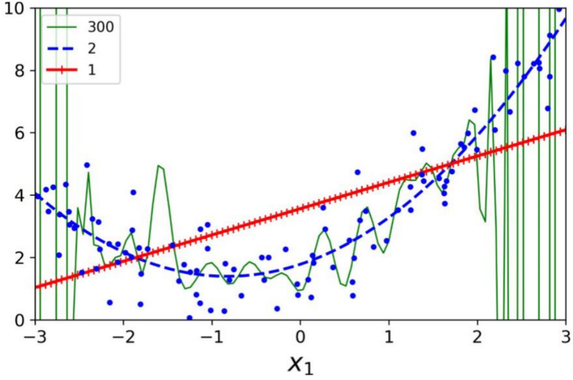

Overfitting

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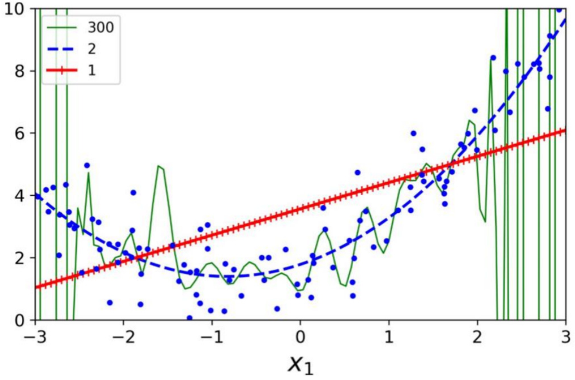
- Regression — use regularization to penalize model complexity
- What about decision trees?
- Deep, complex trees ask too many questions
- Prefer shallow, simple trees



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  - Fix a maximum depth when building the tree
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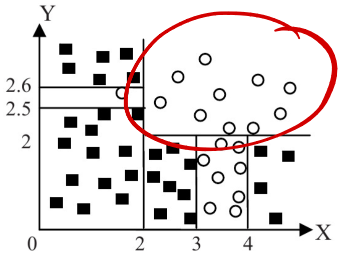
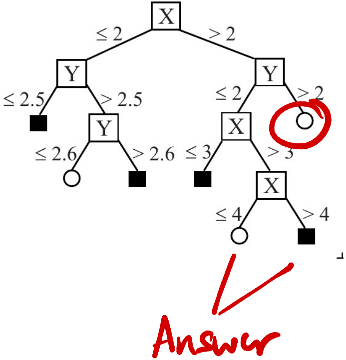
  - How to set the threshold?
- Bottom-up pruning
  - Build the full tree
  - Remove a leaf if the reduced tree generalizes better
  - How do we measure this?

Tree pruning

Overfitted tree





Tree pruning

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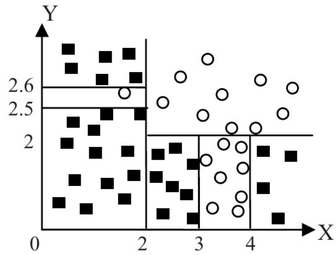
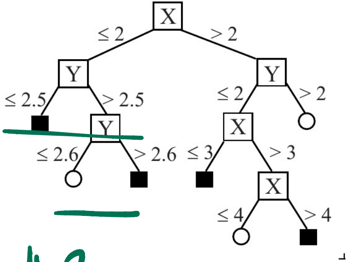
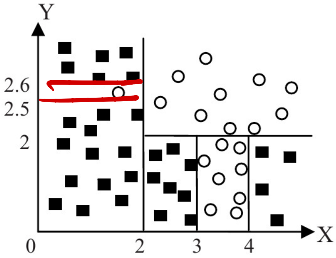
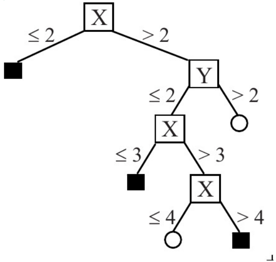




?

Pruned tree





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- Pruning leaves creates a larger impure sample one level above
- Does the confidence interval decrease (improve)?

Example: Predict party from voting pattern [Quinlan]

- Predict party affiliation of US legislators based on voting pattern
  - Read the tree from left to right

- physician fee freeze = n:
  - adoption of the budget resolution = y: democrat (151)
  - adoption of the budget resolution = u: democrat (1)
  - adoption of the budget resolution = n:
    - education spending = n: democrat (6)
    - education spending = y: democrat (9)
    - education spending = u: republican (1)
- physician fee freeze = y:
  - synfuels corporation cutback = n: republican (97/3)
  - synfuels corporation cutback = u: republican (4)
  - synfuels corporation cutback = y:
    - duty free exports = y: democrat (2)
    - duty free exports = u: republican (1)
    - duty free exports = n:
      - education spending = n: democrat (5/2)
      - education spending = y: republican (13/2)
      - education spending = u: democrat (1)
- physician fee freeze = u:
  - water project cost sharing = n: democrat (0)
  - water project cost sharing = y: democrat (4)
  - water project cost sharing = u:
    - mx missile = n: republican (0)
    - mx missile = y: democrat (3/1)
    - mx missile = u: republican (2)

Example: Predict party from voting pattern [Quinlan]

- Predict party affiliation of US legislators based on voting pattern
  - Read the tree from left to right
- After pruning, drastically simplified tree

δ

```
physician fee freeze = n: democrat (168/2.6)
physician fee freeze = y: republican (123/13.9)
physician fee freeze = u:
|     mx missile = n: democrat (3/1.1)
|     mx missile = y: democrat (4/2.2)
|     mx missile = u: republican (2/1)
```

Example: Predict party from voting pattern [Quinlan]

- Predict party affiliation of US legislators based on voting pattern
  - Read the tree from left to right
- After pruning, drastically simplified tree
- Quinlan's comment on his use of sampling theory for post-pruning

  Now, this description does violence to statistical notions of sampling and confidence limits, so the reasoning should be taken with a large grain of salt. Like many heuristics with questionable underpinnings, however, the estimates it produces seem frequently to yield acceptable results.

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