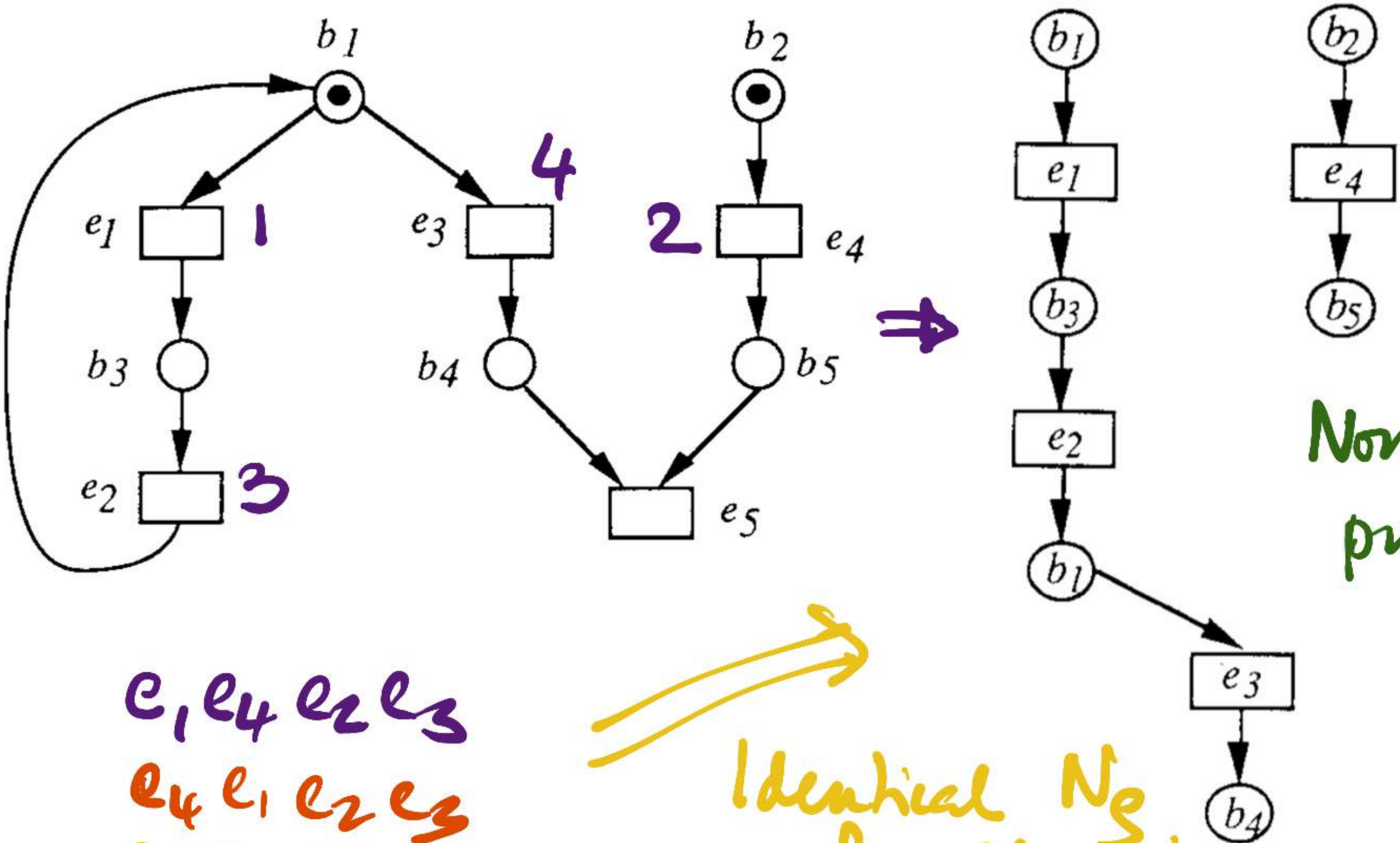
# Concurrency Theory, 6 Sep 2019

Given ENS, for any firing sequence $\varrho$, inductively construct an acyclic net $N_\varrho$




Non sequential process

$e_1 e_4 e_2 e_3$

$e_4 e_1 e_2 e_3$

$e_1 e_2 e_4 e_3$

Identical $N_\varrho$ for all $\varrho' \approx \varrho$

$gu$ & $gv$ are two firing sequences

$N_{gu}$ & $N_{gv}$ will be identical upto $N_g$

These are called <u>causal</u> <u>net</u>

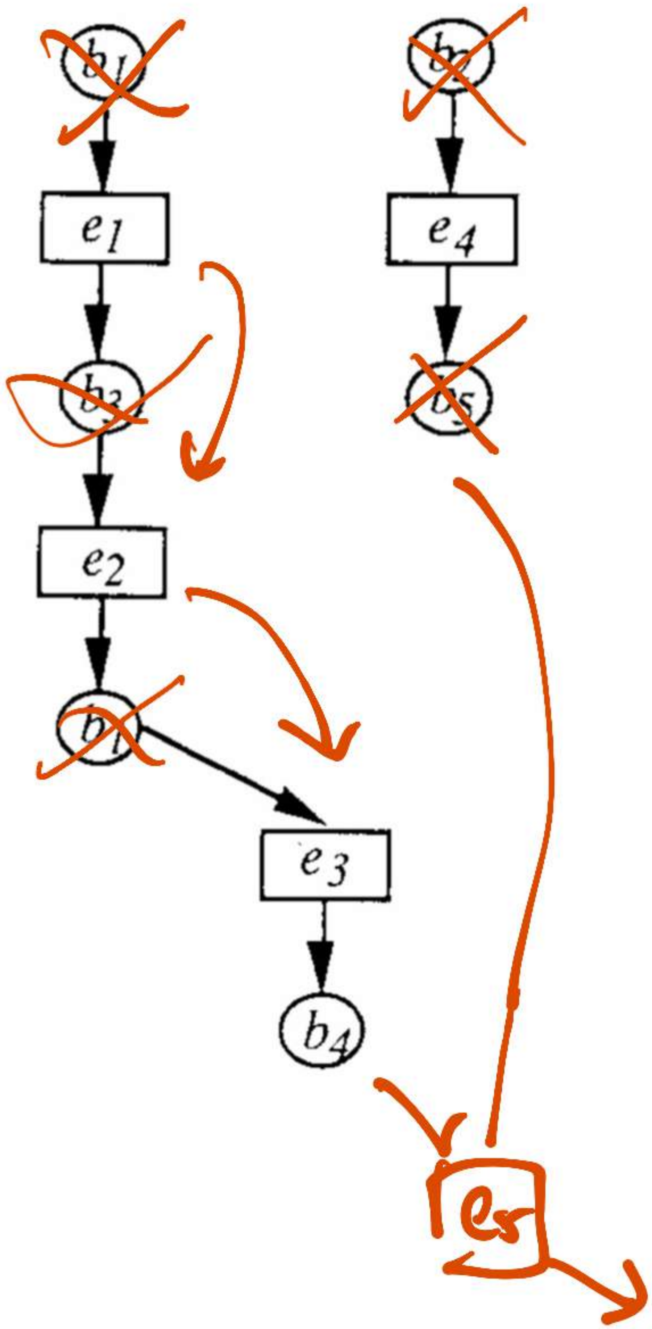
$\forall p.\ |p^\bullet| = 1$ - all choices resolved

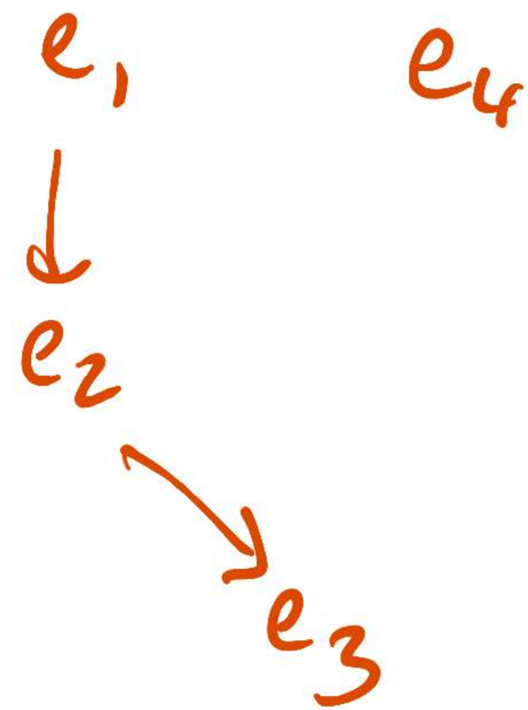
$|{}^\bullet p| = 1$ except minimal $p$

- unique enabling transition

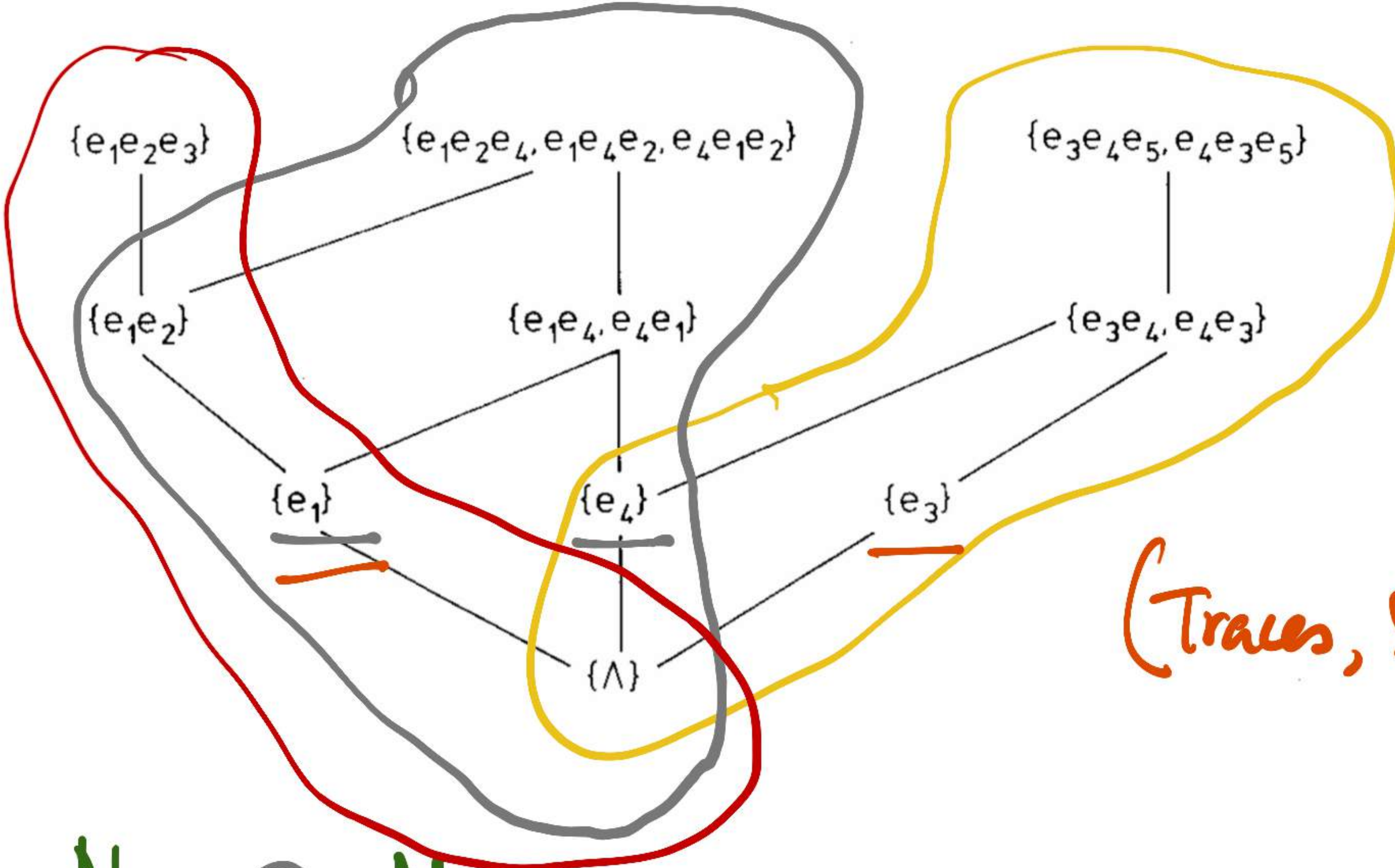
$F^*$ is a partial order

Causal net – discard places, $\leq$ on transitions




= trace over $(T, I_N)$




(Traces, $\subseteq$)

$N_g \subseteq N_{gt}$

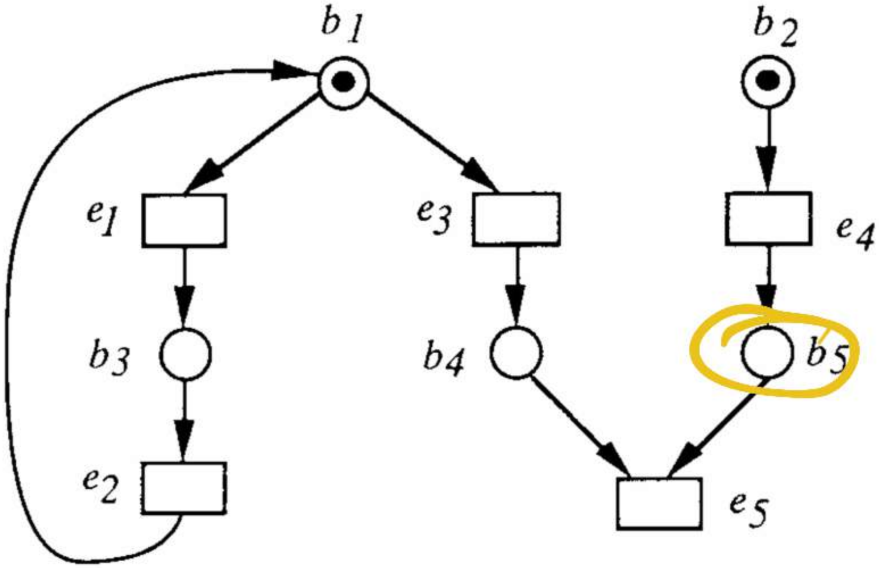
$(P_g, T_g, F_g) \quad (P_{gt}, T_{gt}, F_{gt})$

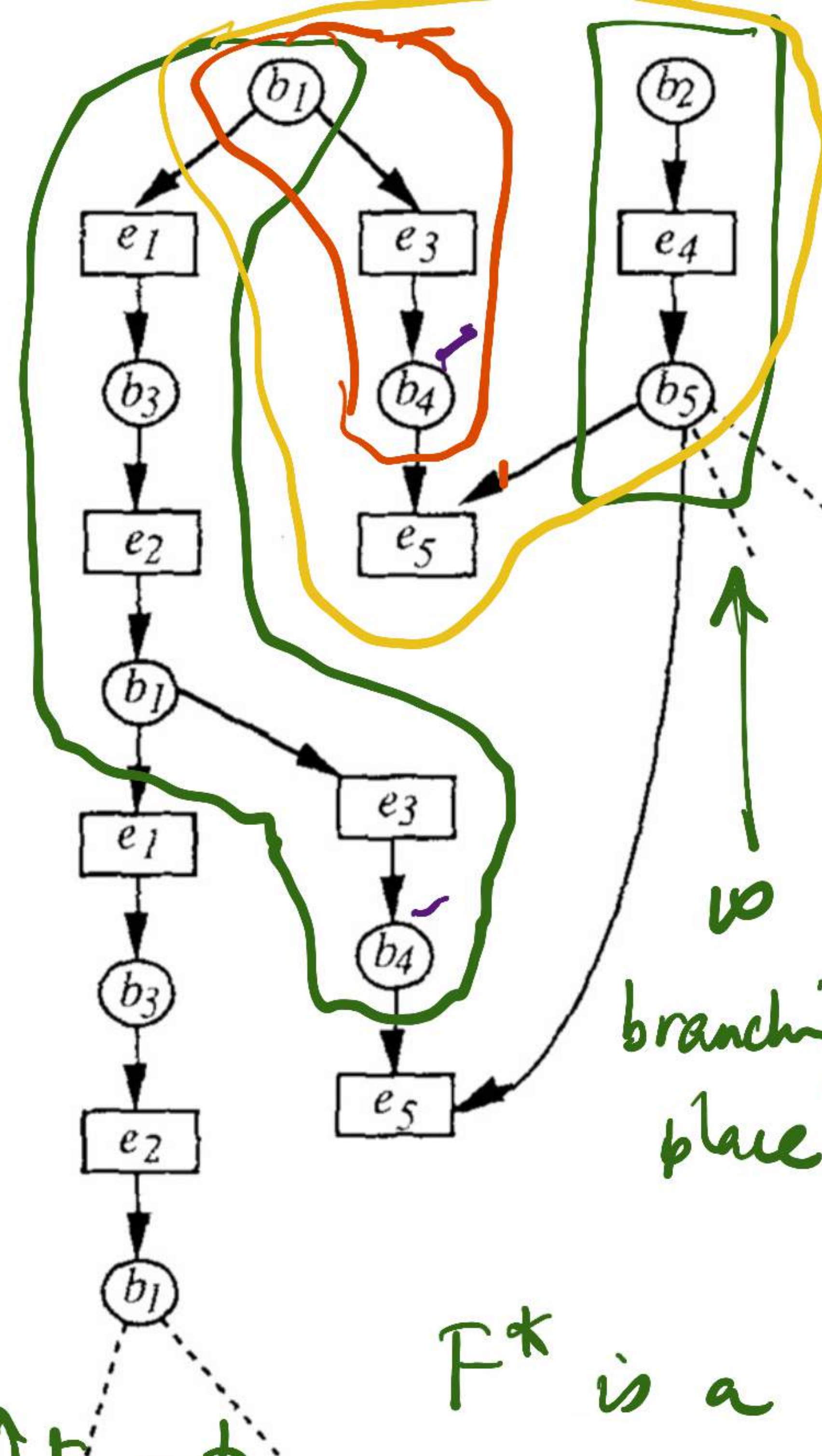
$\subseteq \quad \subseteq \quad \subseteq$

$F_g = F_{gt} \cap (P_g \times T_g \cup T_g \times P_g)$

$$
\begin{tikzcd}
N_s \arrow[r, phantom, "\subseteq"] \arrow[d, "\text{strip place}"] & N_{su} \arrow[d] \\
{[s]} \arrow[r, phantom, "\sqsubseteq"] & {[su]}
\end{tikzcd}
$$

$$\bigcup N_s = \text{set of "branching processes"}$$




no branching place

**Structurally**

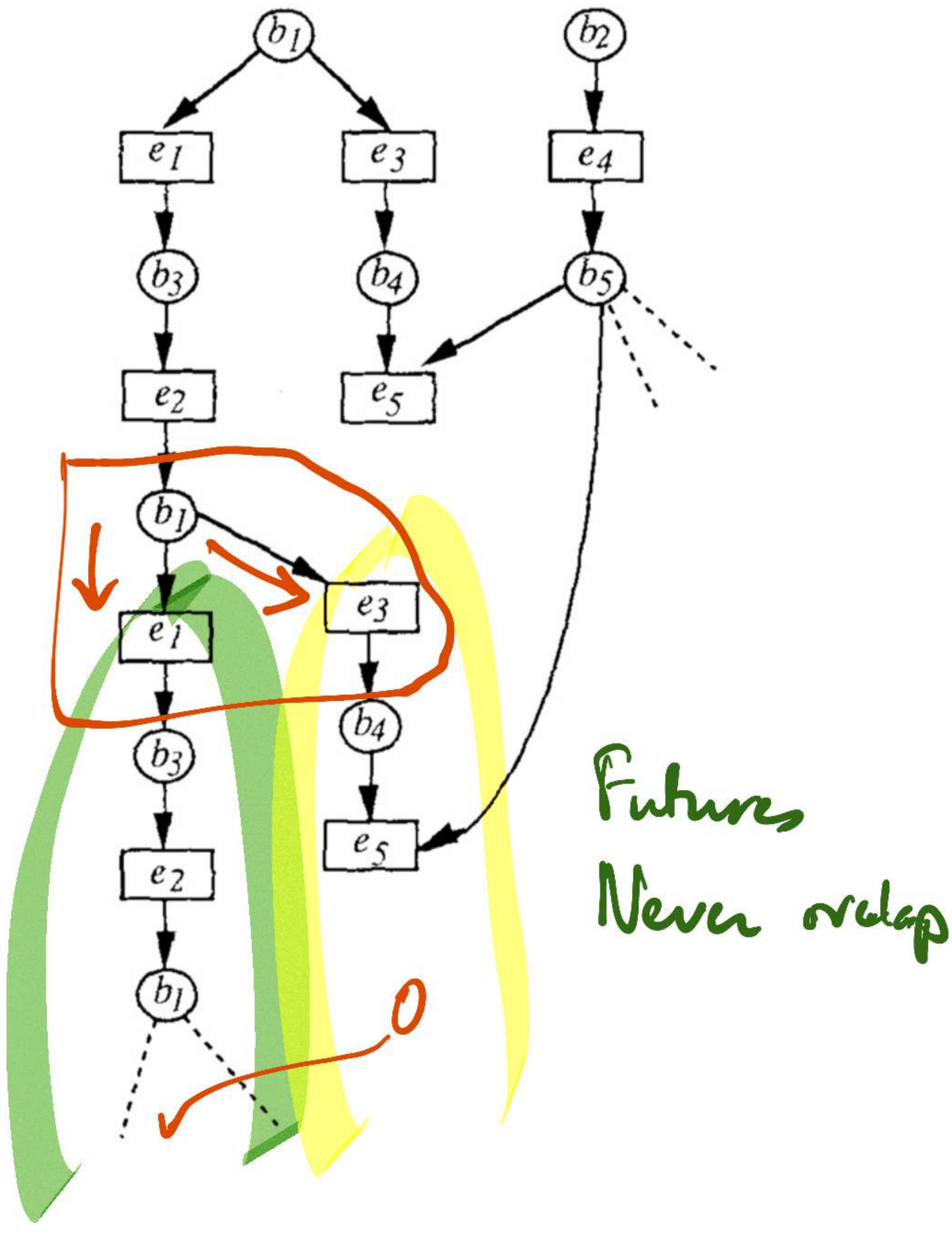
Occurrence Net

$\forall p \quad |{}^\bullet p| \le 1$

$\forall t_1, t_2 \quad t_1 \ne t_2 ,$

$${}^\bullet t_1 \cap {}^\bullet t_2 \ne \phi \Rightarrow \Uparrow t_1 \cap \Uparrow t_2 = \phi$$

$F^*$ is a p.o.


b1
b2
e1
e3
e4
b3
b4
b5
e2
e5
b1
e3
e1
b4
b3
e5
e2
b1
0
Futures
Never overlap

ENS

1-Safe Net $\Rightarrow$ Unfoldings $\rightarrow$ Occurrence Net

What happens if we throw away the places?

$e_1$ & $e_2$ unordered

(1) Compatible & independent (like in a trace)

(2) Incompatible, exclude each other

Underlying transition are "event occurrences"

2 relation

$(E, \leq, \#), \lambda$

causality

conflict

incompatible pairs

(symmetric)

$$\begin{tikzcd} e_1 \arrow[d, phantom, "\#"] \arrow[dr, dash, dashed, "\#"] & \\ e_2 \arrow[r] & e_3 \end{tikzcd}$$

\# is inherited via $\leq$

$$\forall e_1, e_2, e_3 \quad e_1 \# e_2,\ e_2 \leq e_3 \Rightarrow e_1 \# e_3$$

Event Structure

Tree is an event structure where

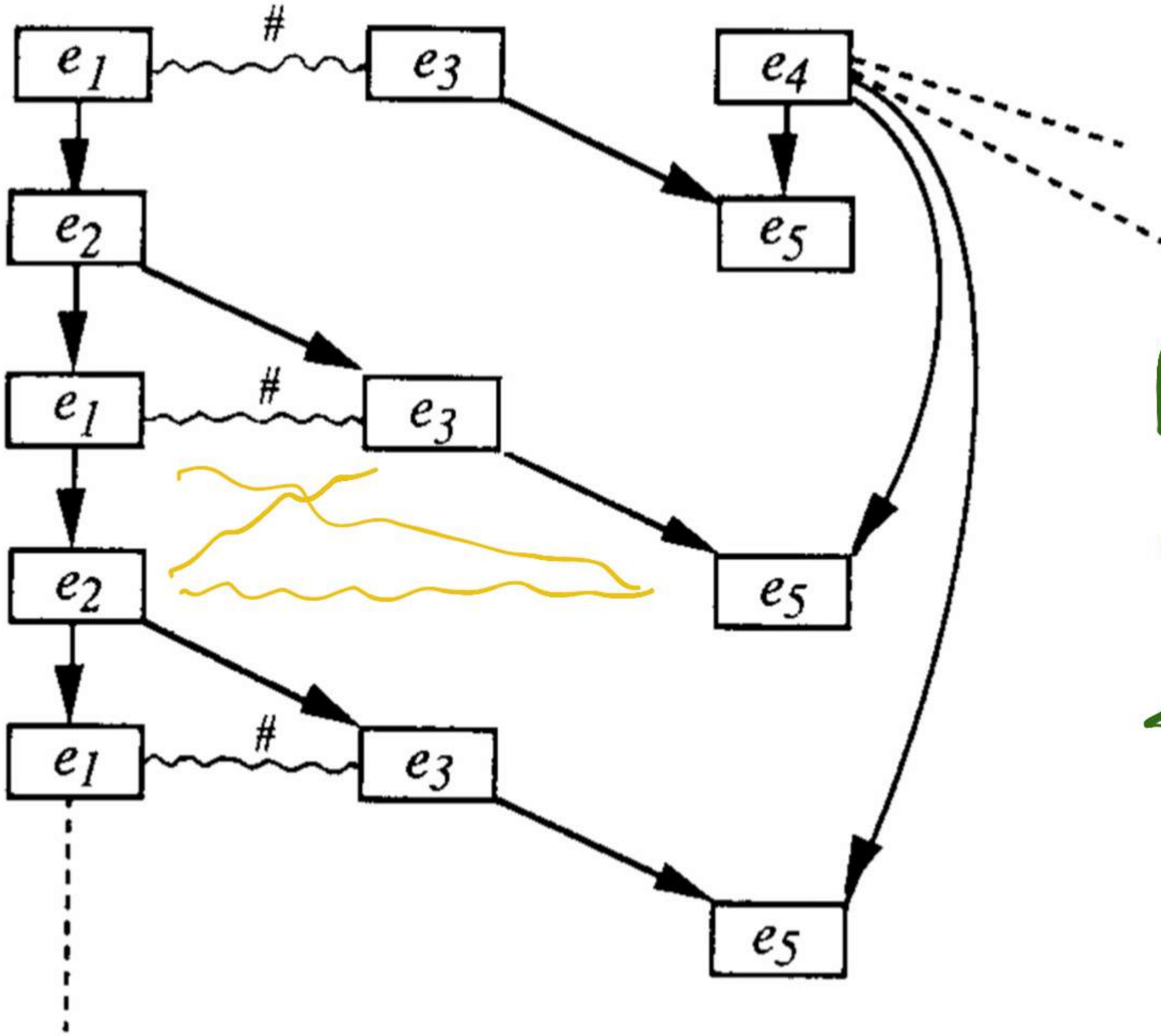
$$e \not\leq e' \ \& \ e' \not\leq e \implies e \# e'$$

In Event Structure

$$e \not\leq e' \wedge e' \not\leq e \wedge e \cancel{\#} e'$$

$$\implies e \ co \ e'$$

concurrent




Minimal conflicts

Recover computations from ES

$c \subseteq E$ is a computation if "configurations"

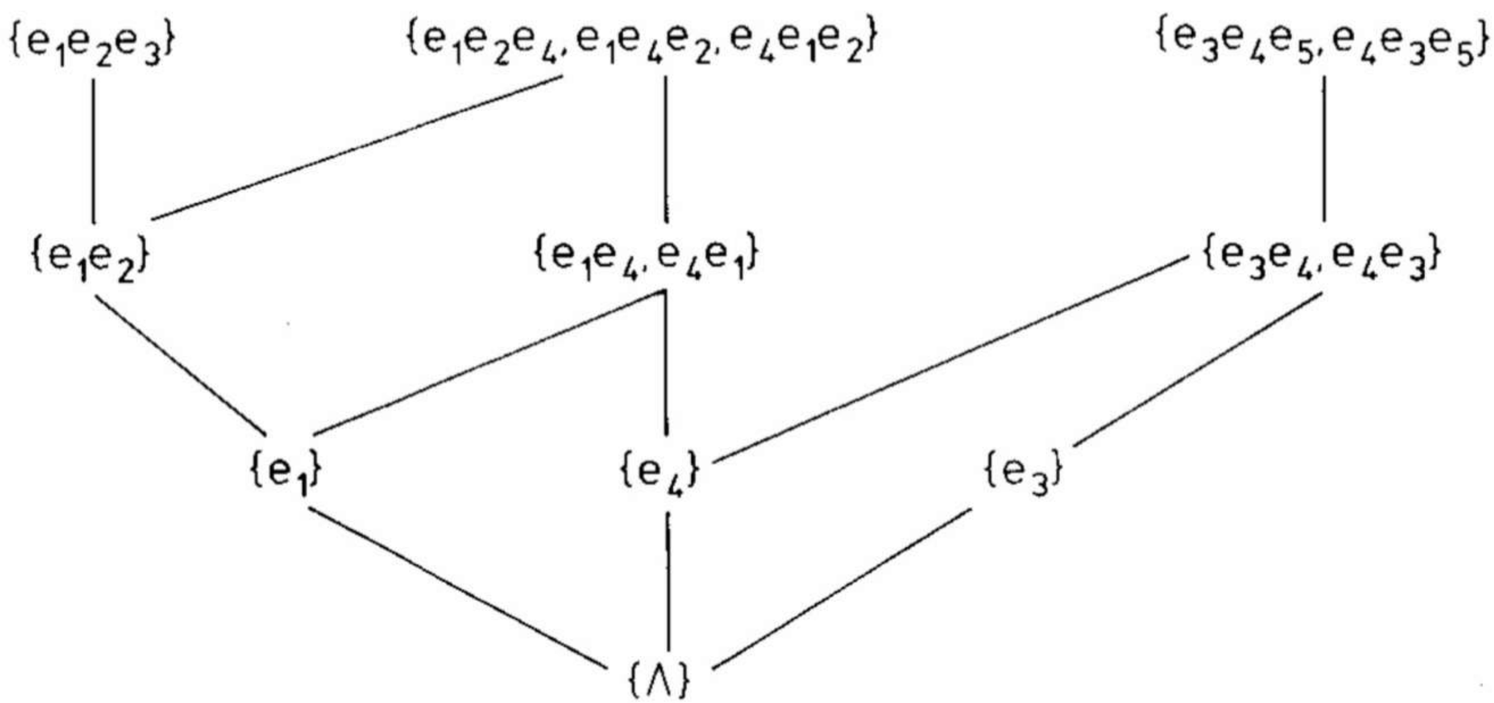
$c = \downarrow c = \{e \mid \exists e' \in c, e \leq e'\}$

$(c \times c) \cap \# = \phi$

$ES = (E, \leq, \#) \longrightarrow (C_{ES}, \subseteq)$

configuration

prefix closed, conflict free



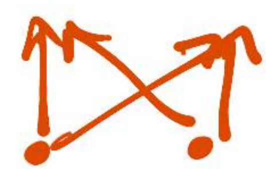
What can we say about $(C_{ES}, \subseteq)$

Has a nice p.o-theoretic characterization

Prime algebraic coherent complete partial order

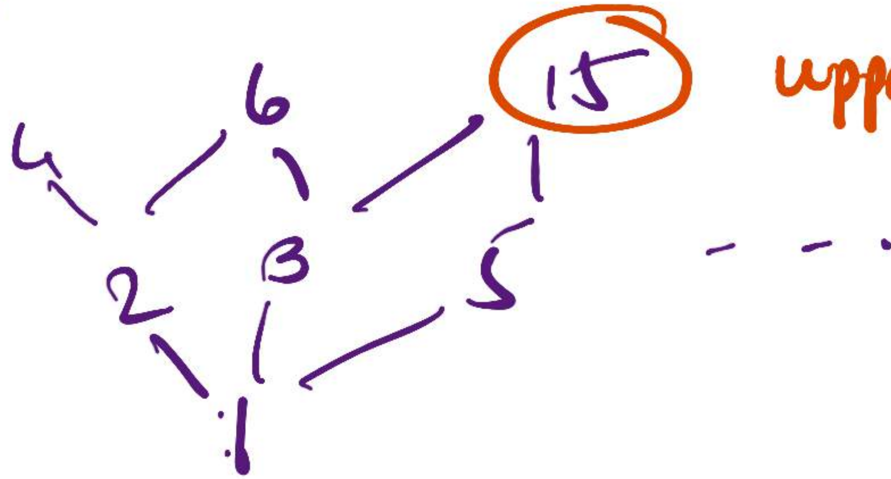
Complete partial order – all directed sets have an lub

Coherent – upper bound $\Rightarrow$ lub



X

## Divisibility

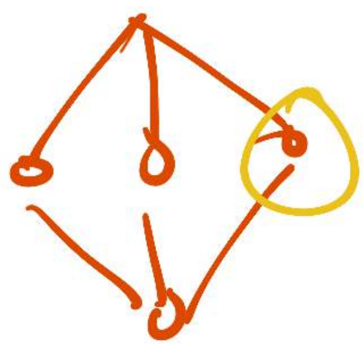



upper bound of the primes below it

## Prime element

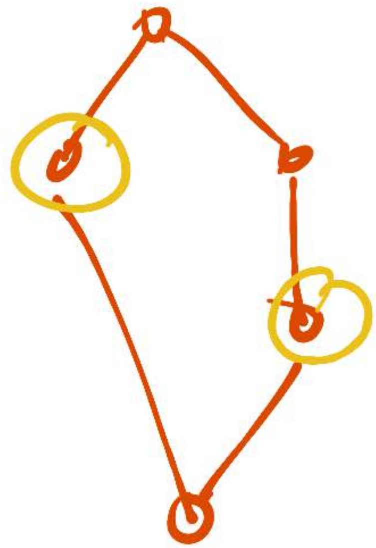
$$p \leq q \sqcup r \implies p \leq q \text{ or } p \leq r$$

Non example



not prim

Example



primes

**Want** every element to be the lub of the primes below it

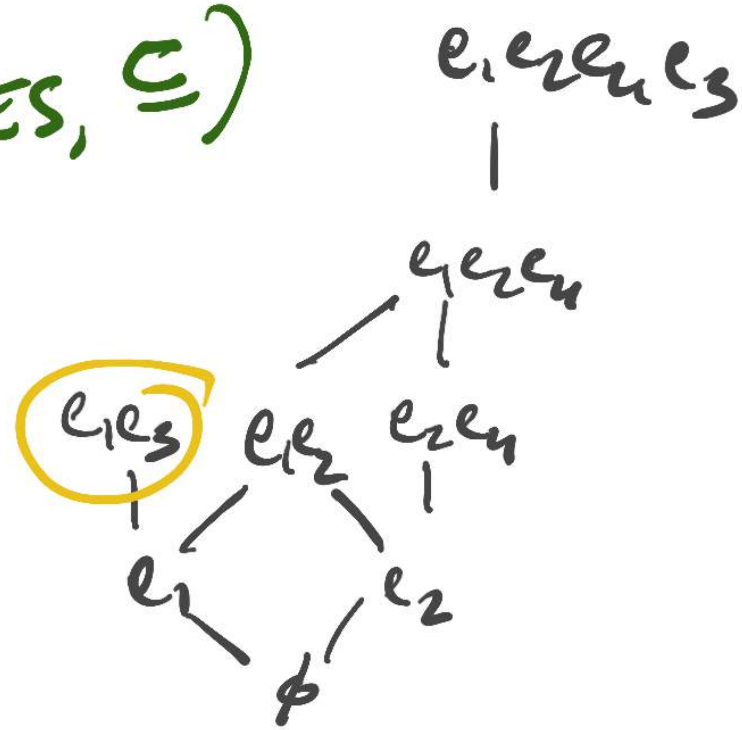
$(C_{ES}, \subseteq)$

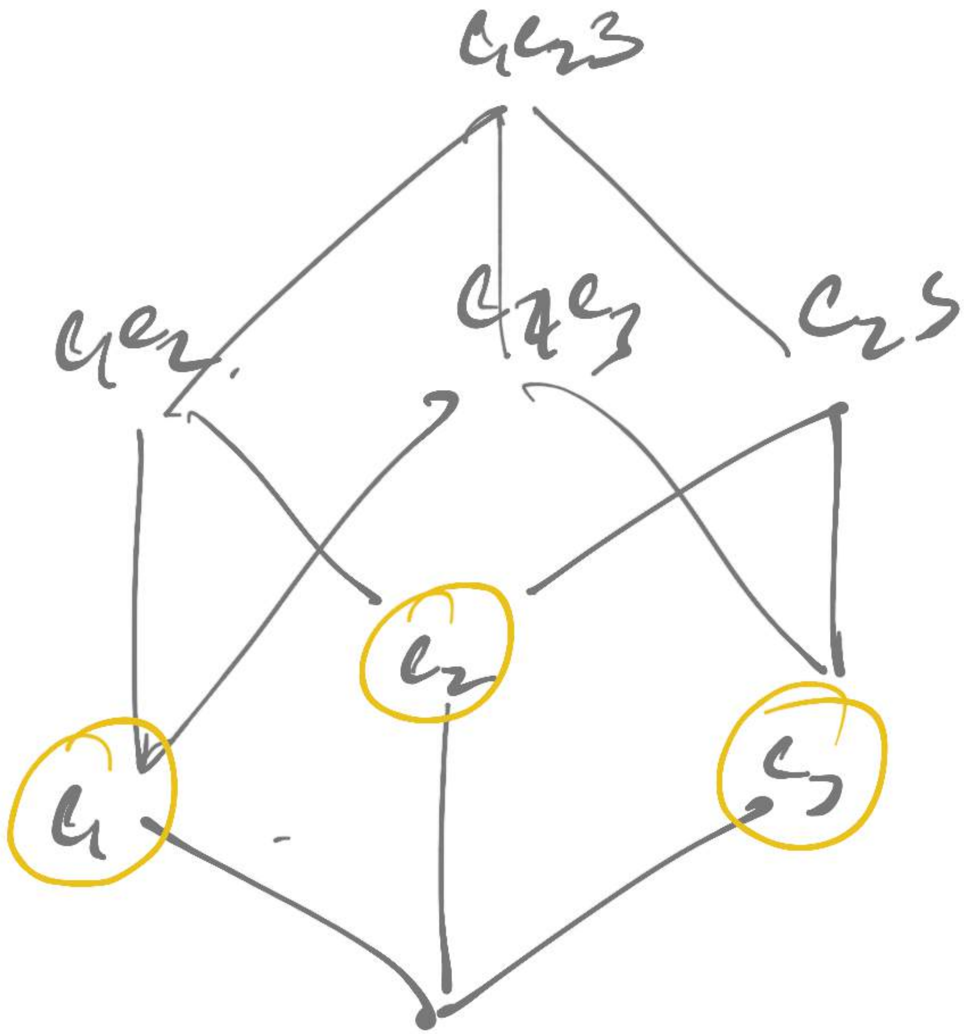
Prime elements are configurations of the form $\downarrow e$ for $e \in E$

$(E, \leq, \#) \longrightarrow (C_{ES}, \subseteq)$

$e_1 \rightarrow e_3$ $\quad \downarrow e_3$

$e_2 \rightarrow e_4$





c1c2c3
c1c2
c1c3
c2c3
c1
c2
c3

$(C_{ES}, \cup)$ is prime algebraic --

$E$ = primes of $(C_{ES}, \subseteq)$

$\leq \; = \; \subseteq$

$\#$ = no common future

$$\begin{tikzcd} E, \leq, \# \arrow[r, bend left] & (C_{ES}, \subseteq) \arrow[l, bend left] \end{tikzcd}$$

$(E, \leq, \#)$ are called <u>prime</u>

event structures

1-safe nets