Concurrency Theory, 16 Aug 2019

Net $N = (P, T, F)$ $F \subseteq (P \times T) \cup (T \times P)$

For $t \in T$, ${}^\bullet t = \{p \mid (p,t) \in F\}$ "predot"

$t^\bullet = \{p \mid (t,p) \in F\}$ "postdot"

Equivalently ${}^\bullet p$, $p^\bullet$

Marking $M : P \to \mathbb{N}_0$ – global state

Firing rule : t is enabled at M, $M \xrightarrow{t}$

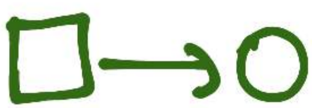
$M(p) \geq 1$ for all $p \in {}^\bullet t$

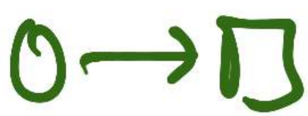
$M \xrightarrow{t} M'$

$$
\begin{aligned}
M'(p) &= M(p) - 1 \quad \text{if } p \in {}^\bullet t \setminus t^\bullet \\
&= M(p) + 1 \quad \text{if } p \in t^\bullet \setminus {}^\bullet t \\
&= M(p) \quad \text{otherwise}
\end{aligned}
$$

Typically, assume

${}^\bullet t,\ t^\bullet \neq \phi$

 always enabled!

 tokens disappear

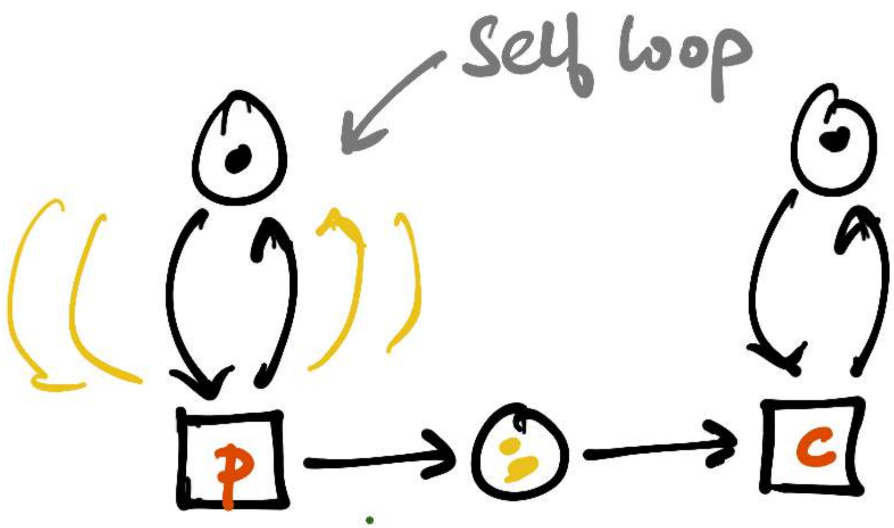
$N = (P, T, F)$ is just a graph

Net system is a net with an initial marking

$$NS = (N, M_{in})$$

Typically no final markings

Producer-Consumer



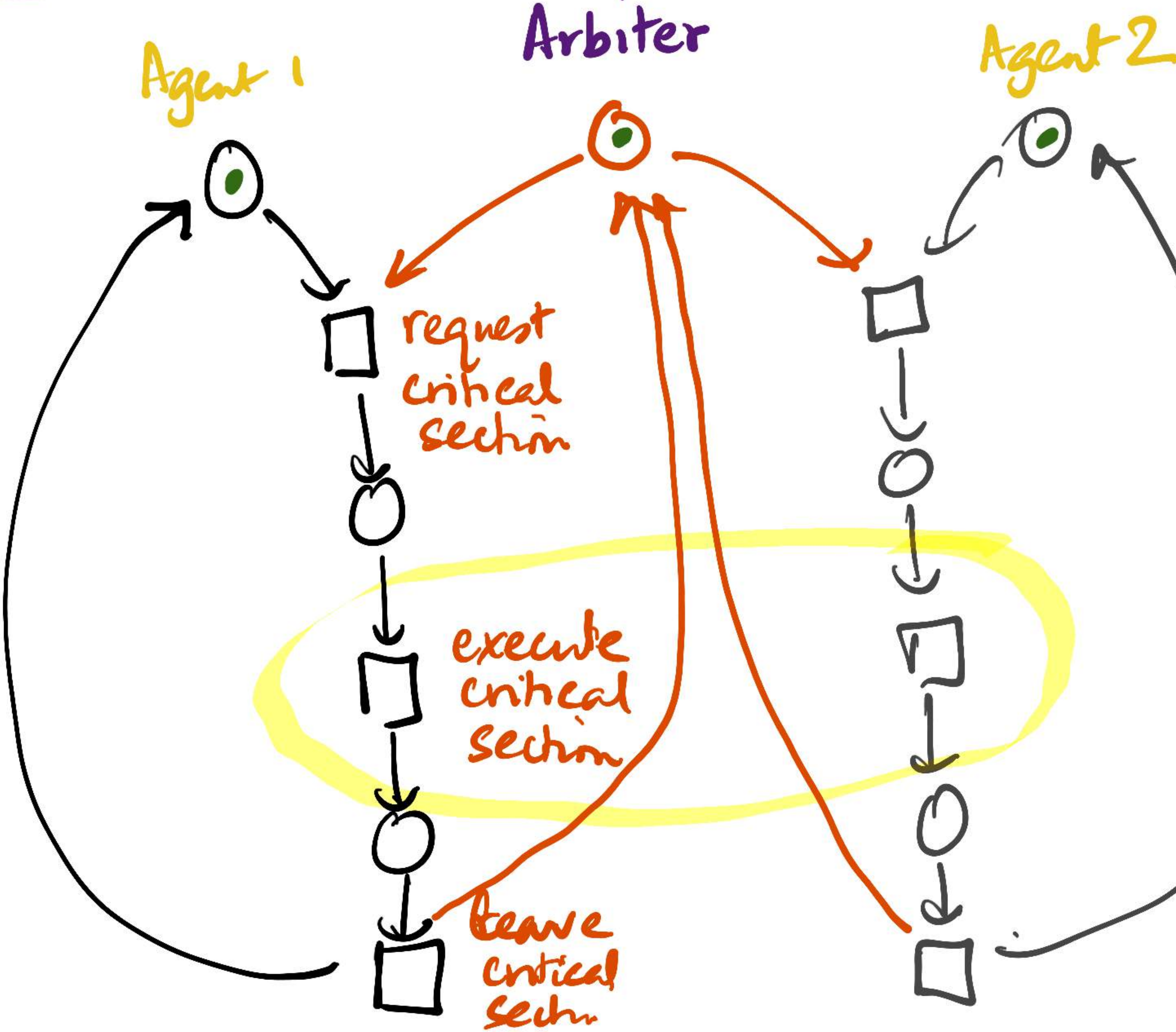
pcpcpc...

ppccpcpc...

$\#p \geq \#c$

Infinite state - states = markings

Mutual Exclusion



State Space of $NS = (N = (P, T, F), M_{in})$

Define set of reachable states (markings)

$Reach(M_{in})$ smallest set of markings s.t

- $M_{in} \in Reach(M_{in})$
- If $M \in Reach(M_{in})$, $M \xrightarrow{t} M'$

$\Rightarrow M' \in Reach(M_{in})$

$$\begin{tikzcd} & M_1 \arrow[r, "t_2"] & M_2 \\ M_{in} \arrow[ur, "t_1"] \arrow[dr, "t_3"'] & & \\ & M_3 \arrow[r, "t_1"] & \cdots \end{tikzcd}$$

Transition system $TS = ((S, \to, s_{in}), \Sigma)$

labelled

$\to \subseteq S \times S$

or $\to \subseteq S \times \Sigma \times S$

<u>LTS</u>

LTS for (N, M_{in})

$$((\mathrm{Reach}(M_{in}), \rightarrow, M_{in}), T)$$

LTS for (N, M_{in}) could have infinitely many states

Finite state nets – every place has an upper bound on the no. of tokens it can have

k-safe : $\forall M \in Reach(M_{in}),\ \forall p \in P,$

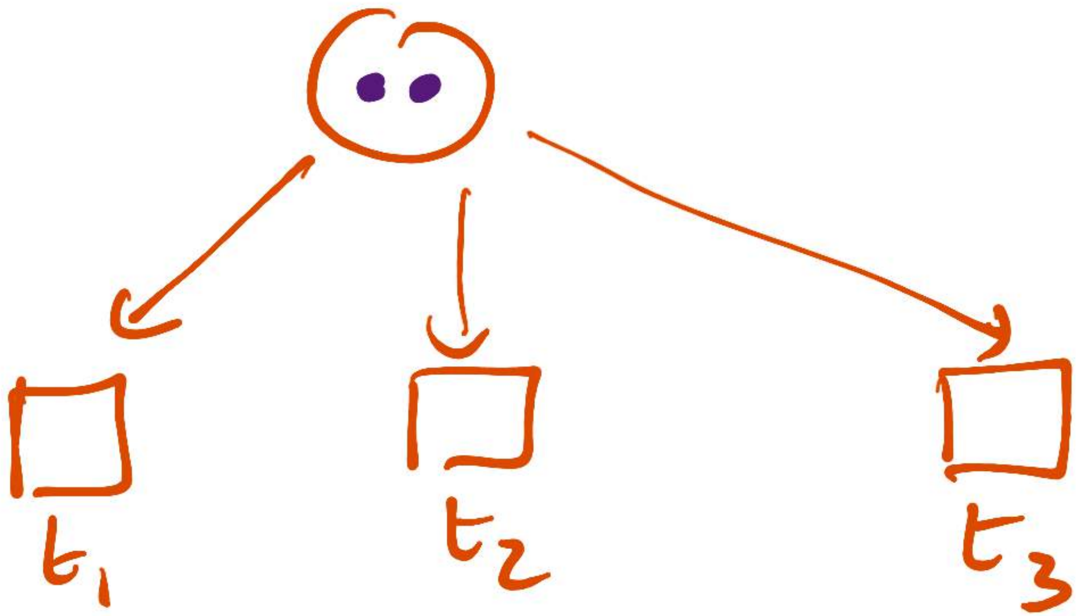
$$M(p) \leq k$$

Trivial : $Reach(M_{in})$ is finite iff (N, M_{in}) is k-safe for some k

1-safe (just **safe**)

Effectively $M : P \to \{0,1\}$

Why allow k-safe, k > 1 ?



2 out of 3 can fire

"There are two printers on the network & 3 users"

Elementary Net Systems

1-safe by definition

$N = (P, T, F)$

Marking: $M : P \to \{0, 1\}$

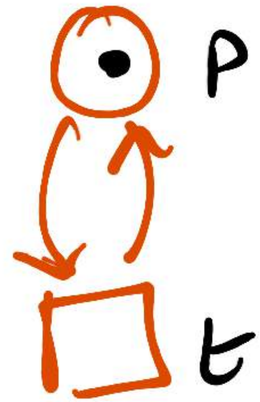
equivalently $M \subseteq P$

Change firing rule

t is enabled at M if $^\bullet t \subseteq M$

and $t^\bullet \cap M = \phi$

Self loops can never fire



$t^\bullet \cap M \neq \phi$

Fundamental relations between actions

Choice / Conflict

Independence / Concurrency

Formally Given M, t_1, t_2 s.t.

$M \xrightarrow{t_1}$ and $M \xrightarrow{t_2}$

- t_1 & t_2 are independent at M if

$$({}^\bullet t_1 \cup t_1^\bullet) \cap ({}^\bullet t_2 \cup t_2^\bullet) = \phi$$

$$(\text{and } {}^\bullet t_1 \subseteq M \text{ and } {}^\bullet t_2 \subseteq M)$$

- t_1 & t_2 are in conflict at M otherwise

Causality

$$M \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2$$

t_2 is causally dependent on t_1

if $t_1^{\bullet} \cap {}^{\bullet}t_2 \neq \phi$

Can we extract this information from the reachability graph?

Whenever t_1 & t_2 are independent

$$\forall M. \quad M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \implies \begin{tikzcd} M_0 \arrow[r, "t_1"] \arrow[dr, "t_2"'] & M_1 \arrow[r, "t_2"] & M_2 \\ & M_1' \arrow[ur, "t_1"'] & \end{tikzcd}$$

"Sideways Diamond"

$$\forall M. \quad \begin{tikzcd} & M_1 \\ M_0 \arrow[ur, "t_1"] \arrow[dr, "t_2"'] & \\ & M_2 \end{tikzcd} \implies \begin{tikzcd} & M_1 \arrow[dr, "t_2"] & \\ M_0 \arrow[ur, "t_1"] \arrow[dr, "t_2"'] & & M_3 \\ & M_2 \arrow[ur, "t_1"'] & \end{tikzcd}$$

"Forward Diamond"

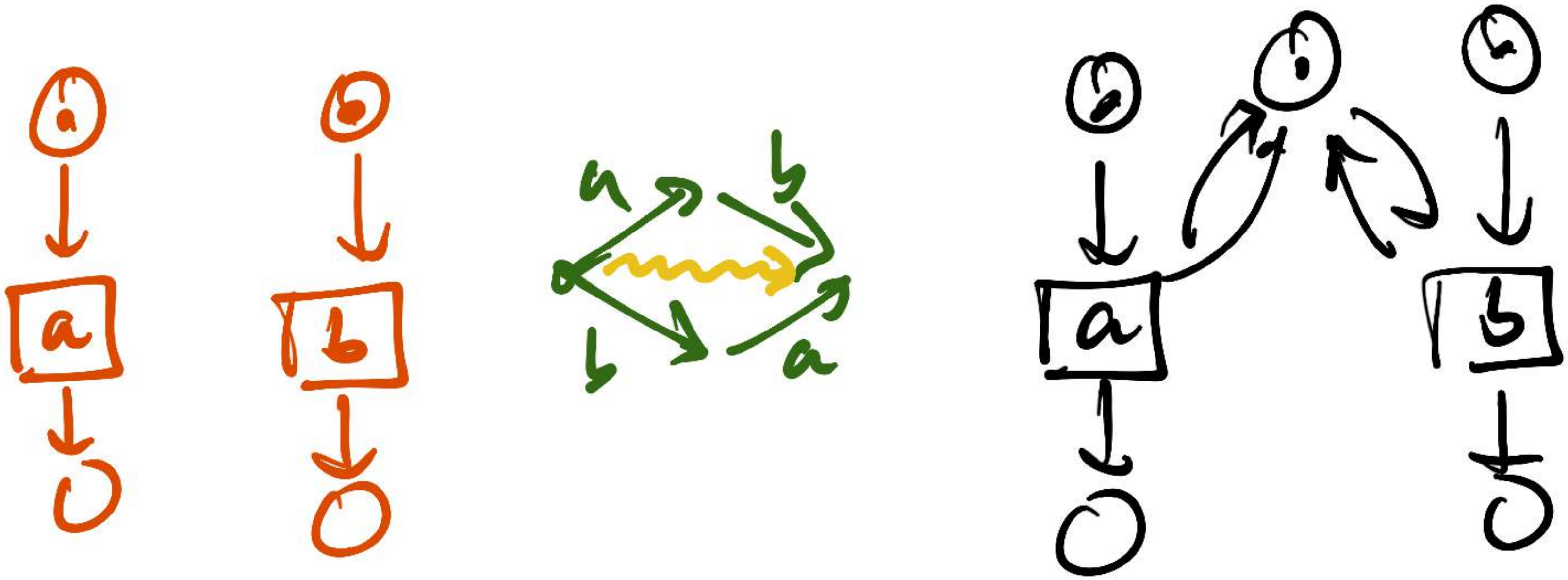
In an ENS, converse is true:

If Reach(M_{in}) contains

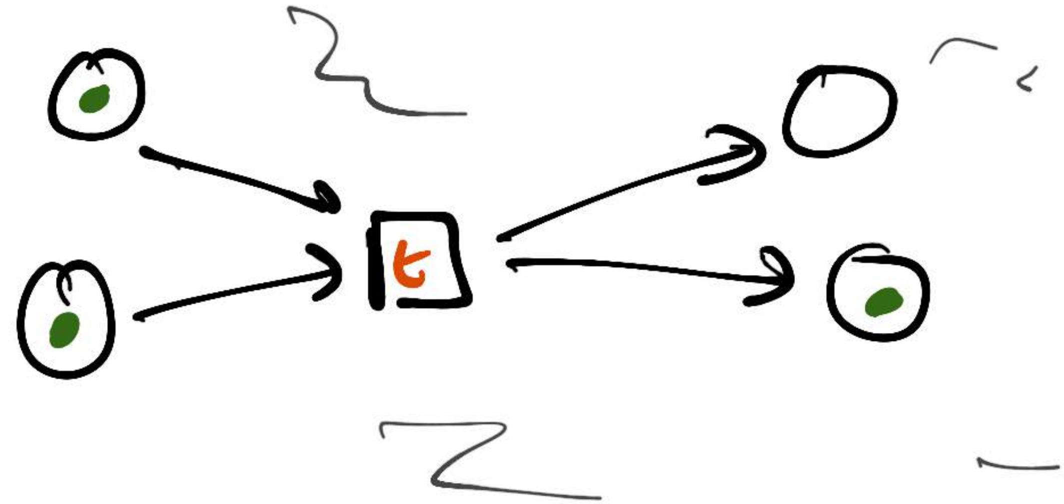
$$
\begin{tikzcd}
 & M_1 \arrow[dr, "t_2"] & \\
M_0 \arrow[ur, "t_1"] \arrow[dr, "t_2"'] & & M_3 \\
 & M_2 \arrow[ur, "t_1"'] &
\end{tikzcd}
$$

then t_1 & t_2 <u>must</u> be independent

This is not true in general.



In ENS



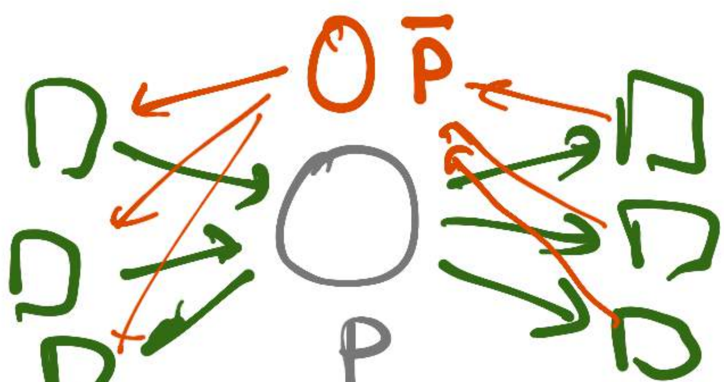
"Contact"

${}^\bullet t \subseteq M$

but $t^\bullet \cap M \neq \emptyset$

Construct a "complementary" place for each $p \in P$

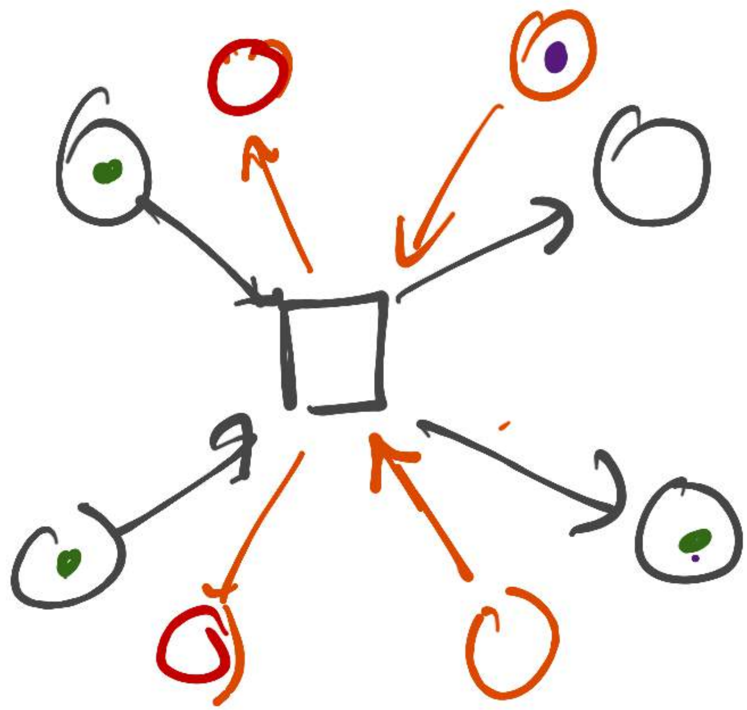
Reverse connections wrt ${}^\bullet p \cup p^\bullet$



$\overline{M_{in}(p)} = M_{in}(\bar{p})$

Start with $((P,T,F), M_{in})$

Saturate with complementary places



Check that — No contact!

— Isomorphic Reach(M_{in})

Self loop