Lecture 11: 28 February, 2022

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Data Mining and Machine Learning
January–May 2022

Limitations of classification models

Recall

- Bias : Expressiveness of model limits classification
- Variance: Variation in model based on sample of training data

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Overcoming limitations

- Bagging is an effective way to overcome high variance
  - Ensemble models
    - Sequence of models based on independent bootstrap samples
    - Use voting to get an overall classifier
- How can we cope with high bias?

Dealing with bias

- A biased model always makes mistakes
    - Build an ensemble of models to average out mistakes

Dealing with bias

- A biased model always makes mistakes
    - Build an ensemble of models to average out mistakes
- Mistakes should be compensated across models in the ensemble
    - How to build a sequence of models, each biased a different way?
    - Again, we assume we have only one set of training data

Boosting

- Build a sequence of weak classifiers $M_1, M_2, \ldots, M_n$ on inputs $D_1, D_2, \ldots, D_n$
    - A weak classifier is any classifier that has error rate strictly below 50%

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  - Going from D_i to D_{i+1} : increase weights where M_i makes mistakes on D_i
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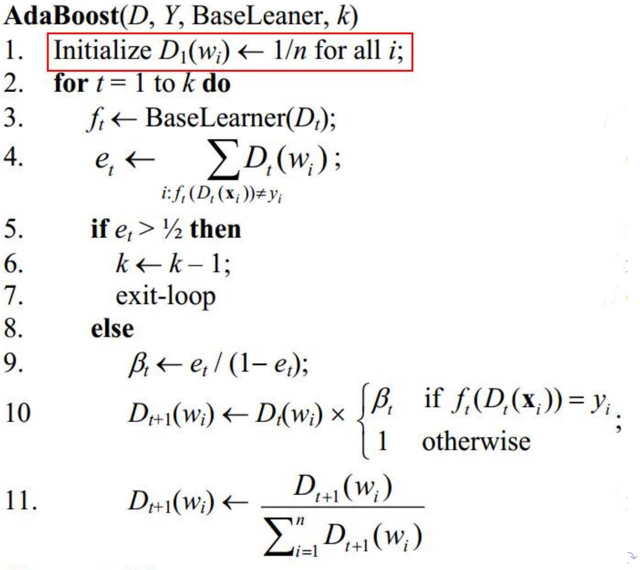
- Also, each model M_i gets a weight α_i based on its accuracy on D_i
- Ensemble output
    - Individual classification outcomes are $\{-1, +1\}$
    - Unknown input x: ensemble outcome is weighted sum $\sum_{i=1}^{n} \alpha_i M_i(x)$
    - Check if weighted sum is negative/positive

The boosting algorithm — Adaboost

- Initially, all data items have equal weight



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Initialize $D_1(w_i) \leftarrow 1/n$ for all $i$;
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    if $e_t > ½$ then
        $k \leftarrow k - 1$;
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    else
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        $D_{t+1}(w_i) \leftarrow D_t(w_i) \times \begin{cases} \beta_t & \text{if } f_t(D_t(\mathbf{x}_i)) = y_i \\ 1 & \text{otherwise} \end{cases}$;
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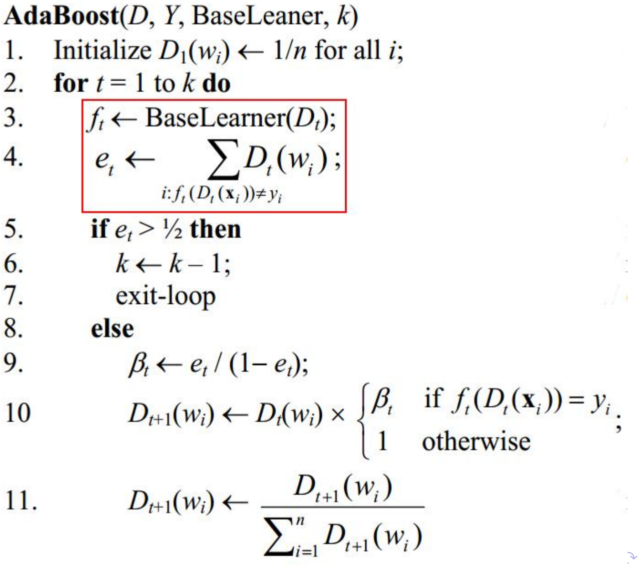

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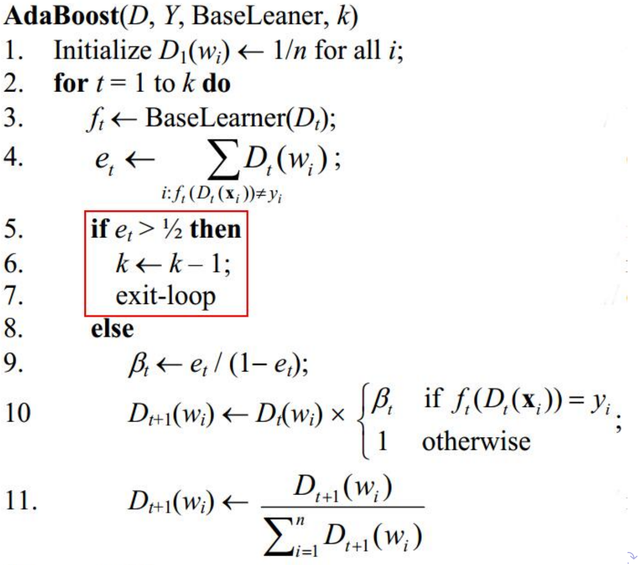

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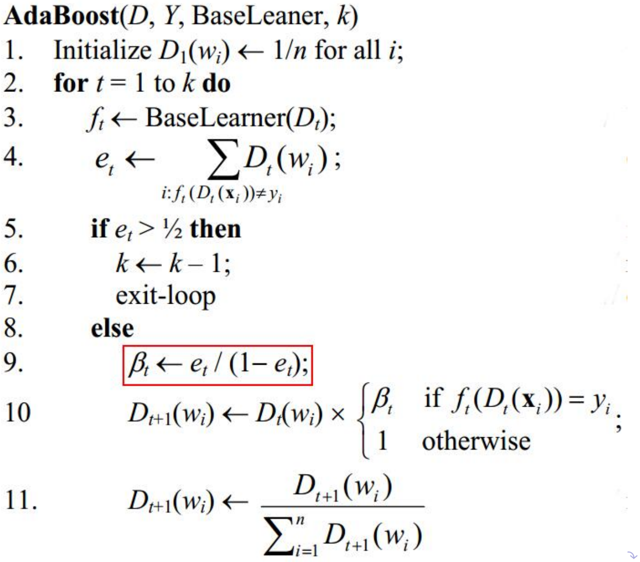

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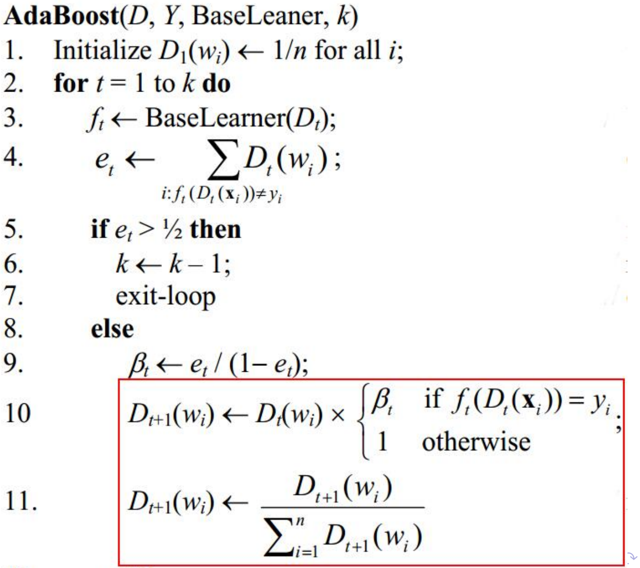
- Reweight data items and normalize



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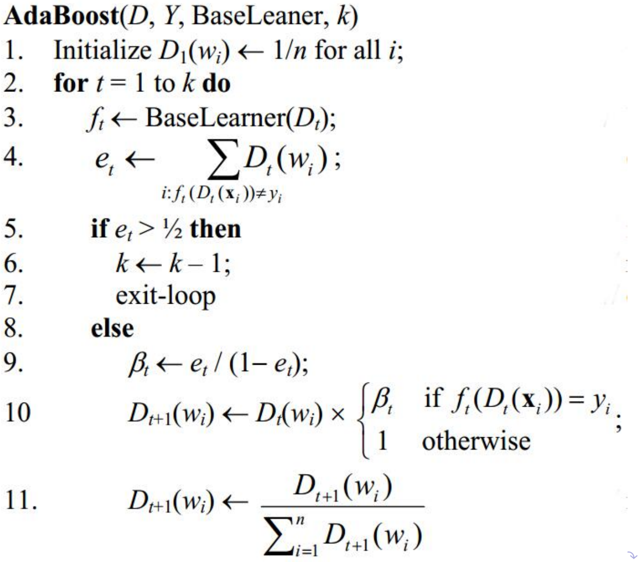
- Discard if error rate is above 50%
- Damping factor — reduce weight of correct inputs
- Reweight data items and normalize
- Final classifier

$$f_{\text{final}}(x) = \arg\max_{y \in Y} \sum_{t: f_t(x) = y} \log \frac{1}{\beta_t}$$



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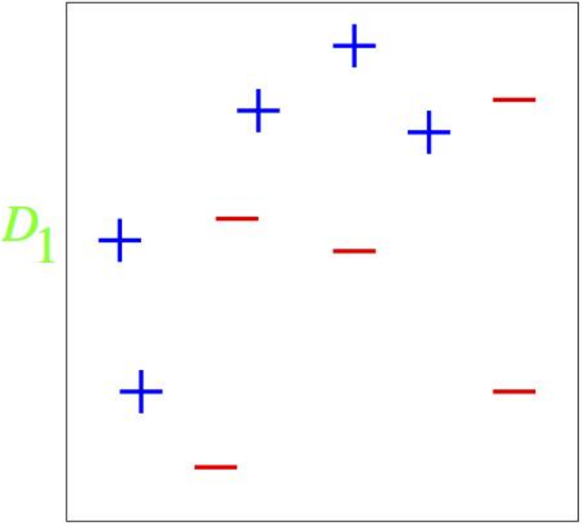
  - Reweight all training data based on error rate of M_{j+1}
- Note that same model M may be picked in multiple iterations, assigned different weights α

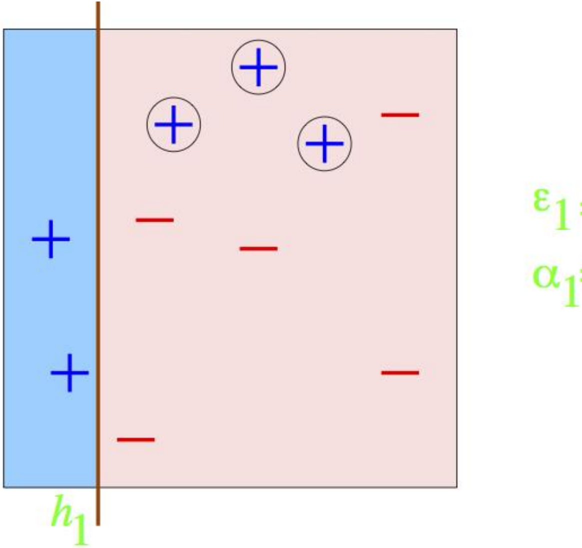
Boosting: An example

- Weak classifiers are horizontal and vertical lines
- Initial training data has equal weights



Boosting: An example

- Weak classifiers are horizontal and vertical lines
- Initial training data has equal weights
- First separator: vertical line



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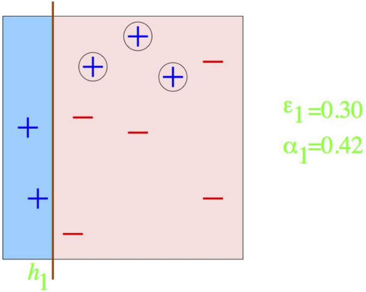

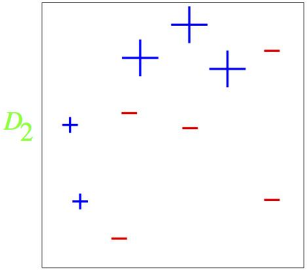

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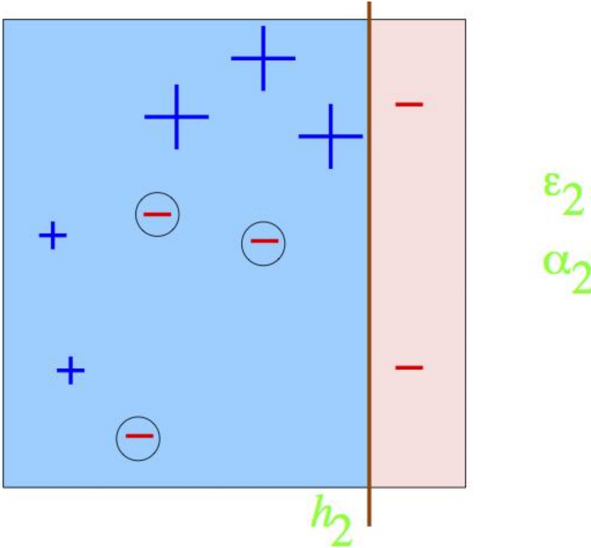

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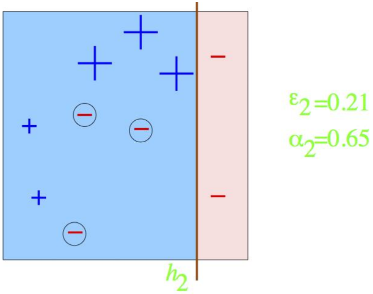

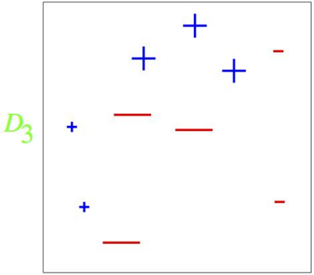

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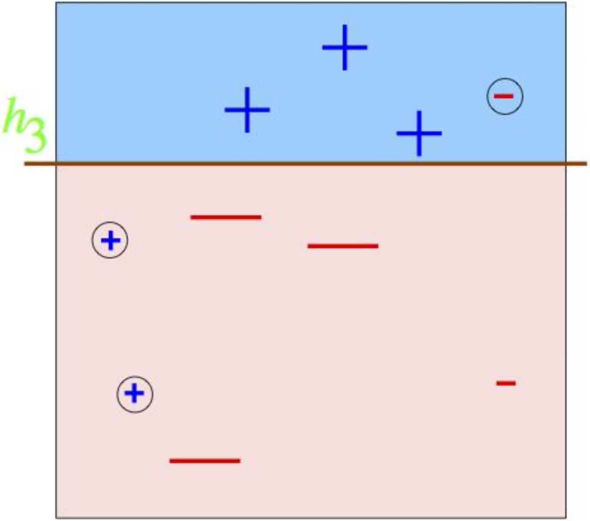
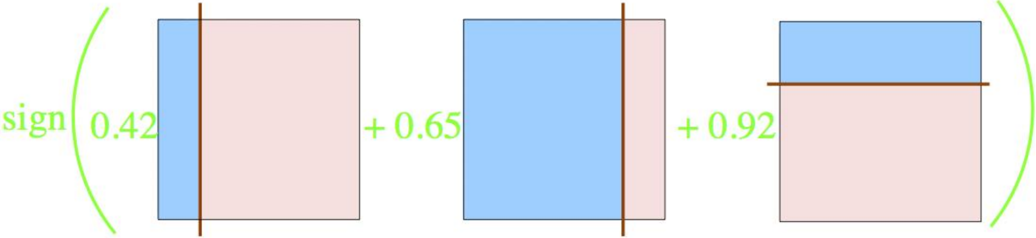
  - Increase weight of misclassified inputs
- Third separator: horizontal line



Boosting: An example

- Final classifier is weighted sum of three weak classifiers



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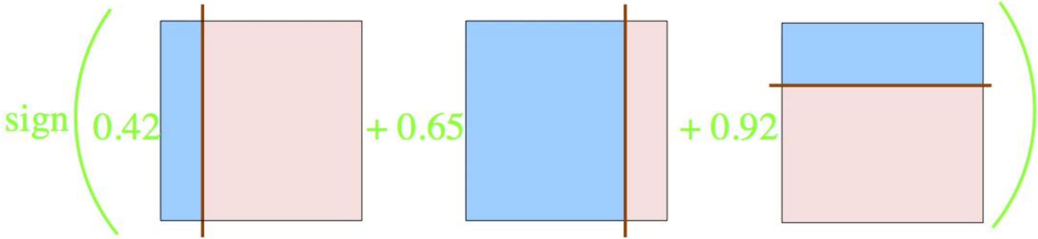
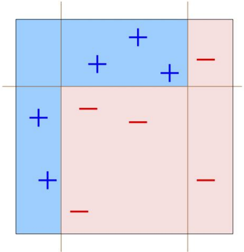


- Pictorially



Gradient Boosting

- AdaBoost uses weights to build new weak learners that compensate for earlier errors
- Gradient boosting follows a different approach
  - Shortcomings of the current model are defined in terms of gradients
  - Gradient boosting = Gradient descent + boosting

Gradient Boosting for Regression

- Training data (x_1, y_1), (x_2, y_2), $\ldots$, (x_n, y_n)
- Fit a model $F(x)$ to minimize square loss

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    - $y_1 = 0.9$, $F(x_1) = 0.8$
    - $y_2 = 1.3$, $F(x_2) = 1.4$
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- Why should this work?

Residuals and gradients

Gradient descent

- Move parameters against the gradient with respect to loss function

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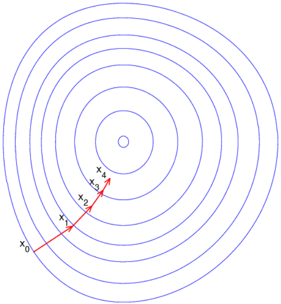

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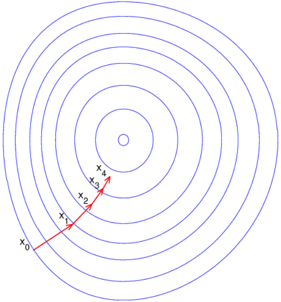

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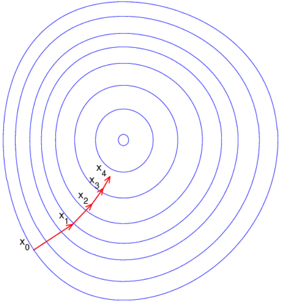

- Individual loss:
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- Minimize overall loss:
  $J = \sum_i L(y_i, F(x_i))$

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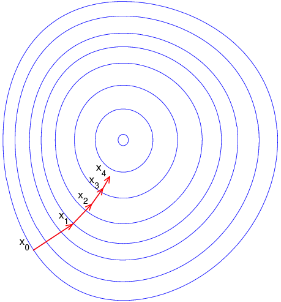

- Individual loss: $L(y, F(x) = (y - F(x))^2/2$
- Minimize overall loss: $J = \sum_i L(y_i, F(x_i))$
- $\frac{\partial J}{\partial F(x_i)} = F(x_i) - y$

Residuals and gradients

Gradient descent

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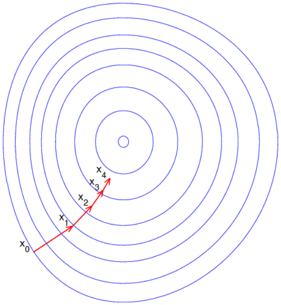

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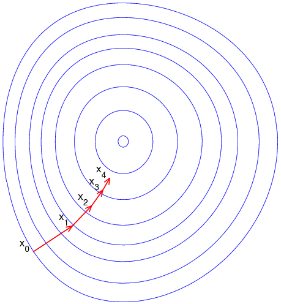

- Individual loss:
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Residuals and gradients

Gradient descent

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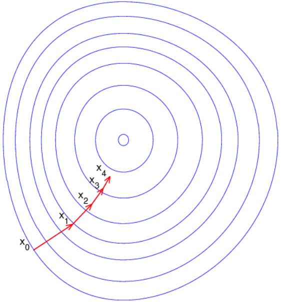

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- Minimize overall loss: $J = \sum_i L(y_i, F(x_i))$
- $\frac{\partial J}{\partial F(x_i)} = F(x_i) - y$
- Residual $y_i - F(x_i)$ is negative gradient
- Fitting h to residual is same as fitting h to negative gradient
- Updating F using residual is same as updating F based on negative gradient

Residuals and gradients

- Residuals are a special case — gradients for square loss

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  - Absolute loss $|y - f(x)|$
  - Huber loss

$$L(y, F) = \begin{cases} \frac{1}{2}(y - F)^2, & |y - F| \leq \delta \\ \delta(|y - F| - \delta/2), & |y - F| > \delta \end{cases}$$

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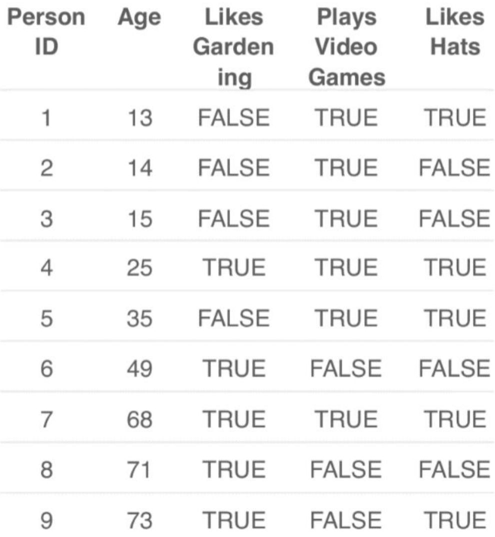
- More generally, boosting with respect to gradient rather than just residuals
- Given any differential loss function L,
  - Start with an initial model F
  - Calculate negative gradients

$$-g(x_i) = \frac{\partial L(y_i, F(x_i))}{\partial F(x_i)}$$

  - Fit a regression tree h to negative gradients $-g(x_i)$
  - Update F to $F + \rho h$
  - ρ is the learning rate

Regression Trees

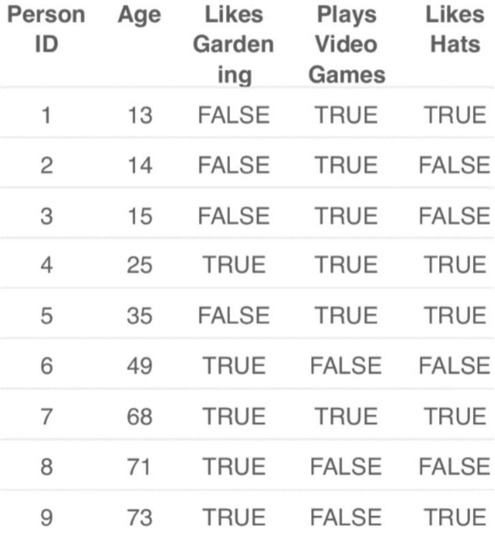
- Predict age based on given attributes



Person ID	Age	Likes Gardening	Plays Video Games	Likes Hats
1	13	FALSE	TRUE	TRUE
2	14	FALSE	TRUE	FALSE
3	15	FALSE	TRUE	FALSE
4	25	TRUE	TRUE	TRUE
5	35	FALSE	TRUE	TRUE
6	49	TRUE	FALSE	FALSE
7	68	TRUE	TRUE	TRUE
8	71	TRUE	FALSE	FALSE
9	73	TRUE	FALSE	TRUE

Regression Trees

- Predict age based on given attributes
- Build a regression tree using CART algorithm



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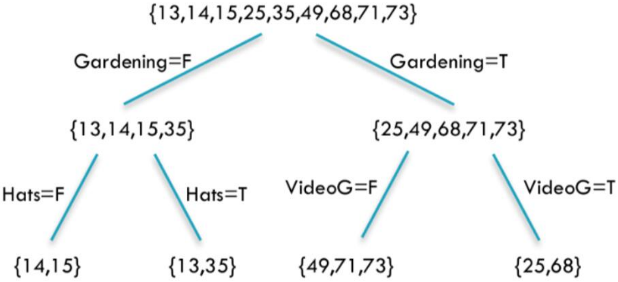
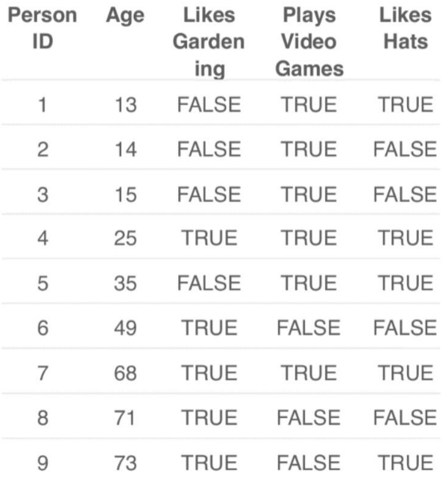
Regression Trees



- **LikesHats** seems irrelevant, yet pops up



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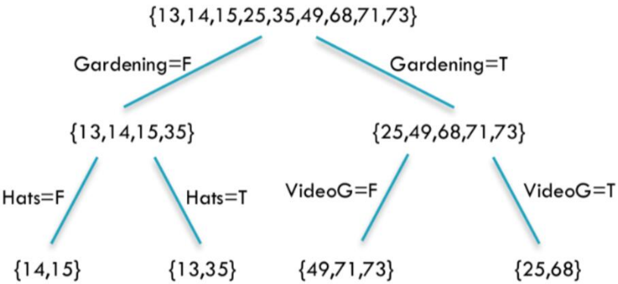
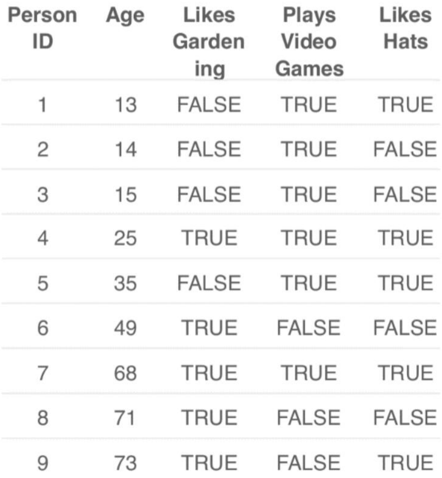
Regression Trees



- **LikesHats** seems irrelevant, yet pops up
- Can we do better?



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8	71	TRUE	FALSE	FALSE
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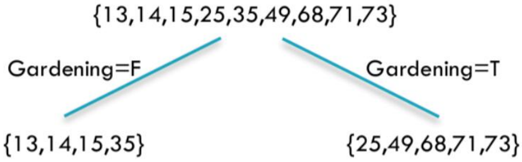
Residuals



{13,14,15,25,35,49,68,71,73}

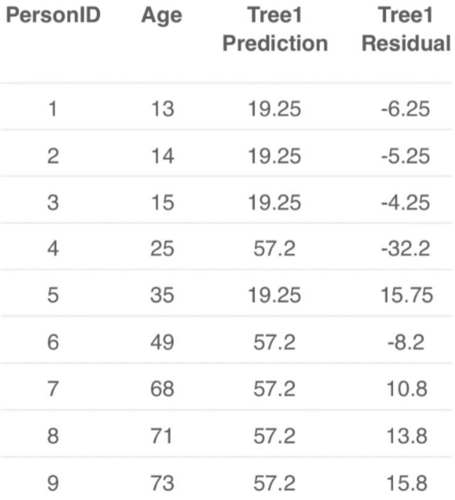
Gardening=F

Gardening=T

{13,14,15,35}

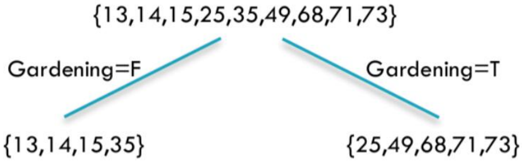
{25,49,68,71,73}

Tree 1

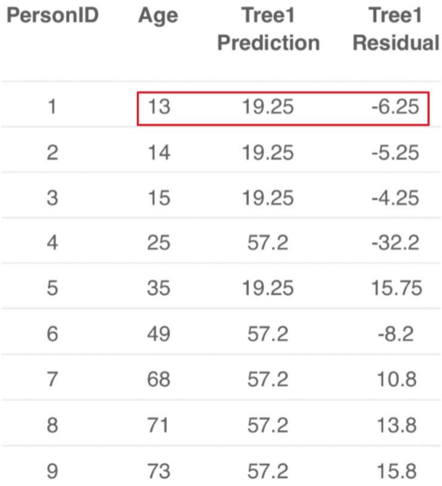


PersonID	Age	Tree1 Prediction	Tree1 Residual
1	13	19.25	-6.25
2	14	19.25	-5.25
3	15	19.25	-4.25
4	25	57.2	-32.2
5	35	19.25	15.75
6	49	57.2	-8.2
7	68	57.2	10.8
8	71	57.2	13.8
9	73	57.2	15.8

Residuals



Tree 1



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8	71	57.2	13.8
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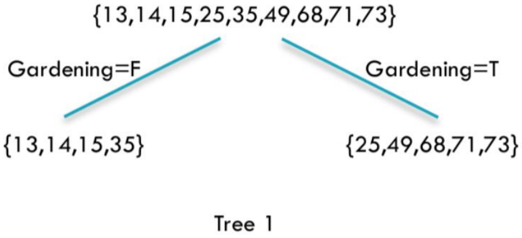
Residuals



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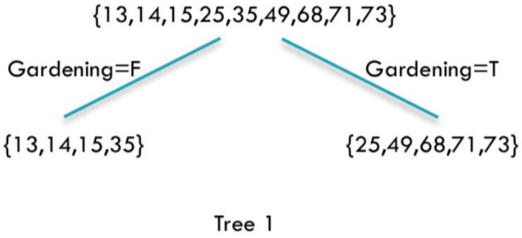
8	71	57.2	13.8
9	73	57.2	15.8

Residuals



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8	71	57.2	13.8
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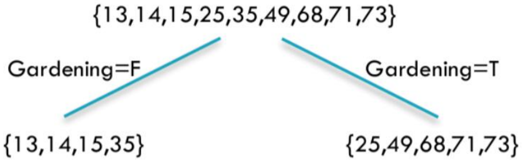
Residuals



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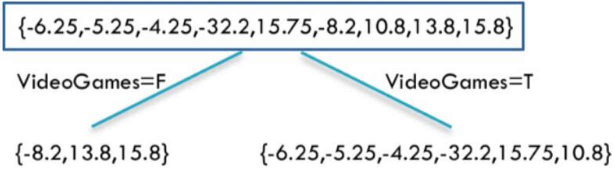

{-6.25,-5.25,-4.25,-32.2,15.75,-8.2,10.8,13.8,15.8}

VideoGames=F

VideoGames=T

{-8.2,13.8,15.8}

{-6.25,-5.25,-4.25,-32.2,15.75,10.8}

Tree 2

PersonID	Age	Tree1 Prediction	Tree1 Residual
1	13	19.25	-6.25
2	14	19.25	-5.25
3	15	19.25	-4.25
4	25	57.2	-32.2
5	35	19.25	15.75
6	49	57.2	-8.2
7	68	57.2	10.8
8	71	57.2	13.8
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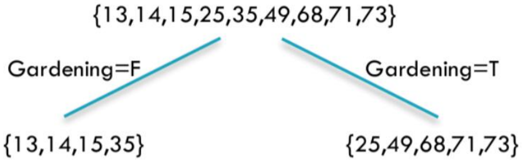
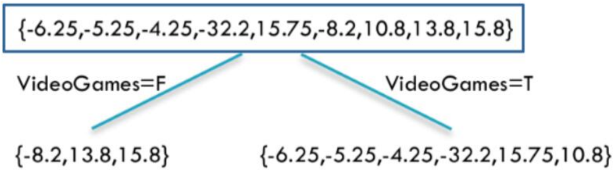
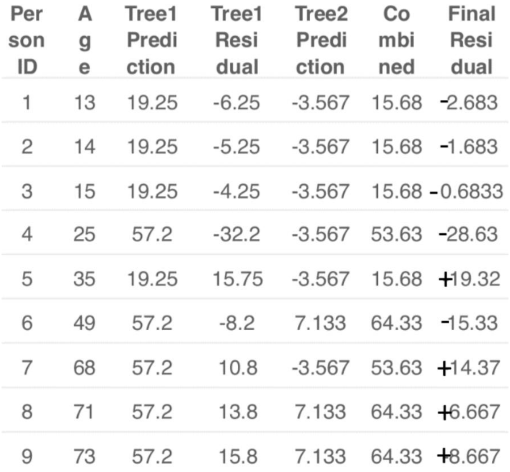
Residuals



Tree 1



Tree 2



Person ID	Age	Tree1 Prediction	Tree1 Residual	Tree2 Prediction	Combined	Final Residual
1	13	19.25	-6.25	-3.567	15.68	-2.683
2	14	19.25	-5.25	-3.567	15.68	-1.683
3	15	19.25	-4.25	-3.567	15.68	-0.6833
4	25	57.2	-32.2	-3.567	53.63	-28.63
5	35	19.25	15.75	-3.567	15.68	+19.32
6	49	57.2	-8.2	7.133	64.33	-15.33
7	68	57.2	10.8	-3.567	53.63	+14.37
8	71	57.2	13.8	7.133	64.33	+6.667
9	73	57.2	15.8	7.133	64.33	+8.667

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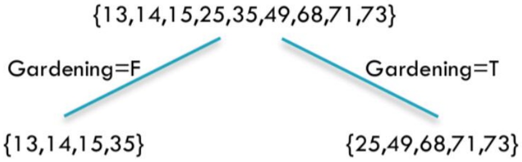

{13,14,15,25,35,49,68,71,73}

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Gardening=T

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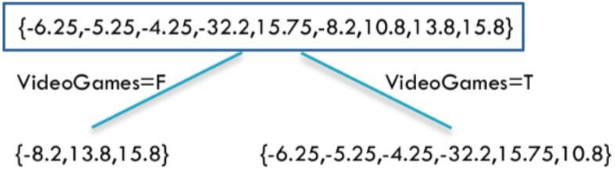

{-6.25,-5.25,-4.25,-32.2,15.75,-8.2,10.8,13.8,15.8}

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{-8.2,13.8,15.8}

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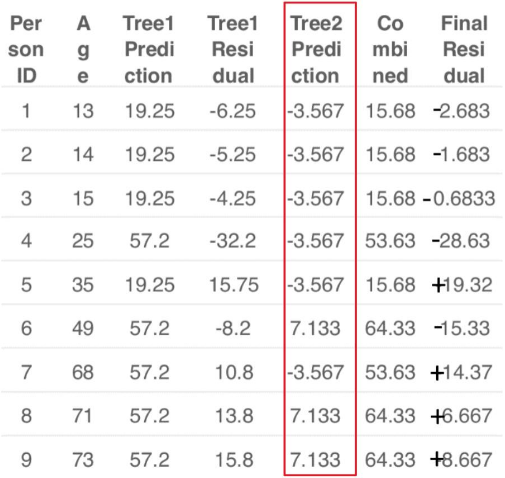
Tree 2



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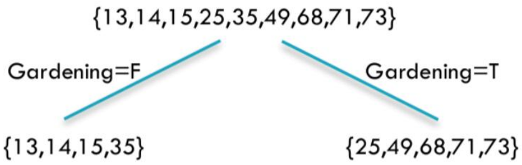
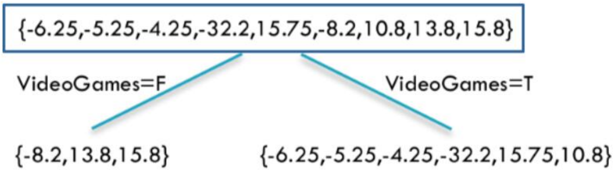
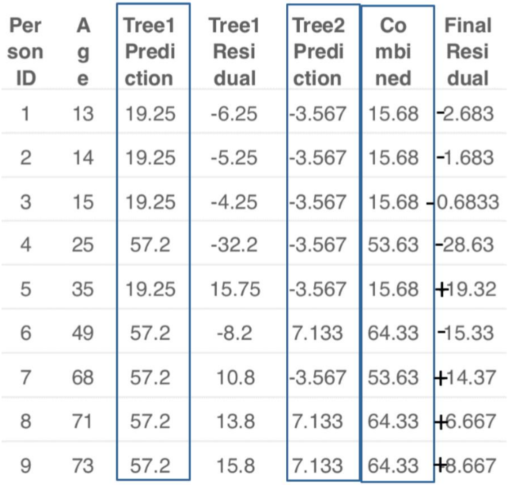


Tree 1



Tree 2



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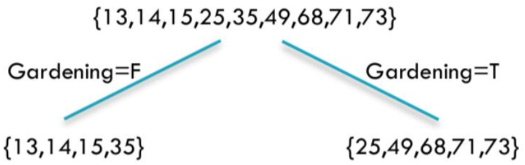
Residuals



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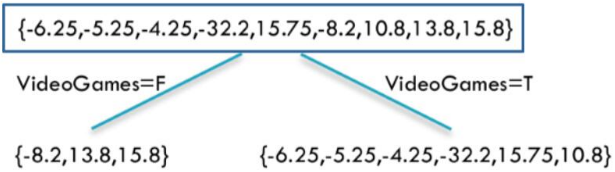
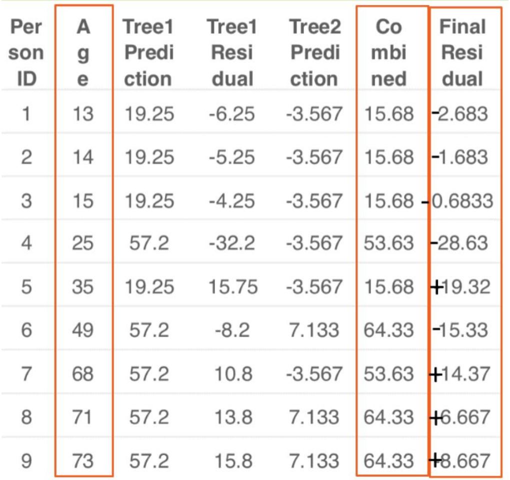


Tree 2



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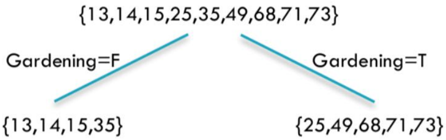
Gradient Boosting

General Strategy



{13,14,15,25,35,49,68,71,73}

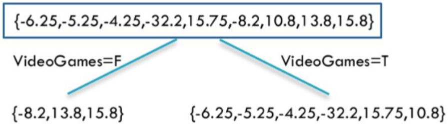
Gardening=F

Gardening=T

{13,14,15,35}

{25,49,68,71,73}

Tree 1



{-6.25,-5.25,-4.25,-32.2,15.75,-8.2,10.8,13.8,15.8}

VideoGames=F

VideoGames=T

{-8.2,13.8,15.8}

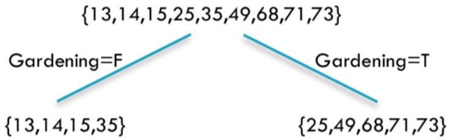
{-6.25,-5.25,-4.25,-32.2,15.75,10.8}

Tree 2

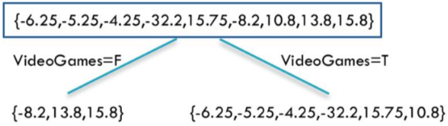
Gradient Boosting

General Strategy

- Build tree 1, F_1



Tree 1

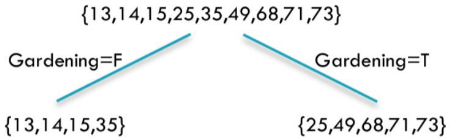


Tree 2

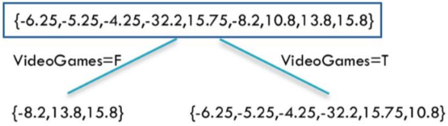
Gradient Boosting

General Strategy

- Build tree 1, F_1
- Fit a model to residuals, $h_1(x) = y - F_1(x)$



Tree 1

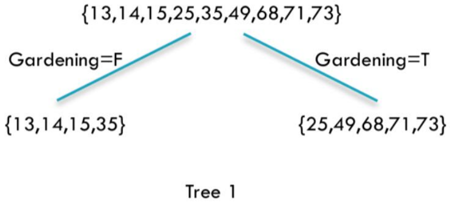


Tree 2

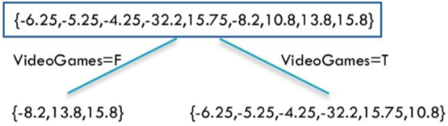
Gradient Boosting

General Strategy

- Build tree 1, F_1
- Fit a model to residuals, $h_1(x) = y - F_1(x)$
- Create a new model $F_2(x) = F_1(x) + h_1(x)$



Tree 1

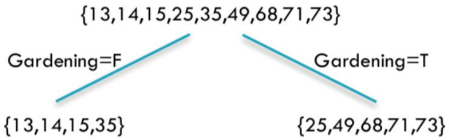


Tree 2

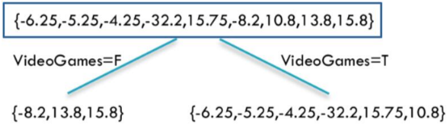
Gradient Boosting

General Strategy

- Build tree 1, F_1
- Fit a model to residuals, $h_1(x) = y - F_1(x)$
- Create a new model $F_2(x) = F_1(x) + h_1(x)$
- Fit a model to residuals, $h_2(x) = y - F_2(x)$



Tree 1

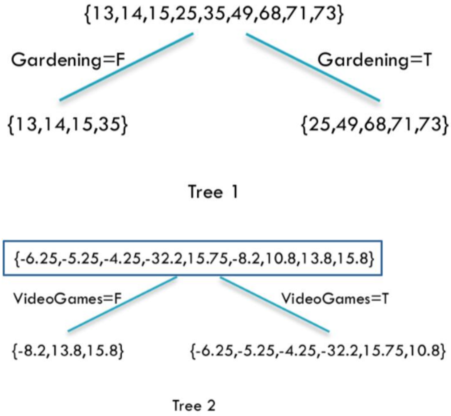


Tree 2

Gradient Boosting

General Strategy

- Build tree 1, F_1
- Fit a model to residuals, $h_1(x) = y - F_1(x)$
- Create a new model $F_2(x) = F_1(x) + h_1(x)$
- Fit a model to residuals, $h_2(x) = y - F_2(x)$
- Create a new model $F_3(x) = F_2(x) + h_2(x)$
- . . .



{13,14,15,25,35,49,68,71,73}

Gardening=F

Gardening=T

{13,14,15,35}

{25,49,68,71,73}

Tree 1

{-6.25,-5.25,-4.25,-32.2,15.75,-8.2,10.8,13.8,15.8}

VideoGames=F

VideoGames=T

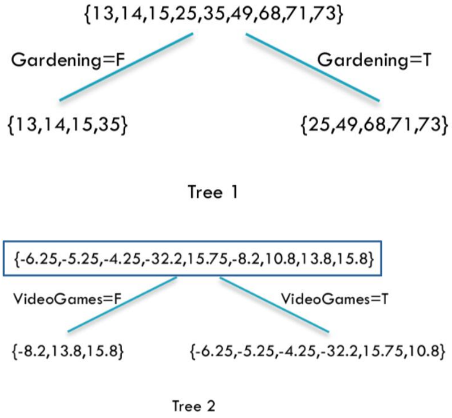
{-8.2,13.8,15.8}

{-6.25,-5.25,-4.25,-32.2,15.75,10.8}

Tree 2

Hyper Parameters

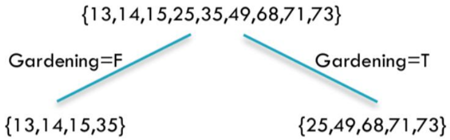
Learning Rate



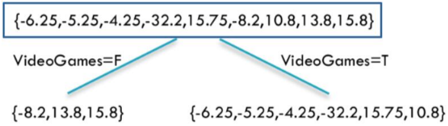
Hyper Parameters

Learning Rate

- h_j fits residuals of F_j



Tree 1

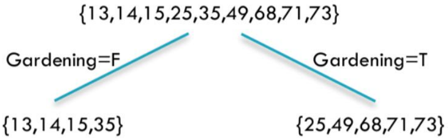


Tree 2

Hyper Parameters

Learning Rate

- h_j fits residuals of F_j
- $F_{j+1}(x) = F_J(x) + LR \cdot h_j(x)$
  - LR controls contribution of residual
  - $LR = 1$ in our previous example



{13,14,15,25,35,49,68,71,73}

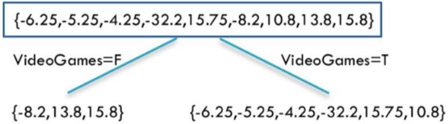
Gardening=F

Gardening=T

{13,14,15,35}

{25,49,68,71,73}

Tree 1



{-6.25,-5.25,-4.25,-32.2,15.75,-8.2,10.8,13.8,15.8}

VideoGames=F

VideoGames=T

{-8.2,13.8,15.8}

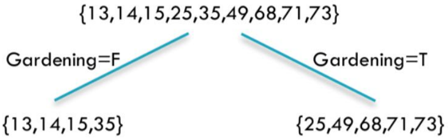
{-6.25,-5.25,-4.25,-32.2,15.75,10.8}

Tree 2

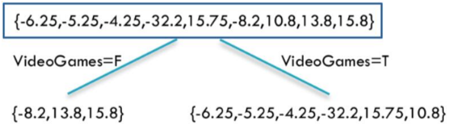
Hyper Parameters

Learning Rate

- h_j fits residuals of F_j
- $F_{j+1}(x) = F_J(x) + LR \cdot h_j(x)$
    - LR controls contribution of residual
    - $LR = 1$ in our previous example
- Ideally, choose LR separately for each residual to minimize loss function
    - Can apply different LR to different leaves



Tree 1



Tree 2