Lecture 9: 21 February, 2022

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Data Mining and Machine Learning
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Bayesian classifiers

- As before
  - Attributes $\{A_1, A_2, \ldots, A_k\}$ and
  - Classes $C = \{c_1, c_2, \ldots c_\ell\}$

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  - $Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i)$

Unknown item $(a_1, .., a_k) \rightarrow$?

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- Given a data item $d = (a_1, a_2, \ldots, a_k)$, identify the best class c for d
- Maximize $Pr(C = c_i \mid A_1 = a_1, \ldots, A_k = a_k)$

Generative models

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  - Choose a class c_j with probability $Pr(c_j)$
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$\Sigma\, Pr(c_i) = 1$

age, house, job { 9/15 – Y, 6/15 – N

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- Generative model has associated parameters $\theta = (\theta_1, \ldots, \theta_m)$
  - Each class probability $Pr(c_j)$ is a parameter
  - Each conditional probability $Pr(a_1, \ldots, a_k \mid c_j)$ is a parameter

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- We need to estimate these parameters

Maximum Likelihood Estimators

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  - N coin tosses, H heads and T tails
  - Why is $\hat{\theta} = H/N$ the best estimate?

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$Pr(T) = 1 - Pr(H)$

- Likelihood
  - Actual coin toss sequence is $\tau = t_1 t_2 \ldots t_N$
  - Given an estimate of θ, compute $Pr(\tau \mid \theta)$ — likelihood $L(\theta)$

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- Likelihood
  - Actual coin toss sequence is $\tau = t_1 t_2 \ldots t_N$
  - Given an estimate of θ, compute $Pr(\tau \mid \theta)$ — likelihood $L(\theta)$
- $\hat{\theta} = H/N$ maximizes this likelihood — $\arg\max_{\theta} L(\theta) = \hat{\theta} = H/N$
  - Maximum Likelihood Estimator (MLE)

Bayesian classification

- Maximize $Pr(C = c_i \mid A_1 = a_1, \ldots, A_k = a_k)$

observation

possible classification

Bayesian classification

- Maximize $Pr(C = c_i \mid A_1 = a_1, \ldots, A_k = a_k)$

- By Bayes' rule,

$$Pr(C = c_i \mid A_1 = a_1, \ldots, A_k = a_k)$$

$$= \frac{Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i) \cdot Pr(C = c_i)}{Pr(A_1 = a_1, \ldots, A_k = a_k)}$$

Generating model

$Pr(c_j)$

$Pr(a_1 \ldots a_n \mid c_j)$

$P(C|A)$

$= \frac{P(A|C) \cdot P(C)}{P(A)}$

Bayesian classification

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Bayesian classification

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Actual quantity is not relevant

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- Denominator is the same for all c_i, so sufficient to maximize

$$Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i) \cdot Pr(C = c_i)$$

Example

- To classify $A = g, B = q$

A	B	C
m	b	t
m	s	t
g	q	t
h	s	t
g	q	t
g	q	f
g	s	f
h	b	f
h	q	f
m	b	f

Example

- To classify $A = g, B = q$

- $Pr(C = t) = 5/10 = 1/2$
- $Pr(A = g, B = q \mid C = t) = 2/5$

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$\frac{2}{5} \cdot \frac{1}{2}$

A	B	C
m	b	t
m	s	t
g	q	t
h	s	t
g	q	t
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- $Pr(C = f) = 5/10 = 1/2$
- $Pr(A = g, B = q \mid C = f) = 1/5$

A	B	C
m	b	t
m	s	t
g	q	t
h	s	t
g	q	t
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g	s	f
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1/5

5/10

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- $Pr(A = g, B = q \mid C = t) \cdot Pr(C = t) = 1/5$
- $Pr(C = f) = 5/10 = 1/2$
- $Pr(A = g, B = q \mid C = f) = 1/5$
- $Pr(A = g, B = q \mid C = f) \cdot Pr(C = f) = 1/10$

$\frac{1}{5} \cdot \frac{1}{2}$

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- $Pr(A = g, B = q \mid C = t) = 2/5$
- $Pr(A = g, B = q \mid C = t) \cdot Pr(C = t) = 1/5$

- $Pr(C = f) = 5/10 = 1/2$
- $Pr(A = g, B = q \mid C = f) = 1/5$
- $Pr(A = g, B = q \mid C = f) \cdot Pr(C = f) = 1/10$

- Hence, predict $C = t$

A	B	C
m	b	t
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g	q	t
h	s	t
g	q	t
g	q	f
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h	q	f
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Example . . .

- What if we want to classify $A = m, B = q$?

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Example . . .

- What if we want to classify $A = m, B = q$?
- $Pr(A = m, B = q \mid C = t) = 0$

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Example . . .

- What if we want to classify $A = m, B = q$?
- $Pr(A = m, B = q \mid C = t) = 0$
- Also $Pr(A = m, B = q \mid C = f) = 0$!

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- What if we want to classify $A = m, B = q$?
- $Pr(A = m, B = q \mid C = t) = 0$
- Also $Pr(A = m, B = q \mid C = f) = 0$!
- To estimate joint probabilities across all combinations of attributes, we need a much larger set of training data

A	B	C
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m	s	t
g	q	t
h	s	t
g	q	t
g	q	f
g	s	f
h	b	f
h	q	f
m	b	f

$$\frac{Pr(A|C) \cdot Pr(C)}{Pr(A)}$$

$$0 \Leftarrow Pr(A)$$

$$Pr(A|t)\,Pr(t) + Pr(A|f)\,Pr(f)$$

Naïve Bayes classifier

- Strong simplifying assumption: attributes are pairwise independent

$$Pr(A_1 = a_1, \ldots, A_k = a_k \mid C = c_i) = \prod_{j=1}^{k} Pr(A_j = a_j \mid C = c_i)$$

  - $Pr(C = c_i)$ is fraction of training data with class c_i
  - $Pr(A_j = a_j \mid C = c_i)$ is fraction of training data labelled c_i for which $A_j = a_j$

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- Final classification is

$$\arg\max_{c_i} \; Pr(C = c_i) \prod_{j=1}^{k} Pr(A_j = a_j \mid C = c_i)$$

$Pr(A_1 = a_1, \ldots A_k = a_k) \cdot P(C = c_i)$

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- For instance, text classification
  - Items are documents, attributes are words (absent or present)
  - Classes are topics
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  - Meaning of words is clearly affected by relative position, ordering

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  - Meaning of words is clearly affected by relative position, ordering
- However, naive Bayes classifiers work well in practice, even for text classification!
  - Many spam filters are built using this model

Example revisited

- Want to classify $A = m, B = q$
- $Pr(A = m, B = q \mid C = t) = Pr(A = m, B = q \mid C = f) = 0$

A	B	C
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- $Pr(A = m, B = q \mid C = t) = Pr(A = m, B = q \mid C = f) = 0$
- $Pr(A = m \mid C = t) = 2/5$
- $Pr(B = q \mid C = t) = 2/5$

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- $Pr(A = m \mid C = t) \cdot Pr(B = q \mid C = t) \cdot Pr(C = t) = 2/25$

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1/5 0 4/5 1/2

P(m,q)

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- Hence predict $C = t$

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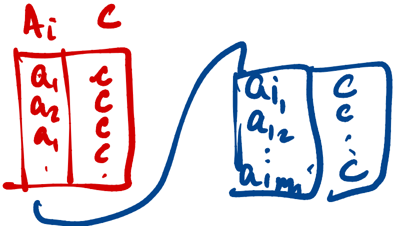
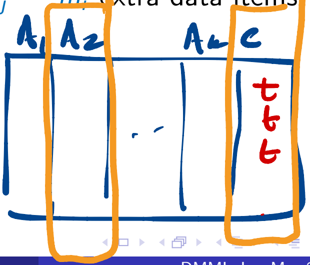
- "Pad" training data with one sample for each value a_j — m_j extra data items





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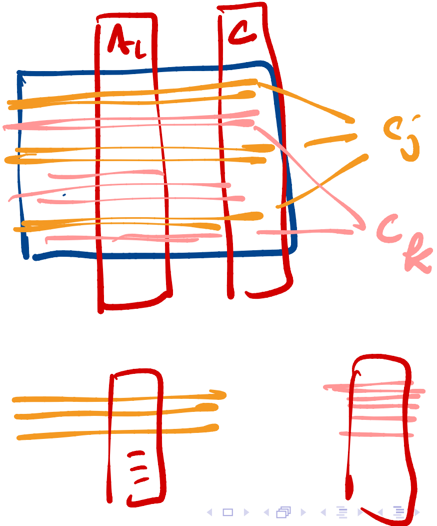
- Assume A_i takes m_i values $\{a_{i1}, \ldots, a_{im_i}\}$
- "Pad" training data with one sample for each value a_j — m_i extra data items
- Adjust $Pr(A_i = a_i \mid C = c_j)$ to $\dfrac{n_{ij} + 1}{n_j + m_i}$ where
  - n_{ij} is number of samples with $A_i = a_i$, $C = c_j$
  - n_j is number of samples with $C = c_j$

- Laplace's law of succession

$$Pr(A_i = a_i \mid C = c_j) = \frac{n_{ij} + 1}{n_j + m_i}$$



Smoothing

- Laplace's law of succession

$$Pr(A_i = a_i \mid C = c_j) = \frac{n_{ij} + 1}{n_j + m_i}$$

- More generally, Lidstone's law of succession, or smoothing

$$Pr(A_i = a_i \mid C = c_j) = \frac{n_{ij} + \lambda}{n_j + \lambda m_i}$$

$\lambda = 1 \Rightarrow$ Laplace

Smoothing

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- More generally, Lidstone's law of succession, or smoothing

$$Pr(A_i = a_i \mid C = c_j) = \frac{n_{ij} + \lambda}{n_j + \lambda m_i}$$

- $\lambda = 1$ is Laplace's law of succession

Text classification

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- Training data has a unique topic label per document — e.g., Sports, Politics, Entertainment
- Want to use a naïve Bayes classifier
- Need to define a generative model
- How do we represent documents?

Set of words model

- Each document is a set of words over a vocabulary $V = \{w_1, w_2, \ldots, w_m\}$

Leave out "stop words" and, not, is, but, ...

w_i = KYC

iPhone13

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- Each topic c has probability $Pr(c)$

Pr(sports) = # sports arts / total # artls

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- Each topic c has probability $Pr(c)$
- Each word $w_i \in V$ has conditional probability $Pr(w_i \mid c_j)$ with respect to each $c_j \in C$

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- Each topic c has probability $Pr(c)$
- Each word $w_i \in V$ has conditional probability $Pr(w_i \mid c_j)$ with respect to each $c_j \in C$
- Generating a random document d
  - Choose a topic c with probability $Pr(c)$
  - For each $w \in V$, toss a coin, include w in d with probability $Pr(w \mid c)$

Set of words model

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$Pr(A \mid c)$

$= \prod_i P(a_i \mid c)$

$Pr(c \mid d)$

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- Training set $D = \{d_1, d_2, \ldots, d_n\}$
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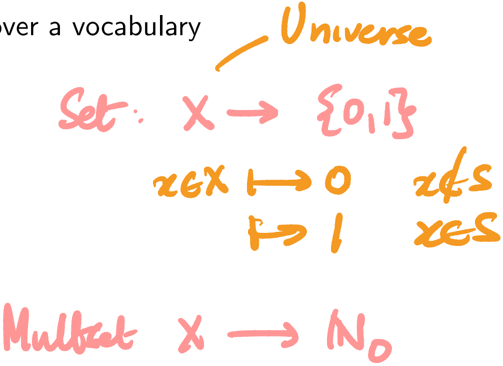

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  - Note that $\sum_{i=1}^{m} Pr(w_i \mid c_j) = 1$
  - Assume document length is independent of the class

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  - Choose a document length ℓ with $Pr(\ell)$
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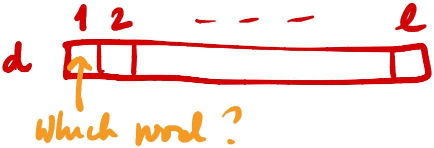
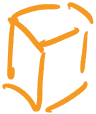


$$P(w_i | c)$$



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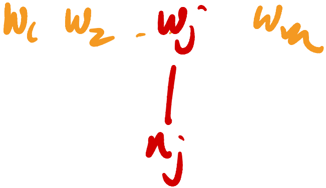

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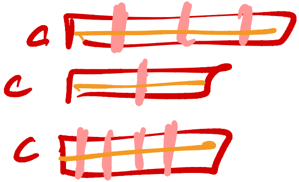

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  - $$Pr(w_i \mid c_j) = \frac{\sum_{d \in D_j} n_{id}}{\sum_{t=1}^{m} \sum_{d \in D_j} n_{td}} = \frac{\sum_{d \in D} n_{id} \, Pr(c_j \mid d)}{\sum_{t=1}^{m} \sum_{d \in D} n_{td} \, Pr(c_j \mid d)},$$

    since $Pr(c_j \mid d) = \begin{cases} 1 & \text{if } d \in D_j, \\ 0 & \text{otherwise} \end{cases}$

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