Concurrency Theory, 11 Sep 2019

Prime Event Structures

$ES = (E, \leq, \#)$

$C \subseteq E$ is a configuration iff $(C \times C) \cap \# = \phi$

Thm $(\mathcal{C}_{ES}, \subseteq)$ is a prime algebraic cpo.

$\downarrow e$ are the primes — define, $\leq \; = \; \subseteq$

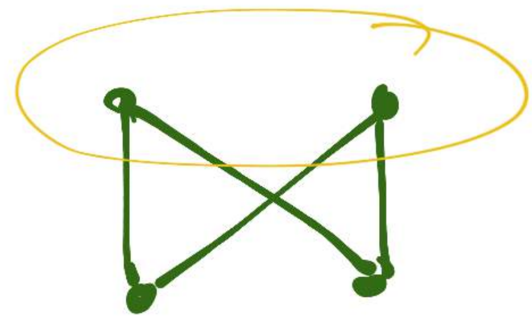
$\# =$ incompatible

Reconstruct $(\{\downarrow e\}, \leq, \#)$

Defns

Coherent - every finite subset that has an upper bound has an lub

Non-example



2 upper bounds, no lub

Directed set - every finite subset has an upper bound within the set

Complete P.O - every directed set has lub

Complete lattice - every subset has lub

Play an important role in "Domain Theory"

Dana Scott

Denotational Semantics for programs

program — computes a function

$f : I \to O$

Non termination

Undefined outputs

Represent I & O as some kind of p.o.

"Full Abstraction"

Highly open in 1970's - 1980's

"Solved" using games

Gave rise to LCF → Standard ML

Haskell type inference

Winskel - Event structures

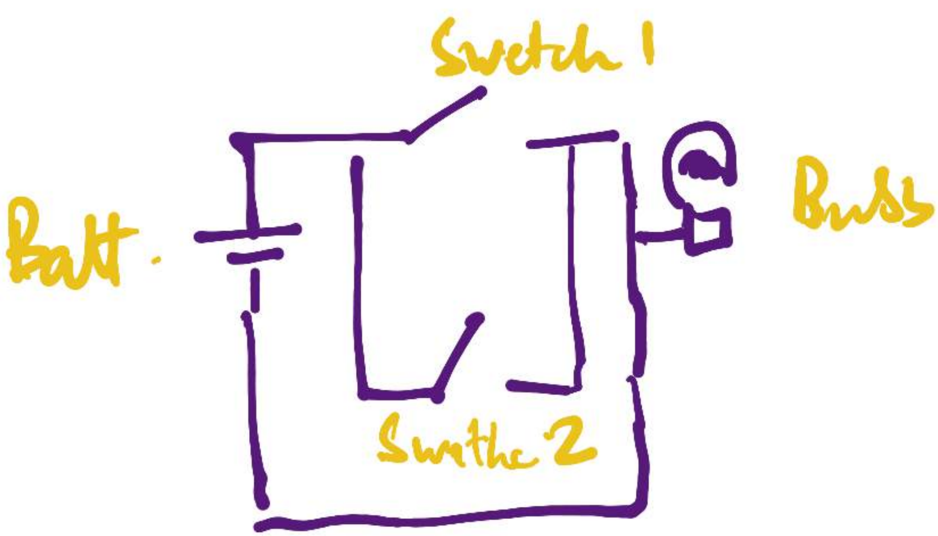
Motivated by "Domain Theory for Concurrency"

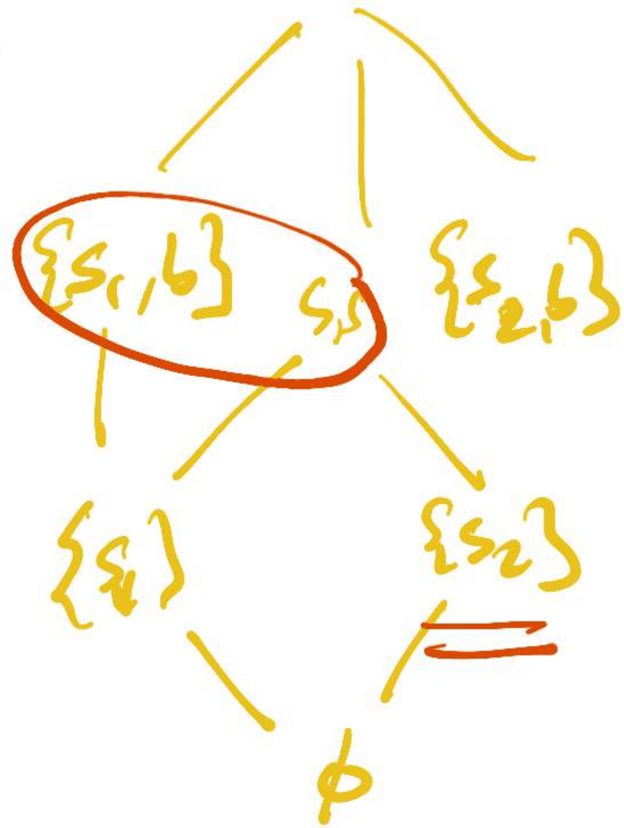
Additional "reasonable" constraints

Finite causes: $\downarrow e$ is always finite

Some situations hard to model

Parallel Switch





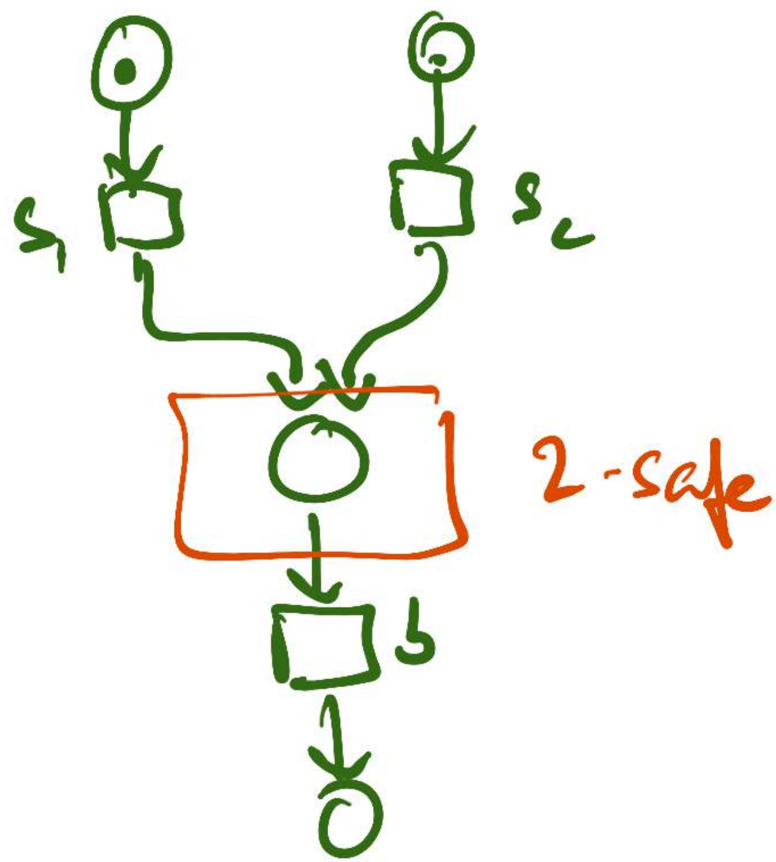
No primes

Prime ES model

Bulb on $\longrightarrow$ b_1 or b_2

via s_1 via s_2

Net rep.



Other event structure models

Focus on $\mathcal{C}_{ES}$ and either $\#$ or $\leq$

Different consistency constraints on

$$(\mathcal{C}_{ES}, \subseteq)$$

Stable event structures — Winskel

Flow event structures — Boudol & Castellani

$\vdots$

A practical application

1-safe net $\xrightarrow{\text{unfold}}$ Occurrence Net $\rightarrow$ ES

Maximal unfolding

What are reachable markings?

Every reachable M occurs in max unfolding

Construct a "small" prefix that has all the info we need

Recall that

$((P, T, F), M_{in})$

Occurrence net – define $\leq$, $\#$, co

between pairs of places / transitions

Configuration is a $\downarrow$-closed $\#$-free

set of transitions

Local configurations $\downarrow t$ — $[t]$

Cut – any maximal pairwise-co subset of $P \cup T$

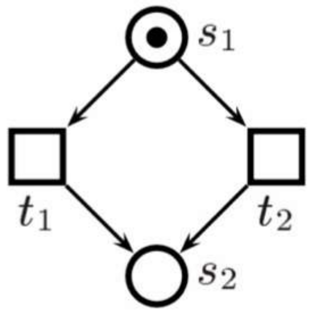
└ usually pairwise-co subset of P

Occurrence net is labelled

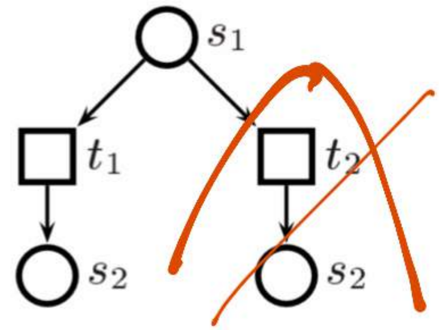
Marking M in original net is <u>represented</u>
in ON if we reach a cut whose labels
correspond to M

Construct a prefix of ON in which every
reachable marking is represented

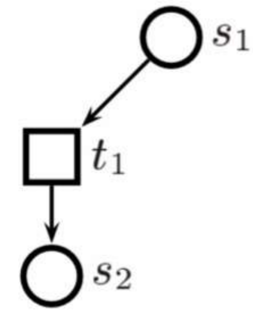
And a bit more



N



ON



Prefix

Both reachable markings are represented

But occurrence of t_2 is not recorded

Complete Prefix

1. Every reachable marking is represented
2. For every reachable M & t s.t. $M \xrightarrow{t}$, there is an occurrence of t

Given original net is 1-safe & finite

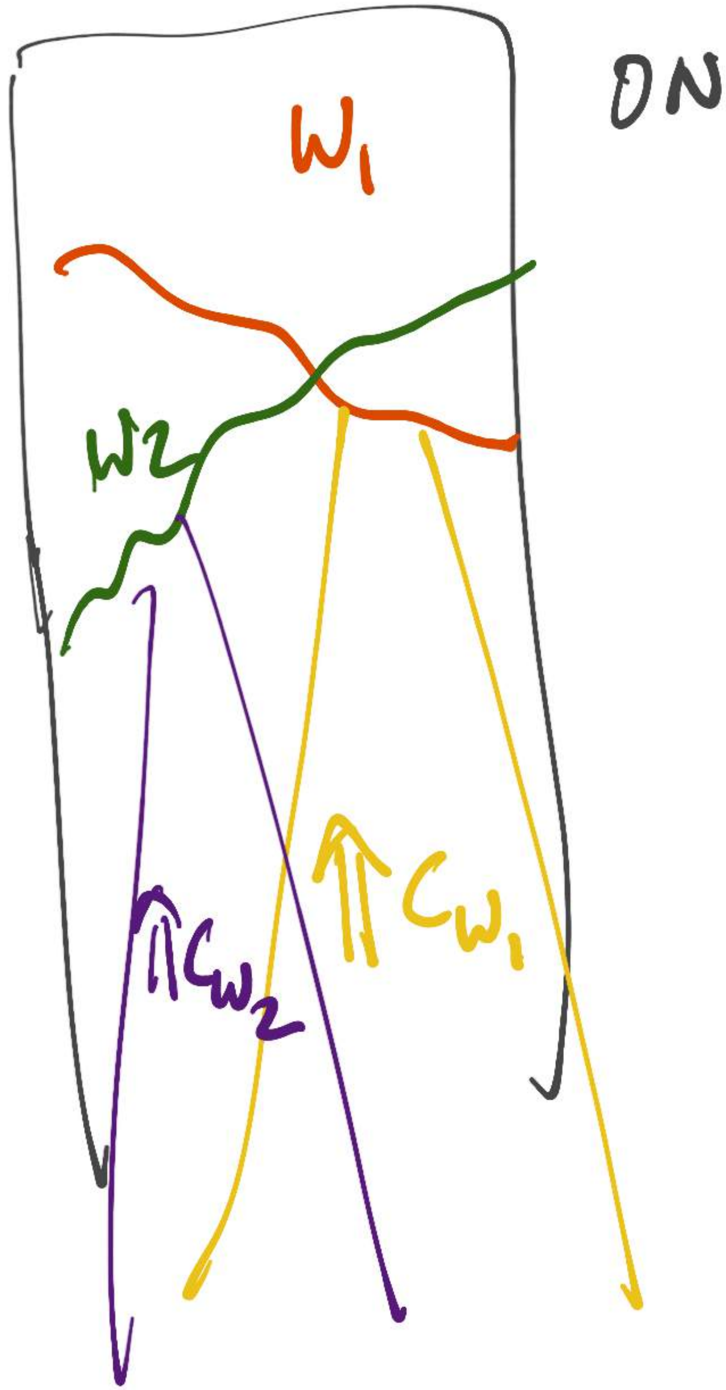
- Must exist a finite complete prefix

Ideally want a complete prefix whose size is linear in the original net

If

$$\begin{tikzcd} M_{in} \arrow[r, squiggly, "w_1"] \arrow[r, squiggly, bend right, "w_2"'] & M \end{tikzcd} \xrightarrow{\text{same future}}$$

C_{w_1} & C_{w_2} have isomorphic futures



Both $\Uparrow C_{w_1}$ & $\Uparrow C_{w_2}$ are isomorphic to the ON generated by

$$\big((P, T, F), M\big)$$

marking after w_1/w_2

In particular, for t_1, t_2 transitions in ON

Suppose $\text{Mark}(\downarrow t_1) = \text{Mark}(\downarrow t_2)$

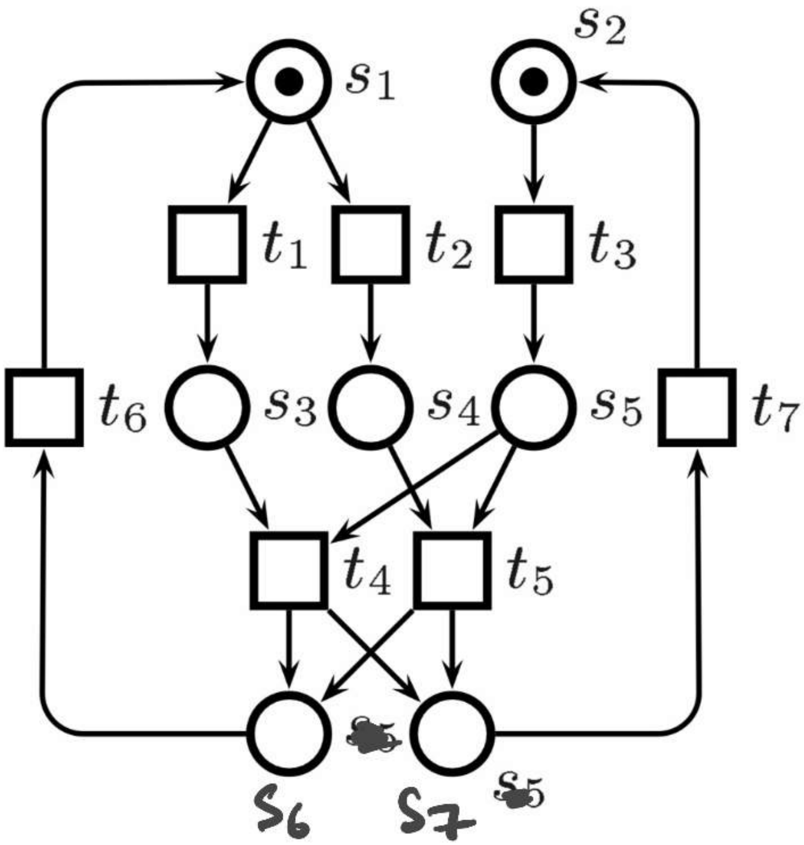
Mark(C) - marking after configuration C
$\Downarrow$
in original net

$\Uparrow(\downarrow t_1)$ isomorphic to $\Uparrow(\downarrow t_2)$

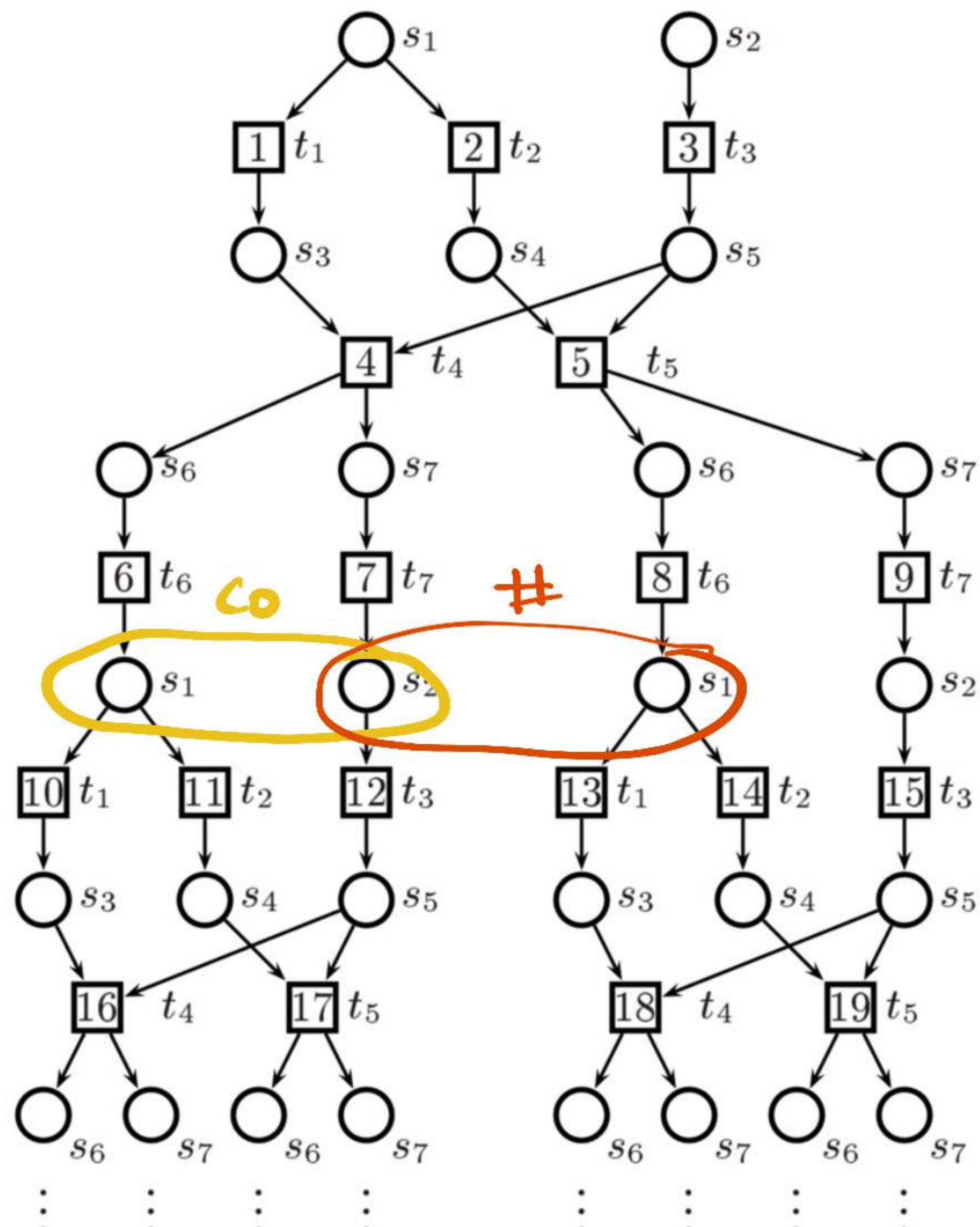
Consider any extension of $\downarrow t_1$ $\quad \downarrow t_1 \oplus E_1$

$\exists E_2 \quad \downarrow t_2 \oplus E_2$ is a configuration

& $E_1 \simeq E_2$ [Isomorphic]



Net



Occurrence Net

When to "truncate" the unfolding?

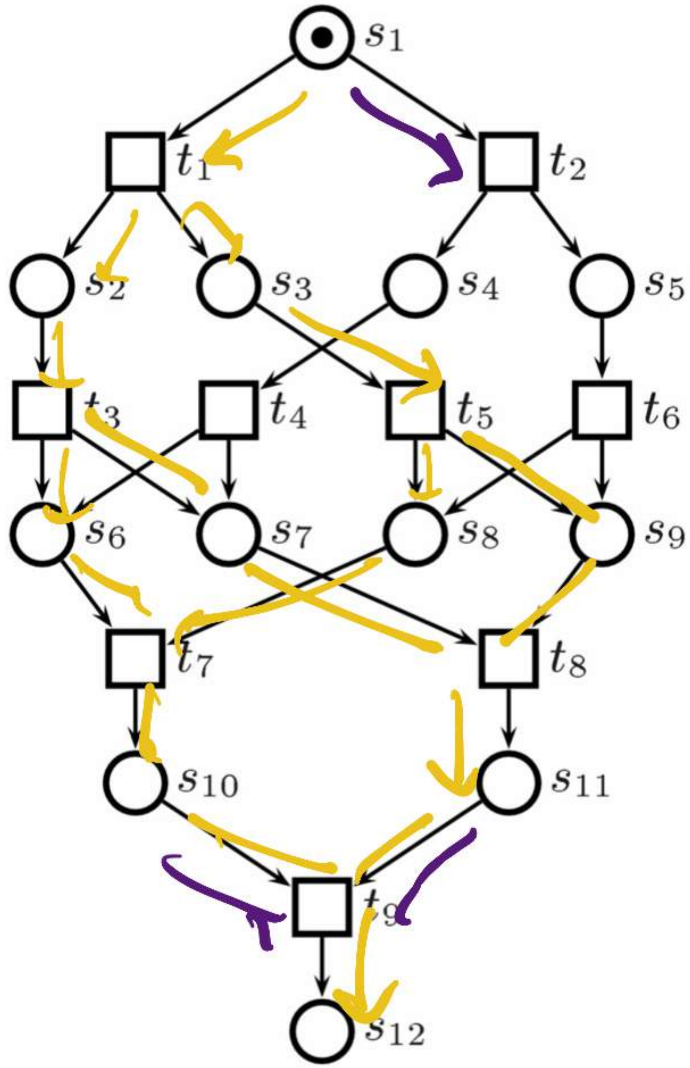
McMillan

Identify cut-off events:

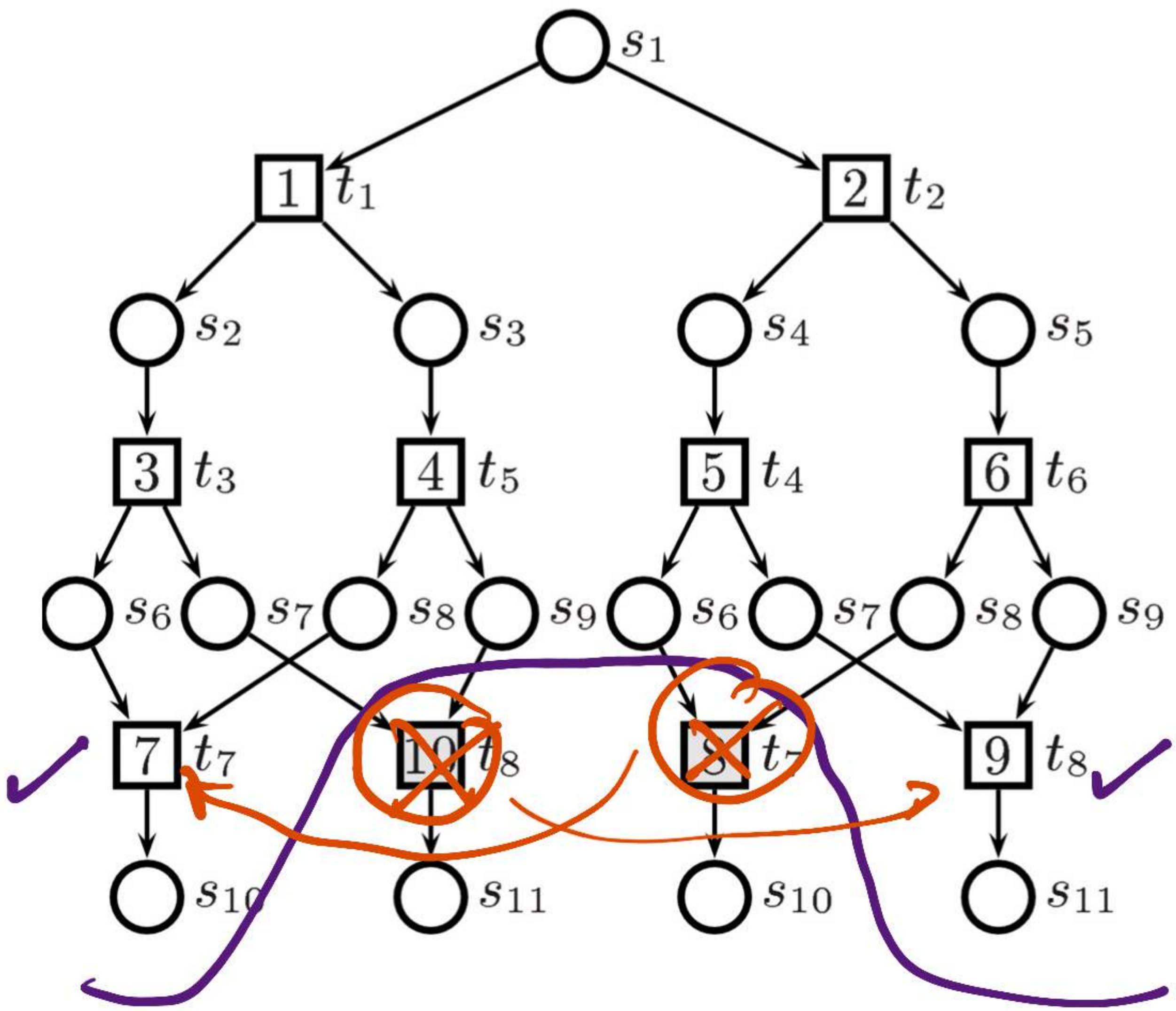
$$\mathrm{Mark}(\downarrow t_1) = \mathrm{Mark}(\downarrow t_2)$$

If we encounter t_2 s.t. $\mathrm{Mark}(\downarrow t_2) = \mathrm{Mark}(\downarrow t_1)$ for previous t_1, designate t_2 a <u>cut-off</u> event

Problems



s1
t1
t2
s2
s3
s4
s5
t3
t4
t5
t6
s6
s7
s8
s9
t7
t8
s10
s11
t9
s12



s1
1 t1
2 t2
s2
s3
s4
s5
3 t3
4 t5
5 t4
6 t6
s6
s7
s8
s9
s6
s7
s8
s9
7 t7
10 t8
8 t7
9 t8
s10
s11
s10
s11

Need an additional constraint to cut-off

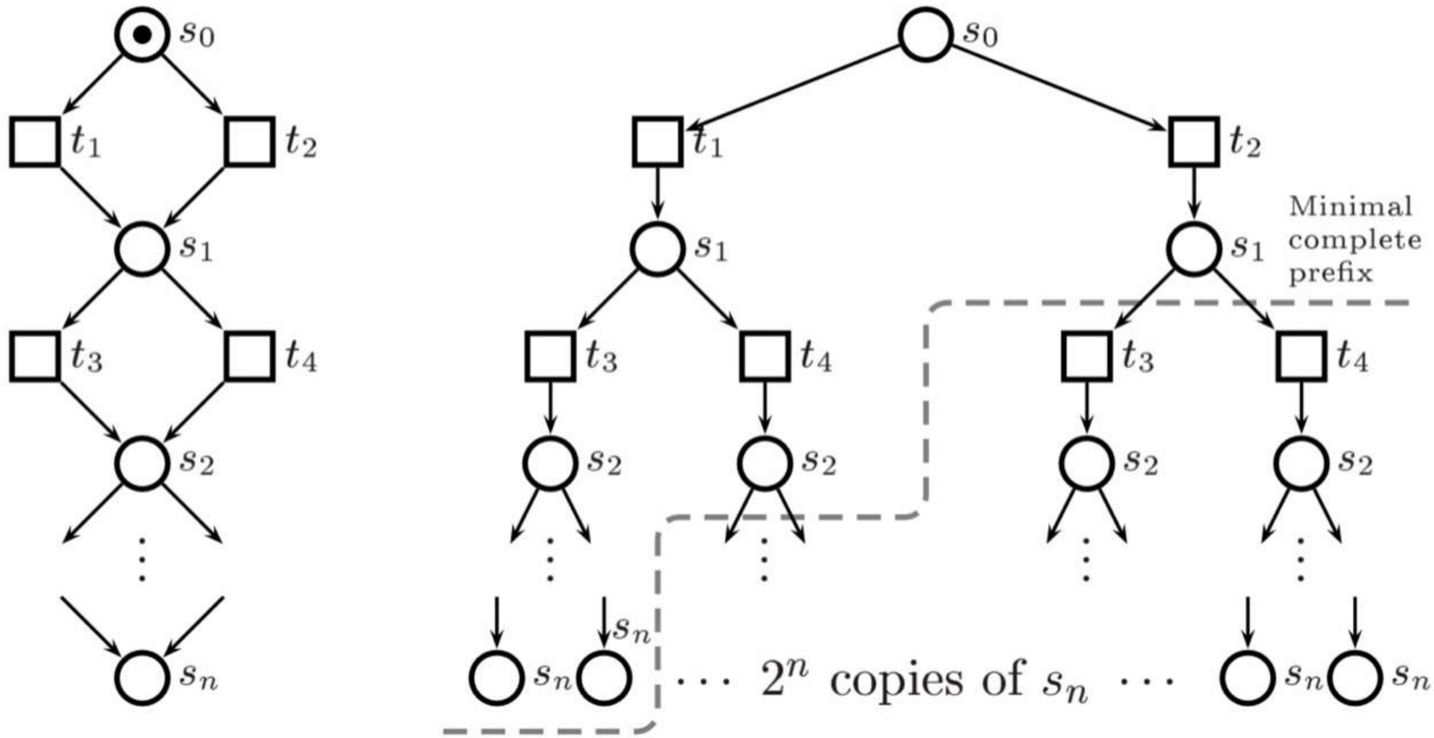
McMillan $\quad Mark(\downarrow t_1) = Mark(\downarrow t_2)$

$\& \quad |\downarrow t_1| < |\downarrow t_2|$

$\Rightarrow$ can call t_2 a cut off event

Solves problem in previous example

There $|\downarrow 10| = |\downarrow 9|$



McMillan's definition cannot cut off anything here

Generalize

Define $\prec$ order on configurations

that is "adequate"

$$C_1 \subset C_2 \rightarrow C_1 \prec C_2$$

well founded

$$C_1 \prec C_2 \quad \& \quad Mark(C_1) = Mark(C_2)$$

$$\Rightarrow \forall E \quad Mark(C_1 \oplus E) \prec Mark(C_2 \oplus E)$$

$|\downarrow t_1| < |\downarrow t_2|$ or

$|\downarrow t_1| = |\downarrow t_2|$ &

$\text{lex}(\text{word to } t_1) \leq \text{lex}(\text{word to } t_2)$

t_1 is enabled by $ababca \Rightarrow a^3b^2c$

$$\vee|$$

t_2 is enabled by $abbaba \Rightarrow a^3b^3$