Concurrency Theory, 28 Aug 2019

Extensions to the Petri net model

- Arc weights $\quad W: (P \times T) \cup (T \times P) \to \mathbb{N}_0$
- Place capacities $\quad K: P \to \mathbb{N}_0$
- Inhibitor arcs

$$(P, T, F, I) \qquad F \subseteq (P \times T) \cup (T \times P)$$

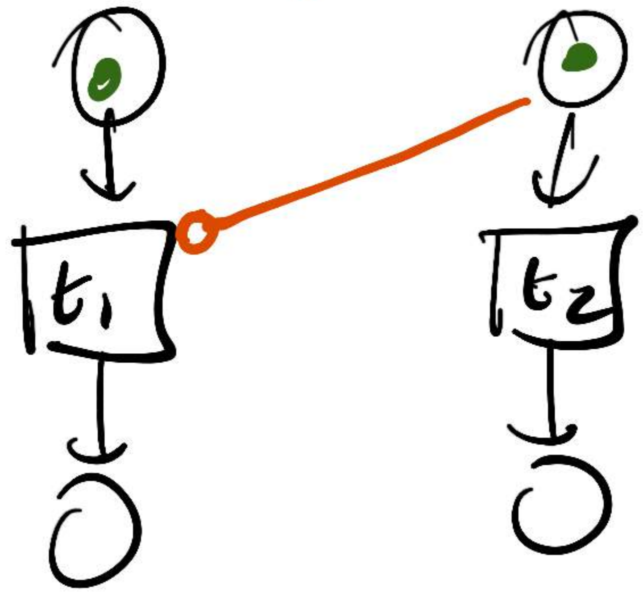
$$I \subseteq P \times T$$

$$M \xrightarrow{t}$$

1. $\forall p \in {}^{\bullet}t,\ M(p) \geq 1$
2. $\forall (p,t) \in I,\ M(p) = 0$

inhibitor arcs

Priority between independent transitions



If $M \xrightarrow{\{t_1, t_2\}}$ either order is possible

Here t_2 happens before t_1

Adding a token can disable a transition — monotonicity fails

Reachability is undecidable

- Places are very similar to counters
  - No possibility to do an action if counter is zero ("test for zero")
- Inhibitor arcs provide a zero test

2 counter Machines : c_1, c_2

$l_1 : i_1$
$l_2 : i_2$
$\vdots$
$l_k : i_k$

default control flow

Instruction

$c_j{+}{+}$

$c_j{-}{-}$ (blocks if 0)

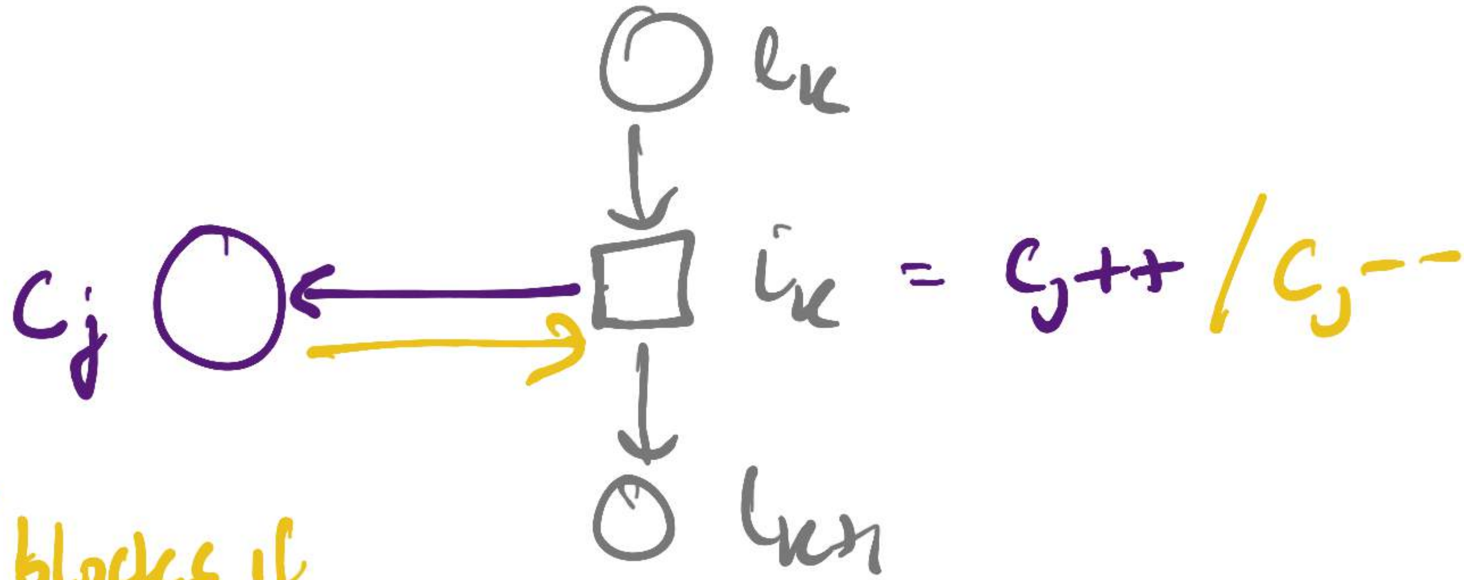
if $c_j == 0$ go to l_k

Theorem: Reachability of l_K is undecidable

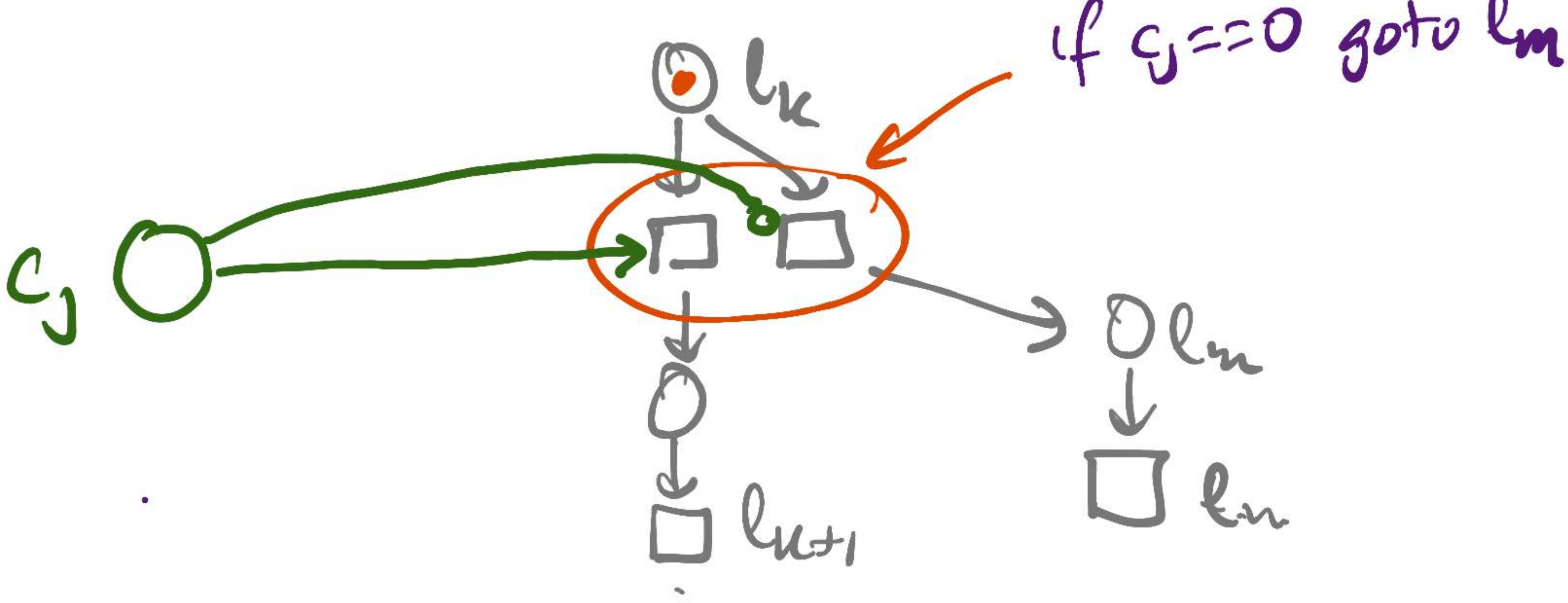
[Minsky] "Finite and Infinite Machines"

Model counters as places

c_j++



c_j-- (blocks if c_j==0)



Reduced reachability in 2 counter machines to reachability in Petri nets + Inhibitor Arcs

Other generalizations

Petri nets as vector addition systems

Nets encode exponentially large transition systems

Class of simple analysis techniques to avoid constructing Reach(Min) explicitly

Invariants

Place invariant - weighted sum of tokens in the net is invariant

$i : P \to \mathbb{Z}$ - invariant weight vector

$$\sum_{p \in P} i(p) \cdot m(p)$$ is invariant for all reachable m

$\forall t \in T$ $$\sum_{p \in {}^\bullet t} i(p) = \sum_{p \in t^\bullet} i(p)$$

$$M \xrightarrow{t} M'$$

$$M(p) - ({}^{\bullet}t) + (t^{\bullet})$$

<u>Example 1</u> If $\sum i(p) M(p) \neq \sum i(p) M_{in}(p)$

then $M \notin Reach(M_{in})$

<u>Example 2</u> If i is non-negative & $i(p_0) > 0$

then p_0 is bounded

$$i(p_0)\, m(p_0) \leq \sum_{p \in P} i(p)\, m(p)$$

$$\underline{\underline{\forall m}} \quad m(p_0) \leq \frac{1}{i(p_0)} \underline{\hspace{6em}}$$

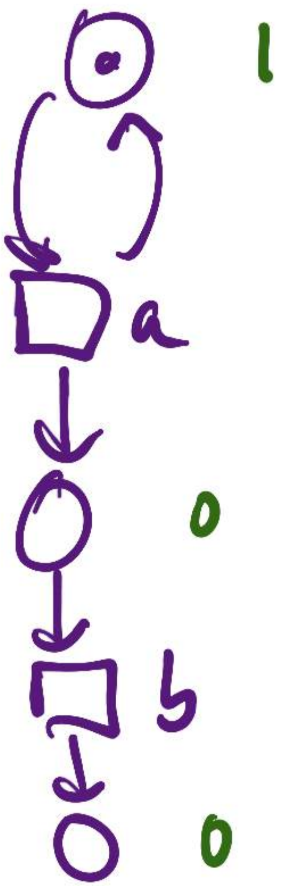
$$\Downarrow$$

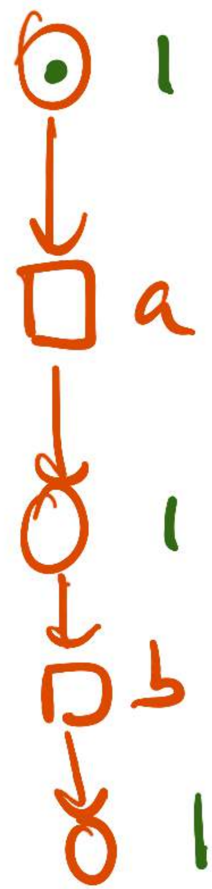
$$\sum_{p \in P} i(p)\, M_{in}(p)$$

Difficulty – finding a place invariant!

Example 3 If we have a strictly positive place invariant, the net is bounded

Finding invariant





Does a nontrivial (not all zero) place invariant always exist?

Transition invariant

$j : T \to \mathbb{N}_0$ → multiset of transitions

$$\forall p \in P \quad \sum_{t \in {}^\bullet p} j(t) = \sum_{t \in p^\bullet} j(t)$$

Suppose $M \xrightarrow{\sigma} M$ is a loop

$\text{Parikh vector}(\sigma) = (n_1, n_2, \ldots, n_m)$

is a transition invariant

Infinite behaviour of bounded net

$$\begin{tikzcd} M_{in} \arrow[r, squiggly, "\sigma_{fin}"] & M \arrow[loop above, "\sigma_\omega"] \end{tikzcd}$$

σ_ω — T_ω

$$T_{fin} = T \setminus T_\omega$$

$$T = T_{fin} \uplus T_\omega$$

Trans inv J: $T_{fin} \downarrow 0$, $T_\omega \downarrow\ > 0$

If N is live, there is a loop where $\sigma_\omega = T$

Languages ?

Regular $\subsetneq$ CFL --

$\mathcal{N} = (N, M_{in})$, $N = (P, T, F)$

Obvious language is over T

(Prefix closed)

Unlabelled net

Add $\lambda : T \to \Sigma$

Σ denotes "observations"

Typically avoid auto-concurrency

t independent of t', $\lambda(t) = \lambda(t')$

$$L(\mathcal{N}) \subseteq T^* \rightsquigarrow \lambda(L(\mathcal{N})) \subseteq \Sigma^*$$

1 (N, M_{in}) is bounded — underlying finite state TS

$$M_{in} \xrightarrow{t_1} M_1 \; \cdots \quad M \xrightarrow{t_1} M' \; \cdots$$

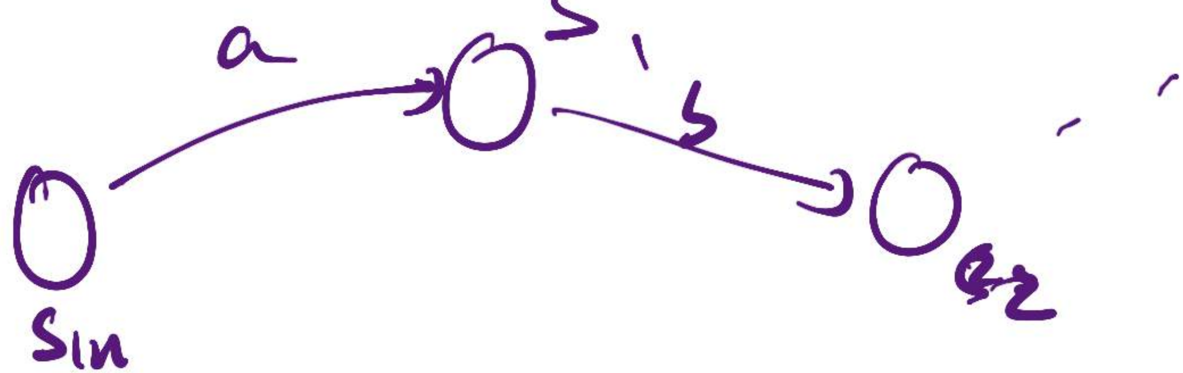
$L(\mathcal{N})$ is a regular language, also $\lambda(L(\mathcal{N}))$ for any λ

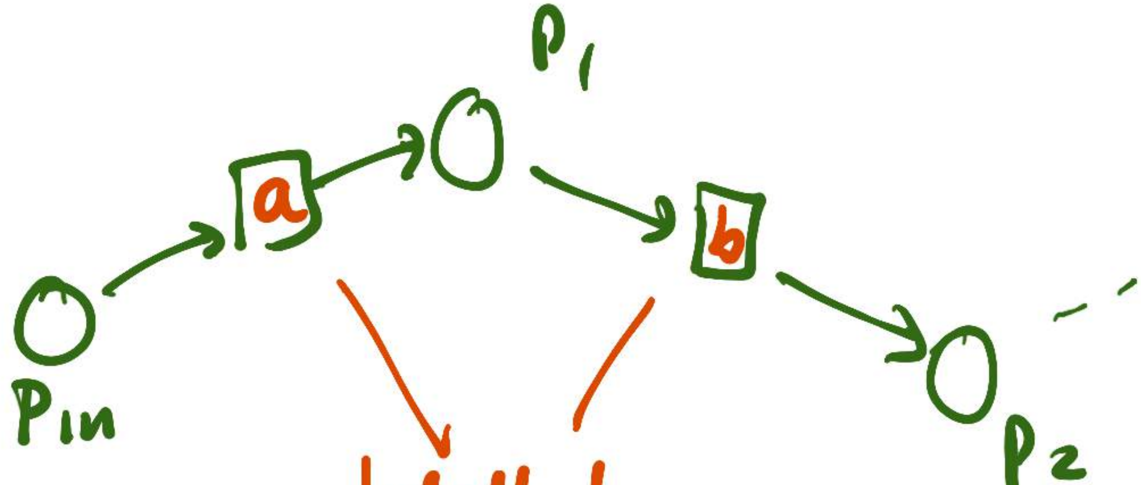
② Given a regular language $L \subseteq \Sigma^*$ (prefix closed)

↓

every state is accepting

Have a DFA/NFA for L





labelled

Can we construct N with $T = \Sigma$?

(unlabelled net recognizing L)

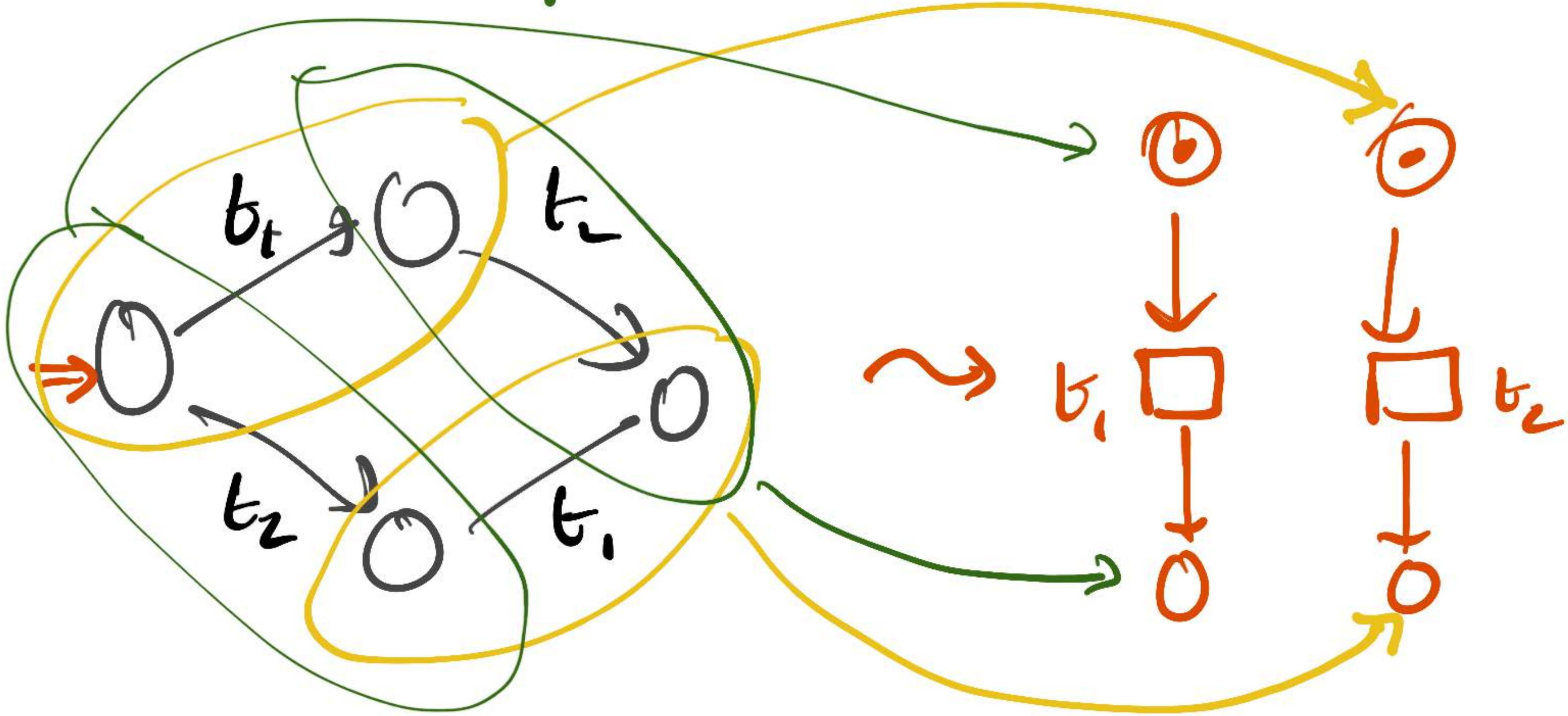
Single word (prefix closed) is trivially regular

<u>Min DFA</u> $w = abbaa$

Unlabelled net over $T = \{a, b\}$ for $L = \{abbaa\}$

Reach(Min) must be isomorphic to the min DFA.

Separately Reconstructing nets from transition systems



A place p $\iff$ Markings where $M(p) = 1$

Consistent subsets — "Regions"