Concurrency Theory, 21 Aug 2019

Concurrency, Conflict & Causality (ENS)

Concurrency

$$
\begin{tikzcd}
 & M_1 \arrow[dr, "t_2"] & \\
M \arrow[ur, "t_1"] \arrow[dr, "t_2"'] & & M_{12} \\
 & M_2 \arrow[ur, "t_1"'] &
\end{tikzcd}
$$

$$M \xrightarrow{\{t_1, t_2\}}$$

$M \xrightarrow{U}$ $\quad U \subseteq T$.

$\forall t \in U \quad M \xrightarrow{t}$

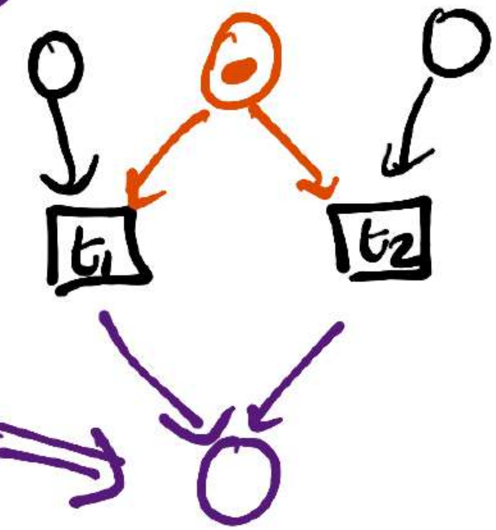
$\forall t_1, t_2 \in U, \quad {}^\bullet t_1^\bullet \cap {}^\bullet t_2^\bullet$

Conflict

$M \xrightarrow{t_1}$, $M \xrightarrow{t_2}$, $M \overset{\{t_1, t_2\}}{\not\longrightarrow}$

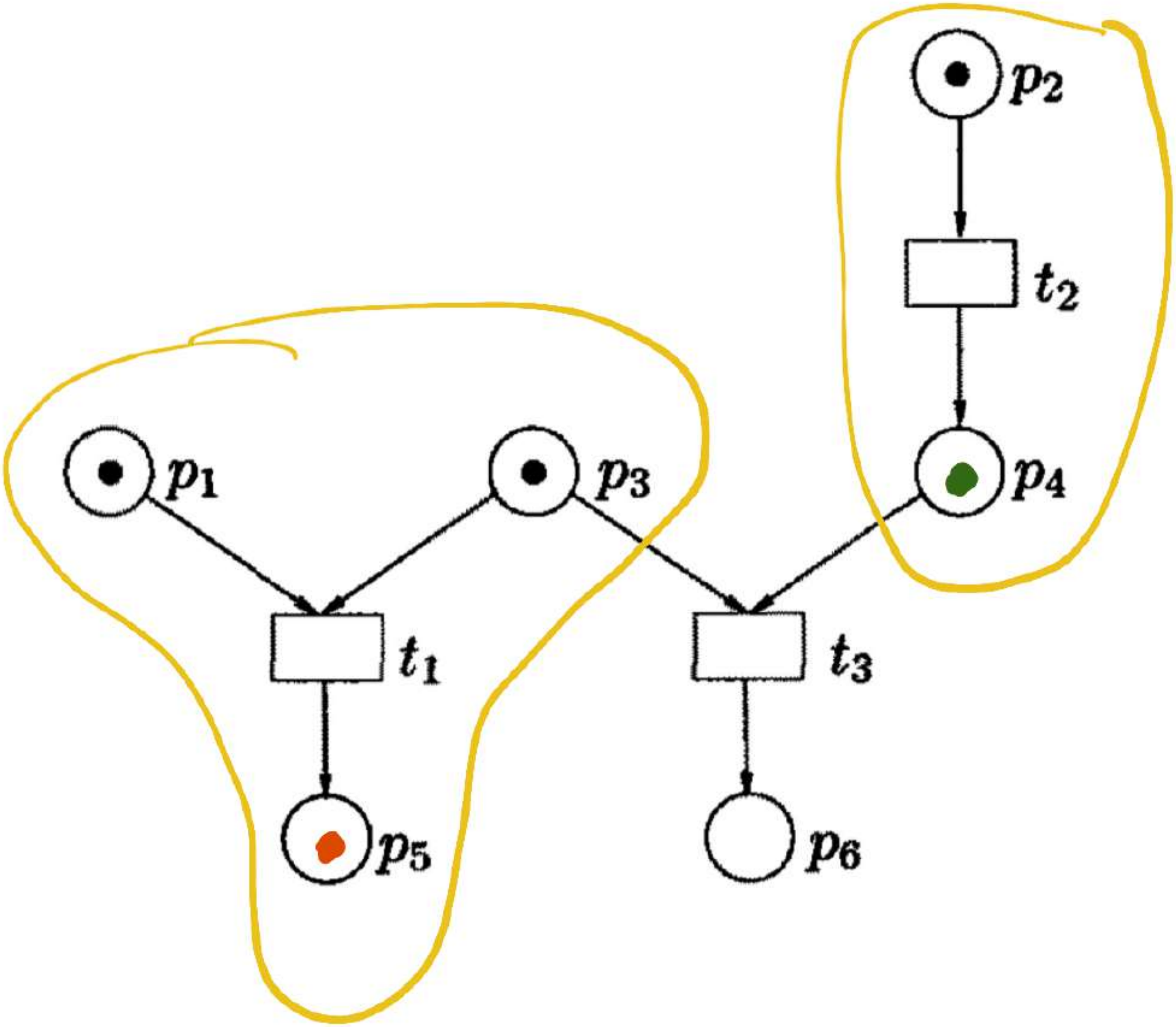
Usually input conflict

ENS – also output conflict $\Longrightarrow$



Causality

$$M \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_{12} \quad \& \quad t_1^{\bullet} \cap {}^{\bullet}t_2 \neq \phi$$



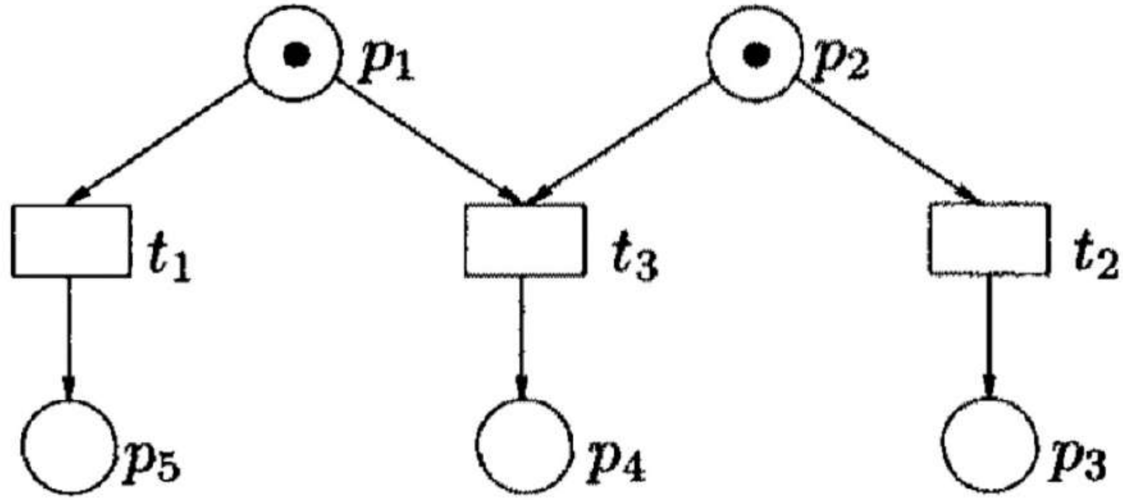
$M \xrightarrow{\{t_1, t_2\}}$

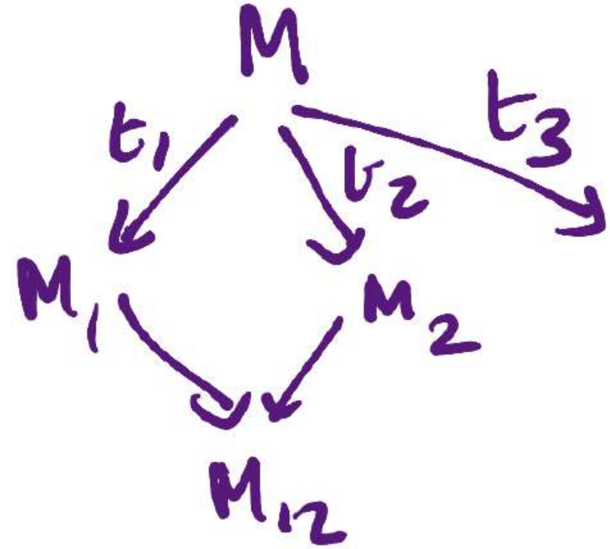
$$\begin{tikzcd}
 & M \arrow[dl, "t_1"'] \arrow[dr, "t_2"] & & \\
M_1 \arrow[dr, "t_2"'] & & M \arrow[dl, "t_1"'] \arrow[dr, "t_3"] & \\
 & M_{12} & & M
\end{tikzcd}$$

$M \xrightarrow{\{t_1, t_2\}} M_{12}$

"CONFUSION"

Did we resolve a conflict between t_1 & t_3?





Conflict wrt t_1 (or t_2) <u>disappears</u>

<u>Confusion</u> Conflict changes due to the occurrence of an independent transition

$$\text{ConflictSet}(M, t) = \{ t' \mid M \xrightarrow{t'}, \ M \not\xrightarrow{\{t, t'\}} \}$$

where $M \xrightarrow{t}$

$\text{Confusion}(M, t_1, t_2)$

$M \xrightarrow{\{t_1, t_2\}}$

$M \xrightarrow{t_2} M_2$ and $\text{ConflictSet}(M, t_1)$

$\neq \text{ConflictSet}(M_2, t_1)$

<u>Example 1</u> $\text{Confusion}(M, t_1, t_2)$ but not ← ASYMMETRIC

$\text{Confusion}(M, t_2, t_1)$

SYMMETRIC

<u>Example 2</u> $\text{Confusion}(M, t_1, t_2)$ & $\text{Confusion}(M, t_2, t_1)$

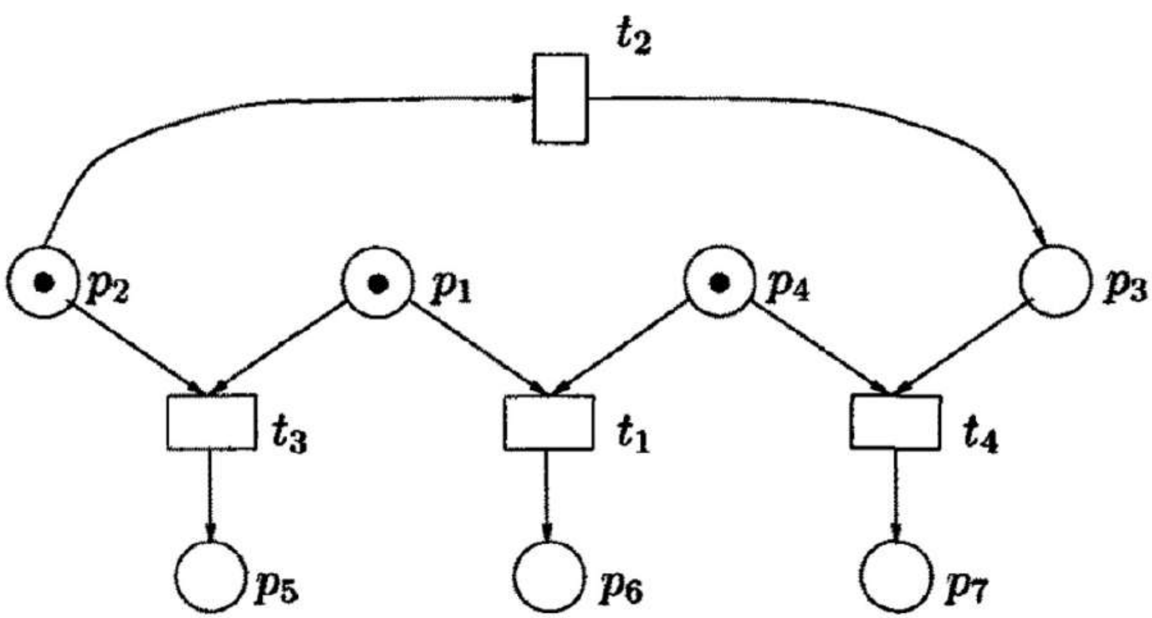
Classification 1 Asymmetric vs Symmetric

Classification 2 Increasing vs Decreasing

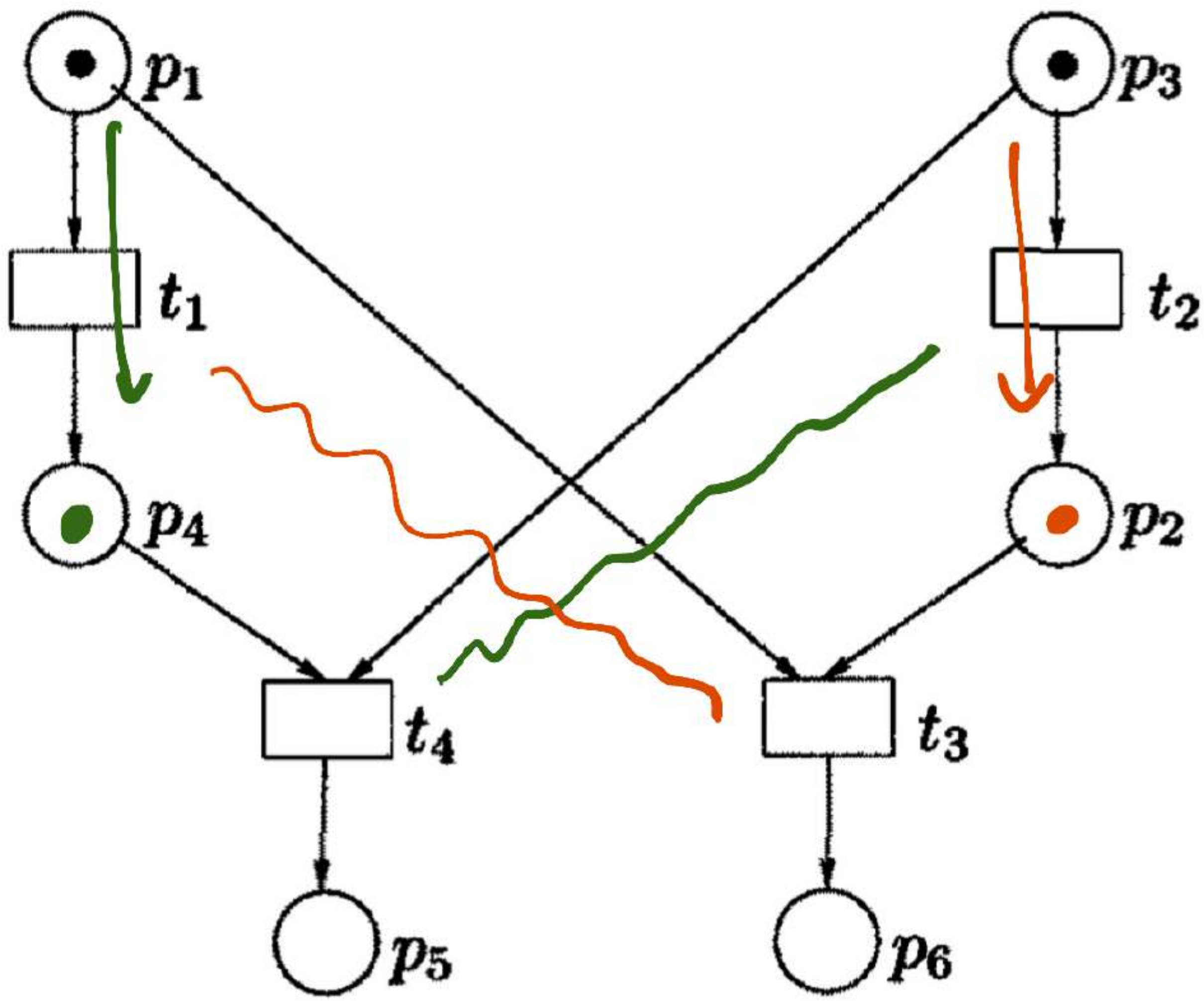
$\Downarrow$

ConflictSet (M, t_1) grows

ConflictSet (M, t_2) shrinks



Neither increasing nor decreasing



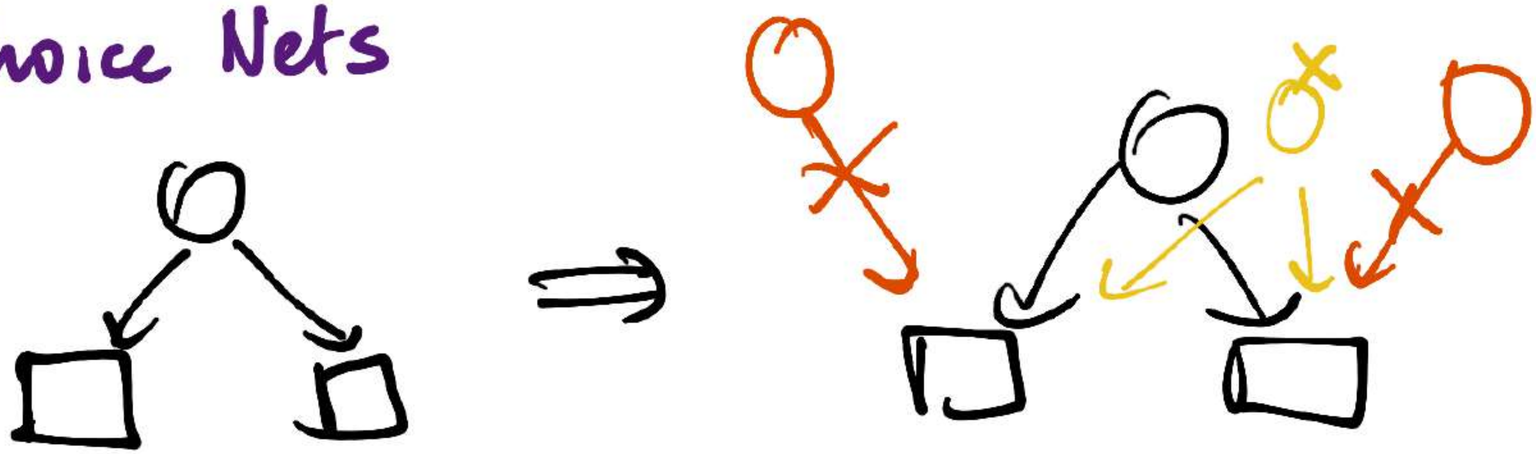
Symmetric but increasing

Theory of Petri Nets suggests that confusion is the source of complexity

Confusion free system admits more efficient algorithms for reachability etc

Syntactic class that guarantees confusion free behaviour

Free Choice Nets



If $p \in {}^\bullet t_1 \cap {}^\bullet t_2$ then ${}^\bullet t_1 = {}^\bullet t_2 = \{p\}$

In particular, our mutual exclusion example was <u>not</u> free choice

Free Choice $\Rightarrow$ Any Min generates confusion free net

Converse is not true

Back to Petri Nets

Bounded Net – k-safe, in every reachable marking at most k tokens on a place

t is dead at M if t is never enabled in Reach(M)

(N, M_{in}) is live if no transition is dead anywhere in Reach(M_{in})

Lot of early work in net theory was to characterize live and safe markings

1-safe

"Structure theory" of nets

Hack's Theorem – characterizes this for free choice nets

More "useful" questions

1. Is (N, M_{in}) k-safe?

2. Is M reachable from M_{in}? – Can both critical sections be enabled together

Is reachability decidable for unbounded nets?

But it is — E Mayr & Rao Kosaraju

~1972

Complexity?

EXPSPACE-hard (Lipton)

↓

Non primitive recursive upper bound (Schmitz, Leroux)

↓ 2018

Non elementary lower bound ✓

Not bounded by a finite tower of exponentials

For infinite state systems, there are only two ways to prove decidability

Petri Net reachability

CFLs

A marking is a vector

A transition is a change vector

$M \xrightarrow{t}$ Resulting marking is $M+t$

Effect of a sequence

$$M \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \cdots \xrightarrow{t_k} M_k$$

$$M + t_1 + t_2 \cdots + t_k$$

Suppose $T = \{\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_m\}$

Each $\hat{t}_i$ occurs n_i times in $t_1 t_2 \ldots t_k$

$$M + \sum_{i=1}^{m} n_i \hat{t}_i$$

Can compute effect of $M \xrightarrow{t_1 t_2 \cdots t_k}$ disregarding the order

Converse is not true

Vector Addition Systems

Linear algebraic analysis

No intermediate marking can have a negative

Coverability

$M \leq M'$ if $\forall p.\ M(p) \leq M'(p)$

Given M_{in}, M is there $M' \in Reach(M_{in})$

s.t $M' \geq M$ [M' covers M]

For verification, this often suffices

e.g. is critical section simultaneously enabled

Monotonicity

$M \xrightarrow{t}$ & $M' \geq M$ then $M' \xrightarrow{t}$

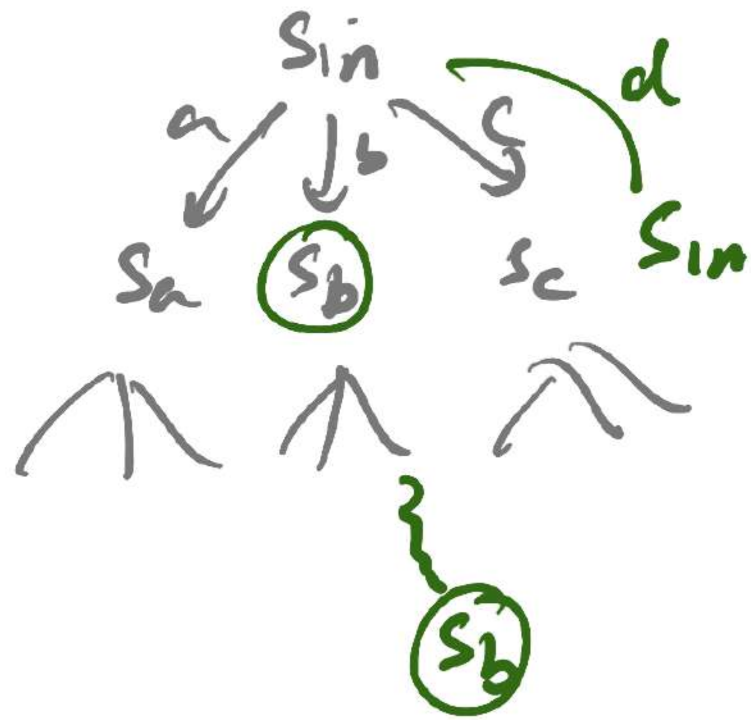
$M \xrightarrow{t} M_1$ & $M' \geq M$, $M' \xrightarrow{t} M_1'$, $M_1' \geq M_1$

Automata & reachability

a) Graph reachability

b) Unfold into a tree

BFS



Apply same pruning to tree generated by an (unbounded) Petri net.