Concurrency Theory, 30 Aug 2019

Language theory of Petri Nets

Given a regular language L over Σ^* is there an unlabelled net (i.e. $T = \Sigma$) with $L(N) = L$

Synthesis problem [Elementary Net Systems – no self loops]

Given $TS = (S, \rightarrow, s_{in})$ over Σ

is there an ENS $((P, T, F), M_{in})$

with $T = \Sigma$ s.t. TS isomorphic to $Reach(M_{in})$?

Given $T = \Sigma$

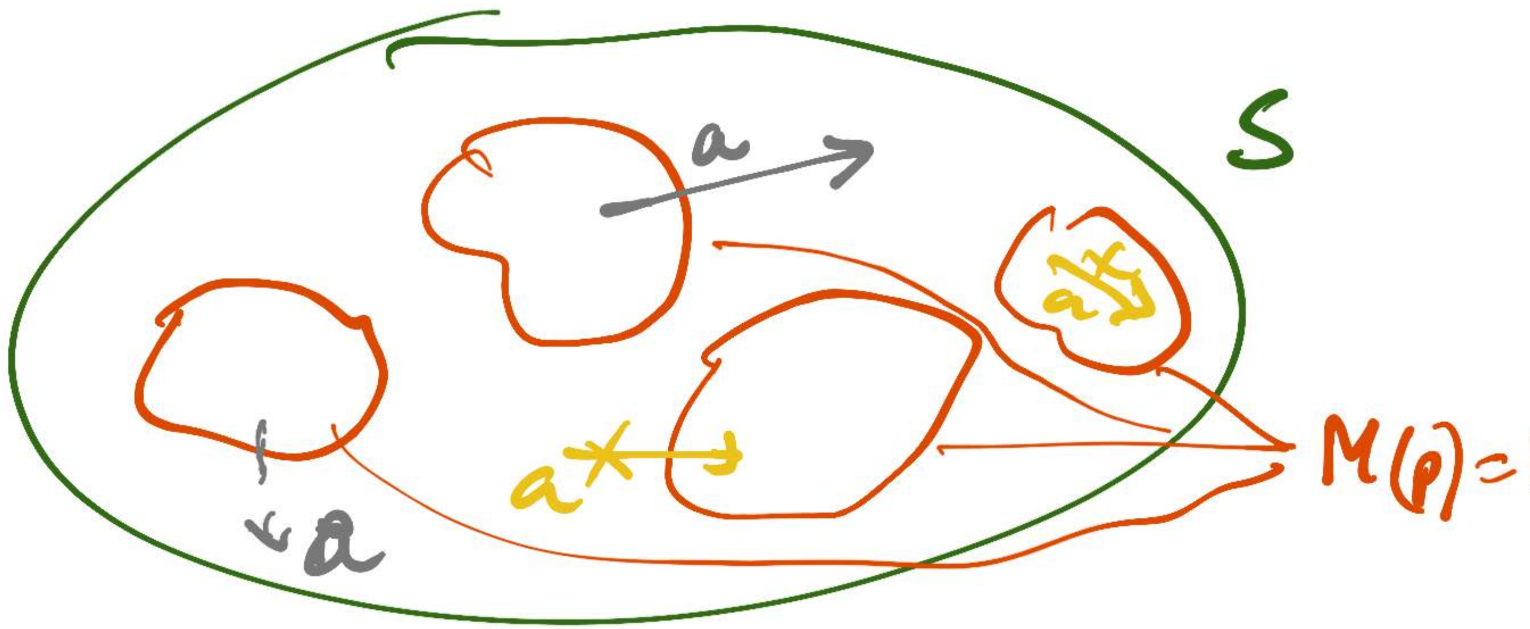
Identify P & F

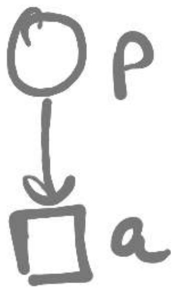
$p \in P$ is marked at some M & not marked at other

$p \longleftrightarrow S_p \subseteq S$, those states/markings where p is marked

$|S| = n$ — potentially $2^n - 1$ places (exclude place never marked)

When is a subset a valid candidate?





Then, consistently, all $\xrightarrow{a}$ arrows start in S_p & end outside

Symmetrically for incoming, non crossing

$R_p \subseteq S$ is a potential "place" if it is consistent wrt crossing pattern of transitions

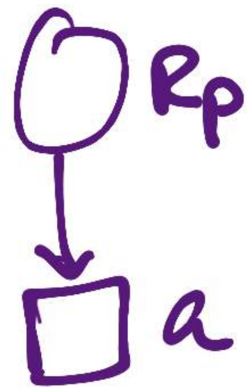
$\forall a$, Every edge $\xrightarrow{a}$ does one of the following

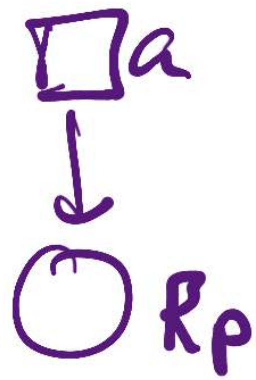
(1) Starts in R_p, ends outside

(2) Starts outside R_p, ends inside

(3) Does not cross an R_p boundary.

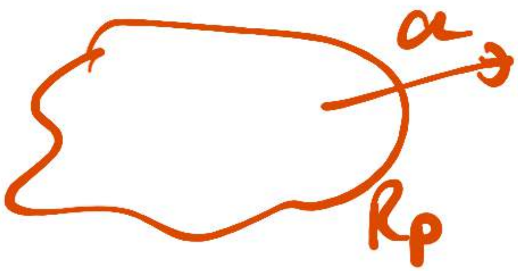
A valid candidate R_p is called a "region"

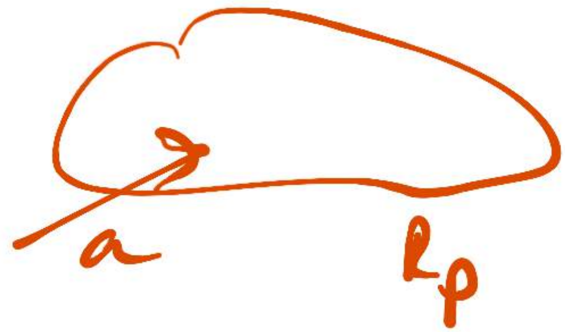




Derive

P, F





① If $s \neq s'$ in S, the "marking" differ

$\exists R_p$ s.t. $s \in R_p$, $s' \notin R_p$ or vice versa

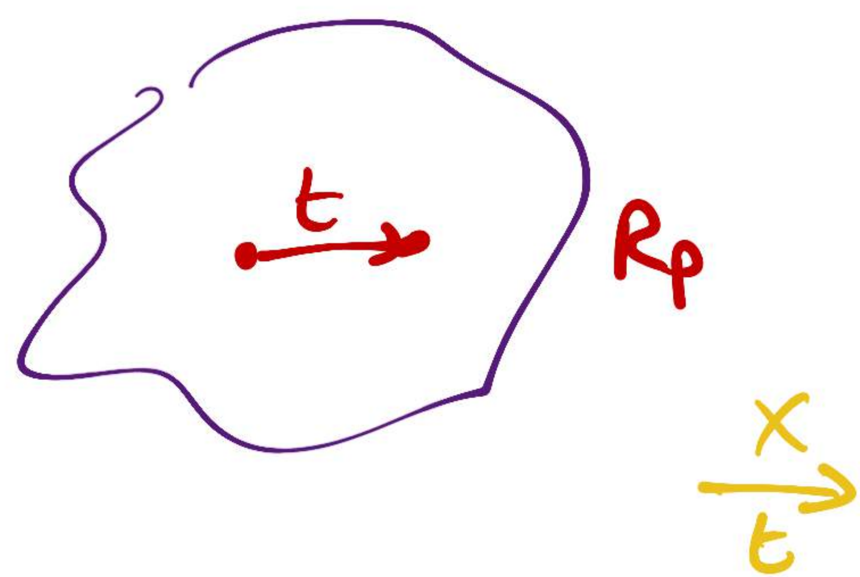
State-State Separate

(2) Wherever $s \overset{a}{\not\rightarrow}$

$\exists R_p \in {}^\bullet a$ s.t. $s \notin R_p$

State Event Separation

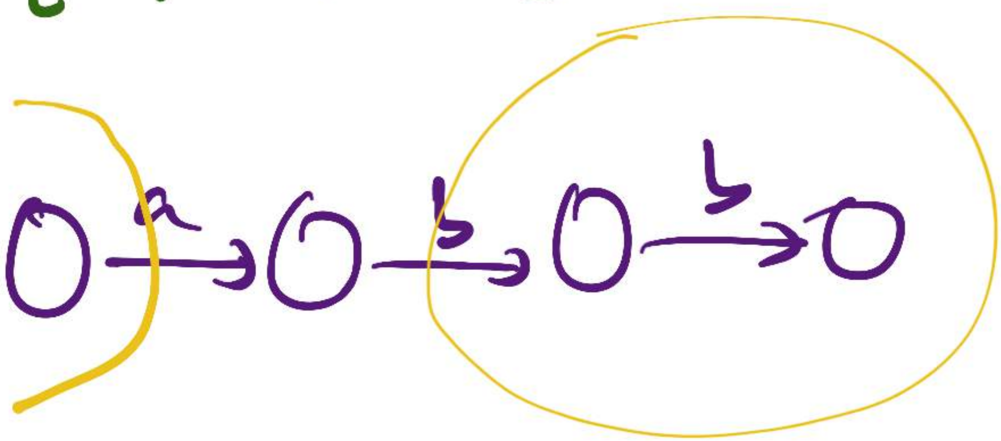
If we allow self loops

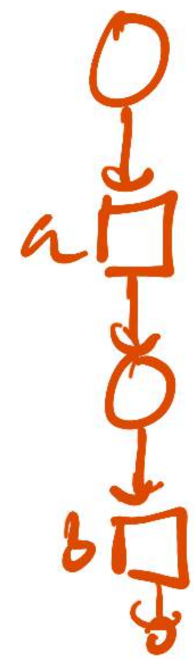


Thm TS is isomorphic to Reach(M_{in}) iff we can find regions to satisfy state-state & state-transition separation

Regular languages → Unlabelled Nets

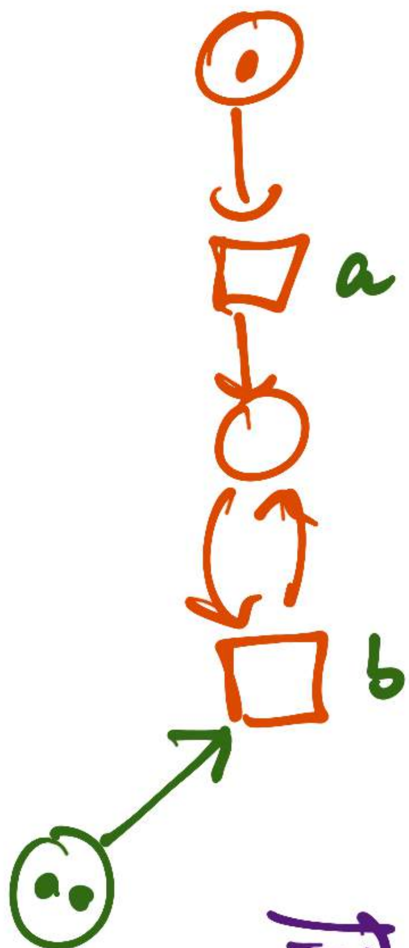
$L = \{a, ab, abb\}$





?

1-safe



ab^k

2-safe

$\Longrightarrow ab^2$

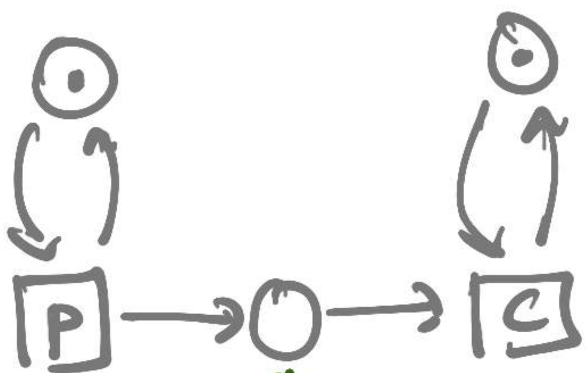
Allow k-safe nets, ab^2 is implementable

abbaa is not implementable

w is <u>solvable</u> if $L = \text{Prefixes}(w)$ can be implemented an an unlabelled safe net

No known algorithmic characterization

Outside safe nets



unsafe

$L(N) = \{w \mid$ Every prefix of w has at least as many p as c

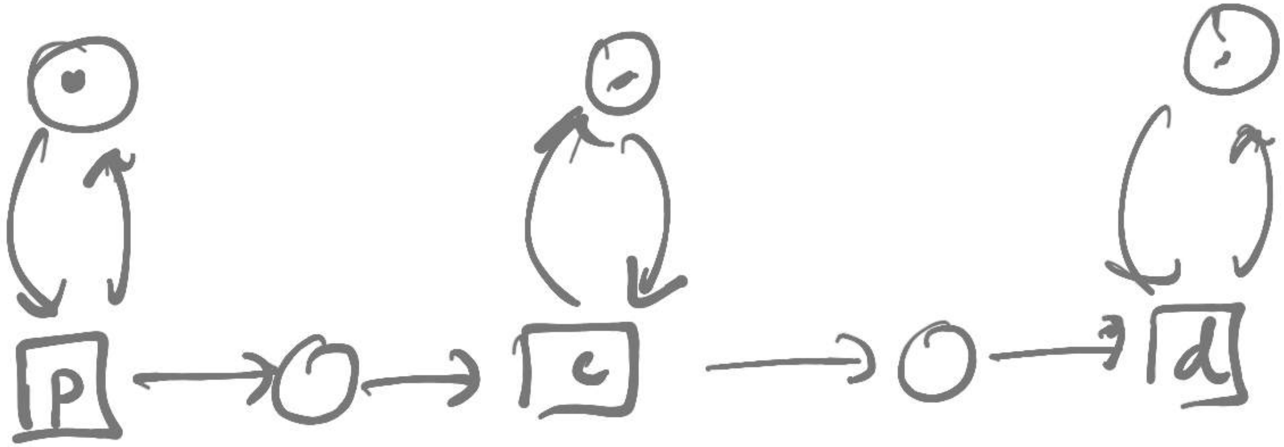
$$\{w \mid \forall v \leq w,\ \#_p(v) \geq \#_c(v)\}$$

$\{w \mid \#_a(w) = \#_b(w)\}$ is not regular

L(N) is also context free

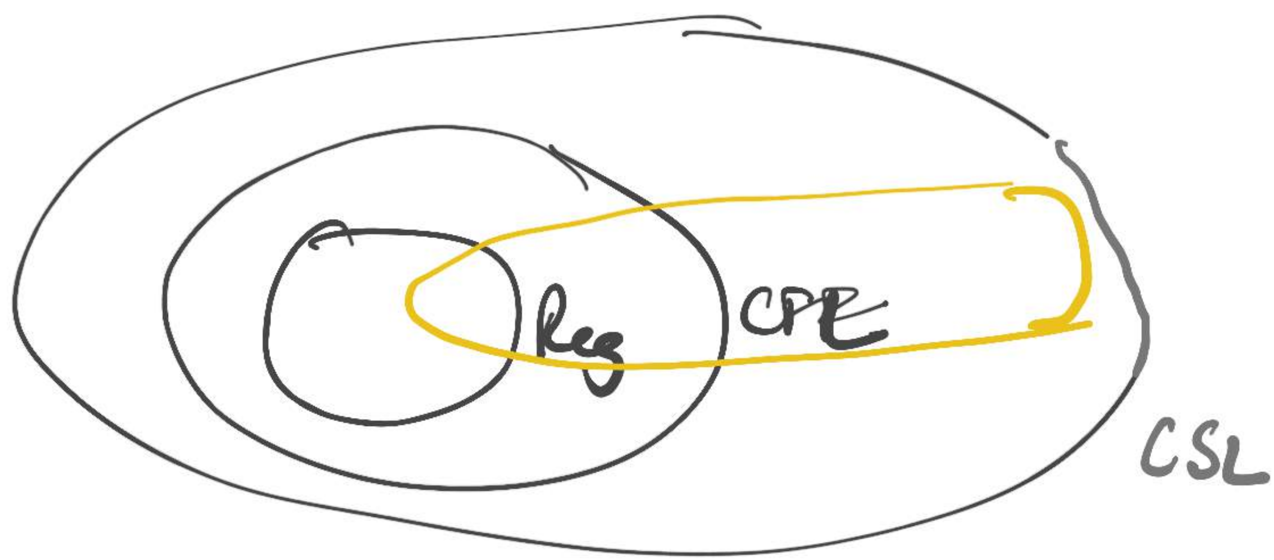
Palindromes are not Petri net languages

└ Combinatorial argument on no. of required materials

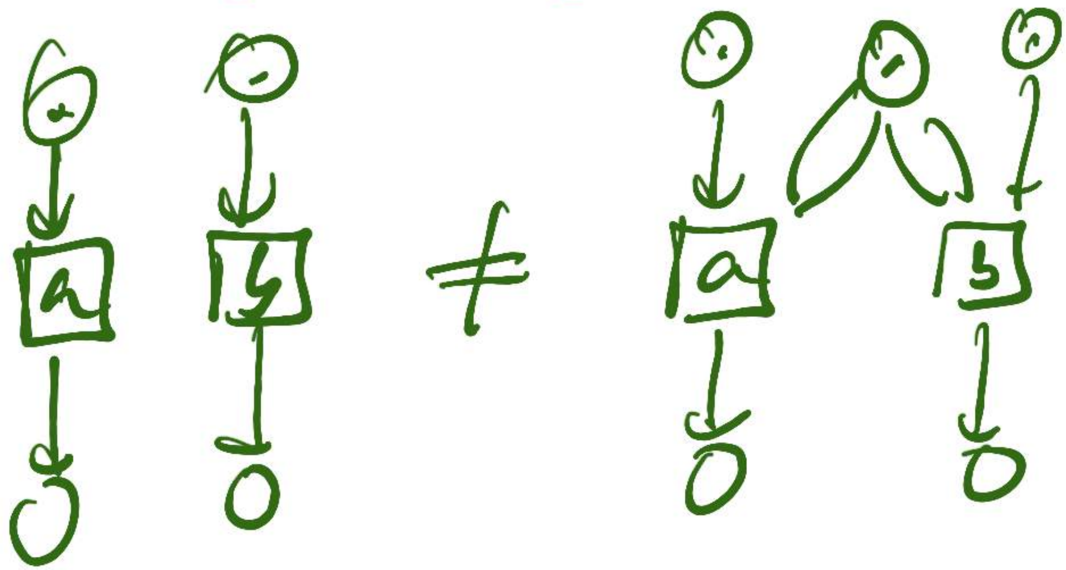


$$\#_p(w) \leq \#_c(w) \leq \#_d(w)$$

Context Sensitive



What is a good language theory for Petri nets?



Mazurkiewicz ~1977

(Σ, I) I is an indepence relation

$I \subseteq \Sigma \times \Sigma$

- Symmetric
- Irreflexive

Simplest interesting case

$$\left(\{a,b,c\},\ \{(a,b),(b,a)\}\right)$$

Some words

abc

acb

bac

bca

cab

cba

Are these the same computation?

No

cb vs bc

? $ab \equiv ba$, so Yes

same

$[\ \{abc, bac\},$
$\{acb\},$
$\{cab, cba\}$
$\{bca\}\ ]$

Formally $w_1 \dot{\sim} w_2$ (1 step equal)

if $w_1 = uabv$ and aIb

$w_2 = ubav$

$$a\underline{ab}b \,\dot{\sim}\, \underline{ab}\underline{ab} \,\dot{\sim}\, baab \,\dot{\sim}\, baba \,\dot{\sim}\, bbaa$$

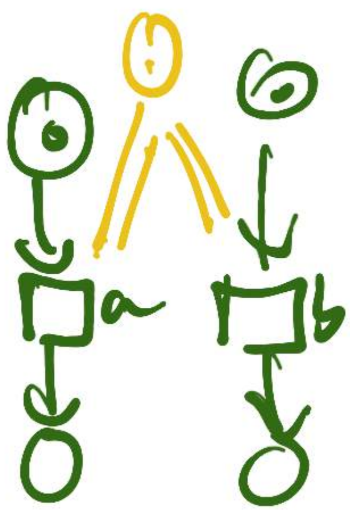
$w_1 \sim w_2$ if there is sequence $u_1, \ldots, u_n$ s.t

$$w_1 = u_1 \dot{\sim} u_2 \dot{\sim} u_3 \dot{\sim} \cdots \dot{\sim} u_n = w_2$$

A Mazurkiewicz trace is an equivalence class wrt $\approx$ ← wrt I

$$[abc]_I = \{abc, bac\} = [bac]$$

$[w]_I$ = trace containing w



One trace

$[ab]$

Two traces

$[ab], [ba]$

Better representation for a trace

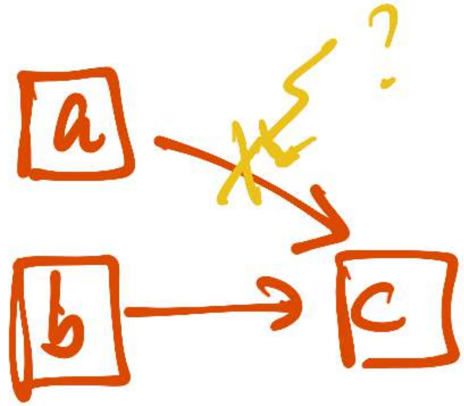
Partially ordered set

$$\left\{ \begin{array}{c} \boxed{a} \\[1em] \boxed{b} \end{array} \right\} \qquad \left\{ \begin{array}{l} \boxed{a} \longrightarrow \boxed{b}, \\[1em] \boxed{b} \longrightarrow \boxed{a} \end{array} \right\}$$

Trace $[w]$ is the set of all linearizations of the underlying p.o.

$(\{a,b,c\}, \{(a,b),(b,a)\})$

$\underline{\underline{abc}}$



bca ×

$w = a_0 a_1 a_2 \cdots a_n$

Represent as $(a_0,0),(a_1,1),\ldots,(a_n,n)$

Labelled Partial Order $(E, \leq, \lambda)$

$$\lambda : E \to \Sigma$$

$$|E| = |w|$$

$$E = \{(a, i)\}$$

$w = aabcab$

$(a,0)(a,1)\ (b,2)(c,3)\ (a,4)(b,5)$

$$\begin{tikzcd}
(a,0) \arrow[r] \arrow[drr] & (a,1) \arrow[dr] & & (a,4) \\
& & (c,3) \arrow[ur] \arrow[dr] & \\
(b,2) \arrow[urr] \arrow[rrr, bend right] & & & (b,5)
\end{tikzcd}$$

$(x, i) \to (y, j)$

$i < j$

$\neg (x\, I\, y)$

$(E, \leq, \lambda)$ represents a valid trace over (Σ, I)

if $\forall (x,y) \in E$

$$x \lessdot y \Rightarrow (\lambda(x), \lambda(y)) \notin I$$

immediat

$<$

$$(\lambda(x), \lambda(y)) \notin I \Rightarrow x \leq y \text{ or } y \leq x$$