

Lecture 22: 21 April, 2022

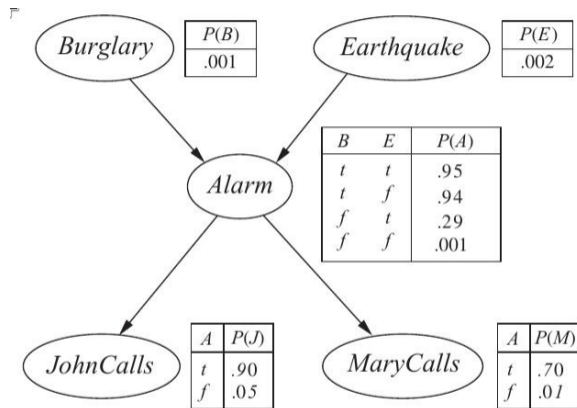
Madhavan Mukund

<https://www.cmi.ac.in/~madhavan>

Data Mining and Machine Learning
January–May 2022

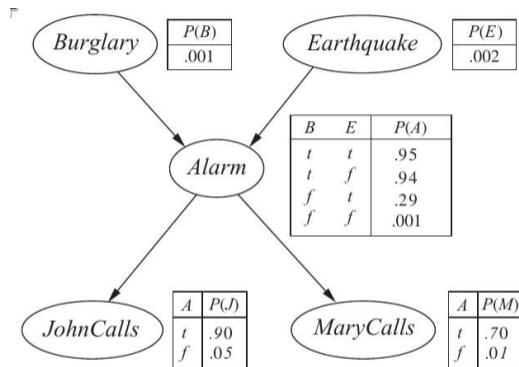
Probabilistic graphical models

- Underlying DAG, no cyclic dependencies
- Each node has a local (conditional) probability table



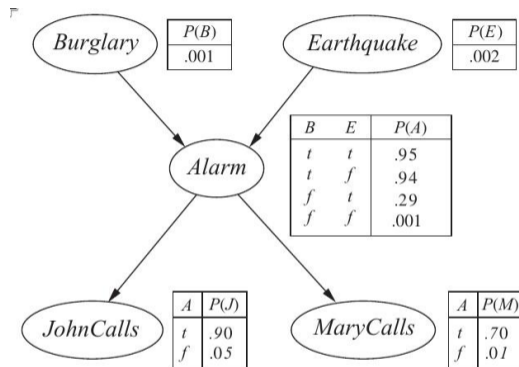
Conditional independence

- $x \perp y$ — x and y are independent
 - $P(x \wedge y) = P(x) \cdot P(y)$



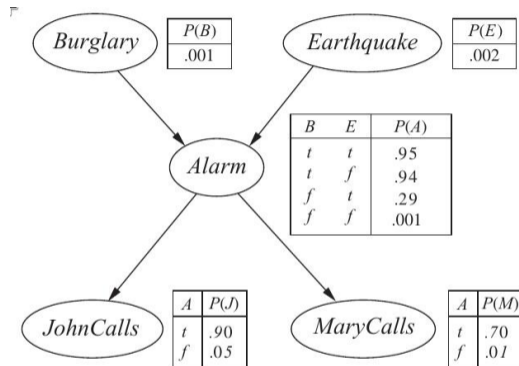
Conditional independence

- $x \perp y$ — x and y are independent
 - $P(x \wedge y) = P(x) \cdot P(y)$
- $x \perp y \mid z$
 - x and y are independent given z



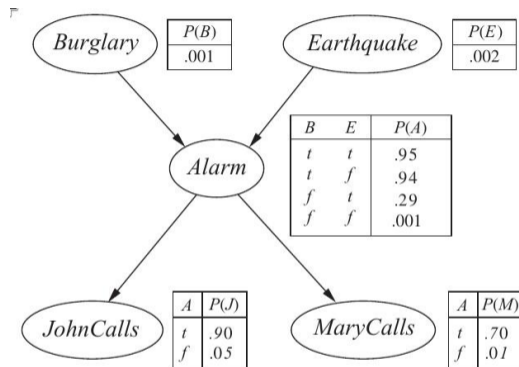
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- Is *JohnCalls* independent of *MaryCalls* ($j \perp m$)?



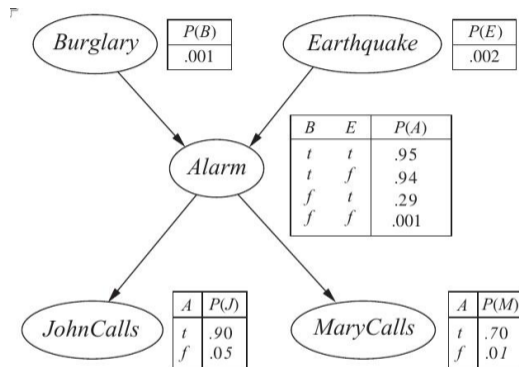
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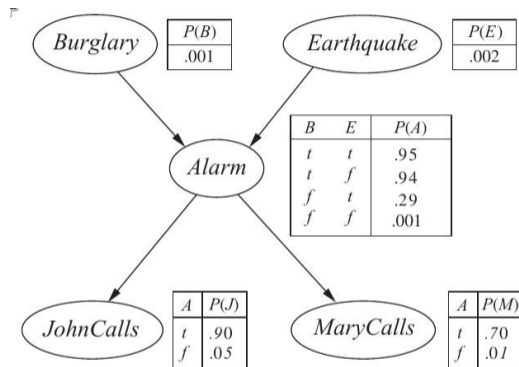
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Conditional independence

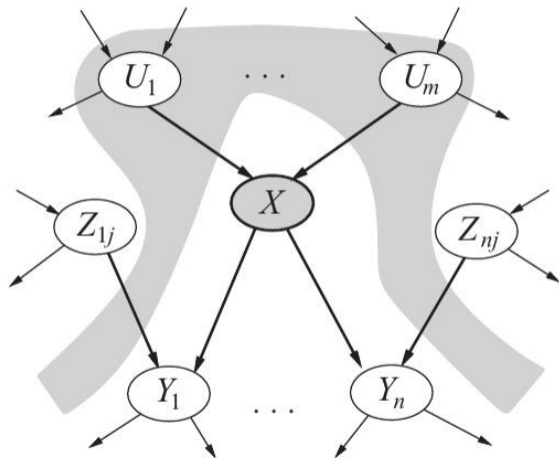
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 - No — value of j tells us something about value of m and vice versa
- Is *JohnCalls* independent of *MaryCalls* given *Alarm* ($j \perp m \mid a$)?
 - Yes — by semantics of network, local independence



Probabilistic graphical models

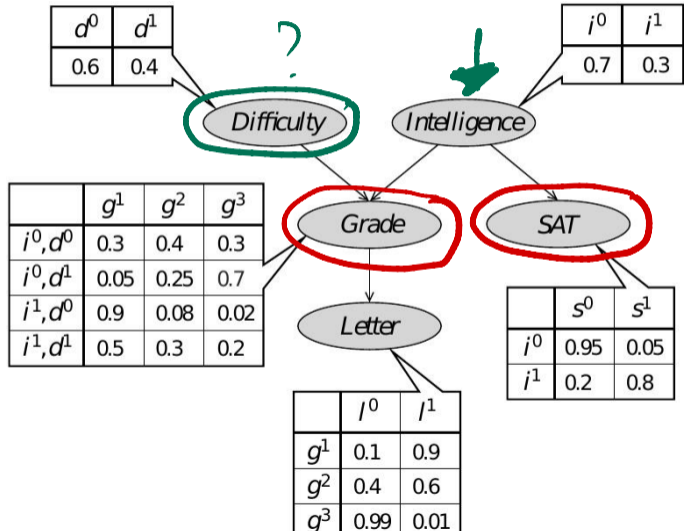
■ Fundamental assumption

A node is conditionally independent of non-descendants, given its parents



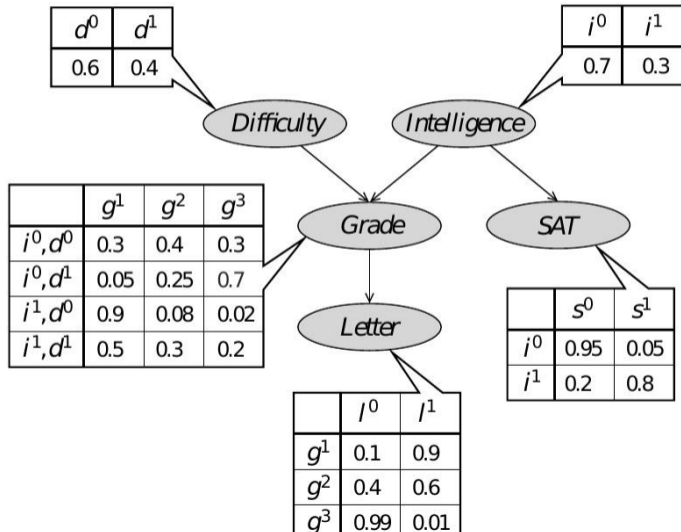
Student example

- $SAT \perp Grade \mid Difficulty$?



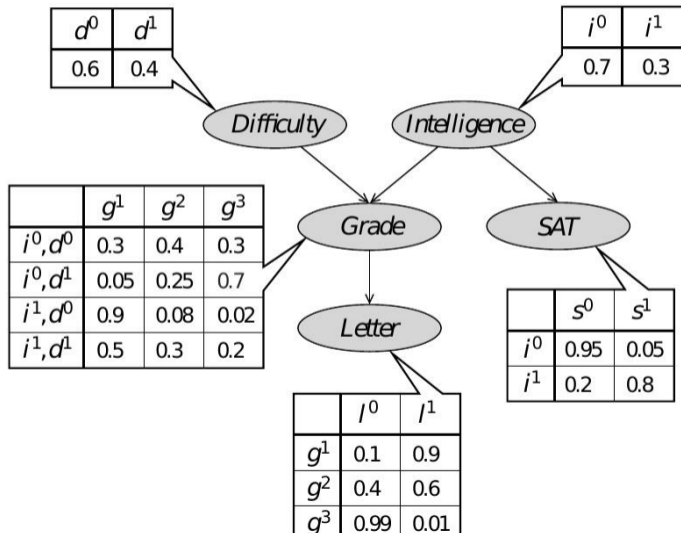
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Student example

- $SAT \perp Grade \mid Difficulty$?
 - No
- Can we calculate conditional independence from the graph?



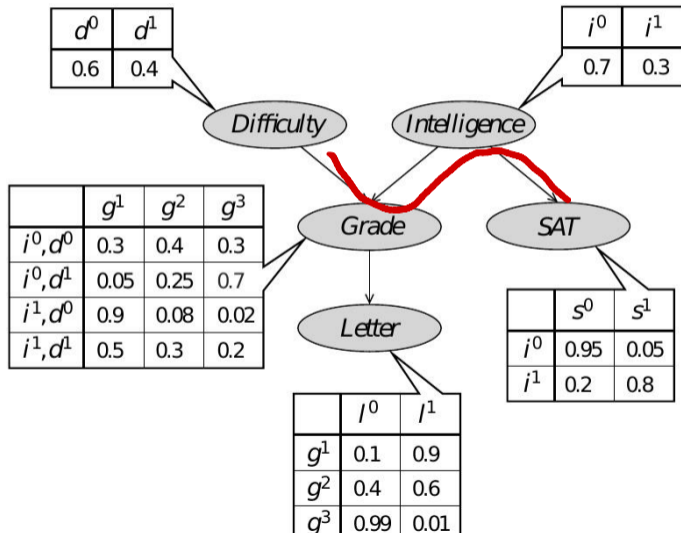
Student example

■ $SAT \perp Grade \mid Difficulty$?

■ No

■ Can we calculate conditional independence from the graph?

■ In general, check if $X \perp Y \mid Z$ for sets of variables X, Y, Z

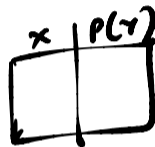


Conditional independence

- How does dependence “flow” through a network?

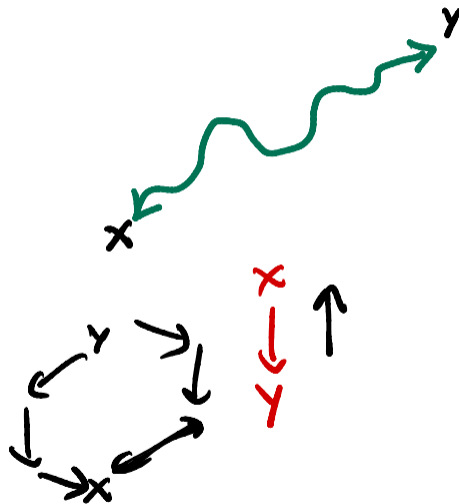
Conditional independence

- How does dependence “flow” through a network?
- For neighbouring nodes, dependence flows both ways
 - If $x \rightarrow y$, knowing x tells us about y and vice versa



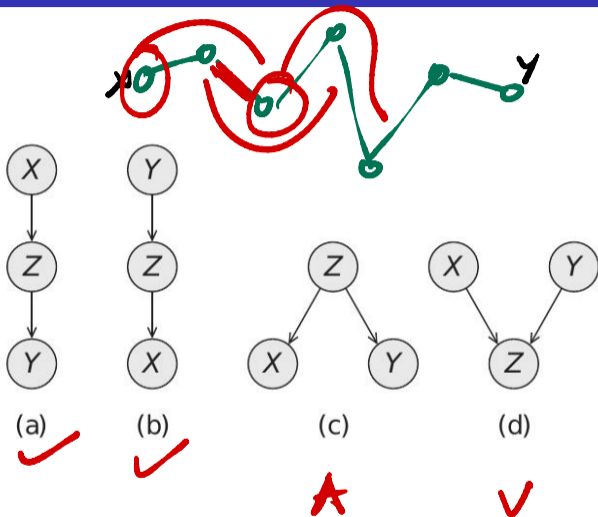
Conditional independence

- How does dependence “flow” through a network?
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- Examine **trails** between nodes
 - Paths in the underlying undirected graph

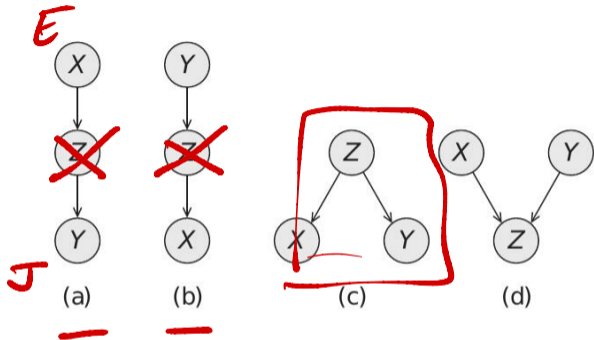


Conditional independence

- How does dependence “flow” through a network?
- For neighbouring nodes, dependence flows both ways
 - If $x \rightarrow y$, knowing x tells us about y and vice versa
- Examine **trails** between nodes
 - Paths in the underlying undirected graph
- **Basic trails** — (undirected) paths of length 2
 - Four basic trails



Basic trails



Basic trails

- (a), (b) and (c): Z blocks flow between X and Y , by semantics of Bayesian networks

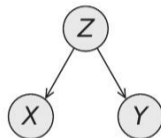
$$\therefore X \perp Y \mid Z$$



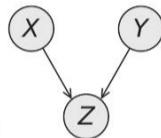
(a)



(b)



(c)



(d)

Basic trails

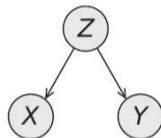
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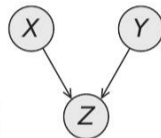
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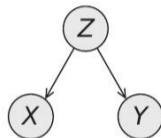
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 - Z : Car does not start
 - X : Low Battery, Y : No Fuel



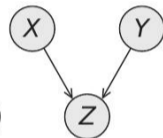
(a)



(b)



(c)



(d)

Basic trails

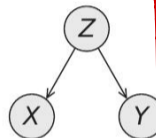
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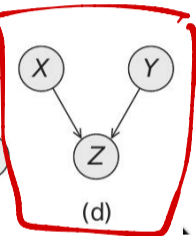
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(d)

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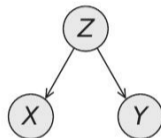
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 - Simplest form of **V-structure**



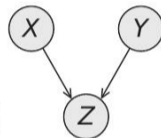
(a)



(b)



(c)



(d)

D-Separation

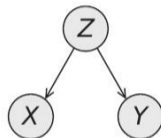
- Check if $X \perp Y \mid Z$



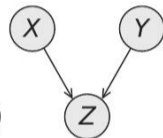
(a)



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(d)

D-Separation

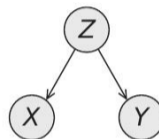
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- Dependence should be blocked on every trail from X to Y



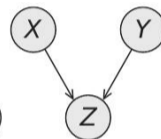
(a)



(b)



(c)



(d)

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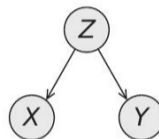
- Check if $X \perp Y \mid Z$
- Dependence should be blocked on every trail from X to Y
 - Each undirected path from X to Y is a sequence of basic trails



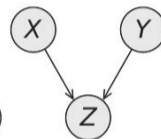
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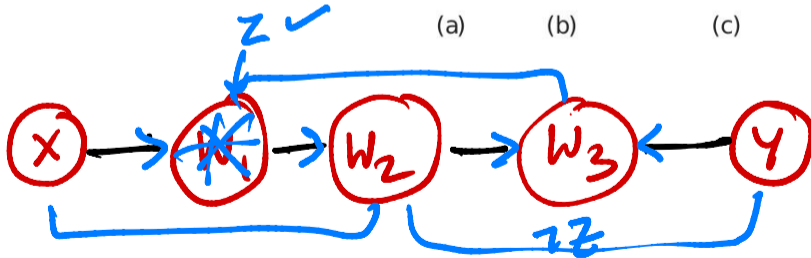
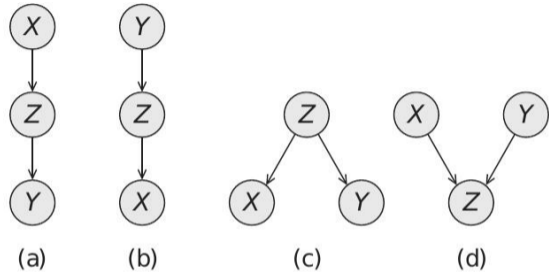
(c)



(d)

D-Separation

- Check if $X \perp Y \mid Z$
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 - For (a), (b), (c), need Z present



D-Separation

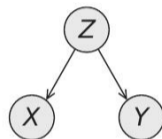
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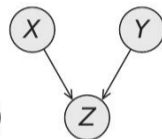
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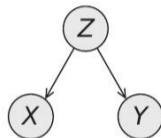
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 - In general, V-structure includes descendants of the bottom node



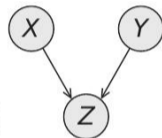
(a)



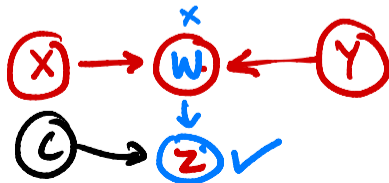
(b)



(c)



(d)



W is known

D-Separation

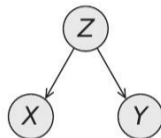
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- x and y are **D-separated** given z if all trails are blocked



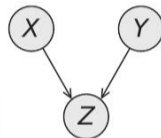
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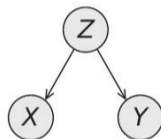
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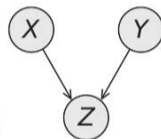
(a)



(b)



(c)



(d)

- x and y are **D-separated** given z if all trails are blocked
- Variation of **breadth first search (BFS)** to check if y is reachable from x through some trail

D-Separation

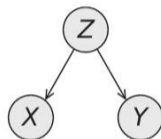
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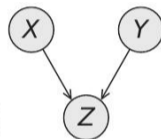
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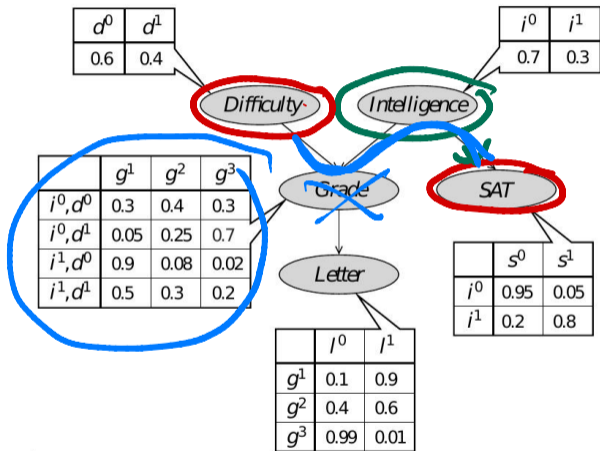


(d)

- x and y are **D-separated** given z if all trails are blocked
- Variation of **breadth first search (BFS)** to check if y is reachable from x through some trail
- Extends to sets — each $x \in X$ is D-separated from each $y \in Y$

Conditional independence, example

- Is **SAT** independent of **Difficulty** given **Intelligence**?
- Yes, **Difficulty** – **Grade** – **Intelligence** – **SAT** trail is blocked at **Grade** (V-structure)



Conditional independence, example

- Is **SAT** independent of **Difficulty** given **Intelligence**?
 - Yes, **Difficulty** – **Grade** – **Intelligence** – **SAT** trail is blocked at **Grade** (V-structure)
- Is **SAT** independent of **Difficulty** given **Grade**?
 - No, **Difficulty** – **Grade** – **Intelligence** – **SAT** trail is open because **Grade** is known (V-structure)

