Concurrency Theory, 14 Aug 2019

Why concurrency?

Parallel vs Distributed vs Concurrent

For efficiency
Under centralized control

Physical Necessity
Decentralized

"parallel"
"Simultaneous"
"Sharing"
"Independence"

Coordination

STRUCTURE

BEHAVIOUR

Concurrency can exist *without* explicit parallelism

- Multi-user system

This course

Formal theories of concurrent systems

Automata, Pushdown, Turing Machines — sequential

How to incorporate concurrency into a formal model

When are two finite state automata equivalent?

Language equivalence

Concurrency / Interaction

$$a(b+c) \stackrel{?}{=} ab + ac \quad ✓$$

a = Join queue

b = Counter 1

c = Counter 2

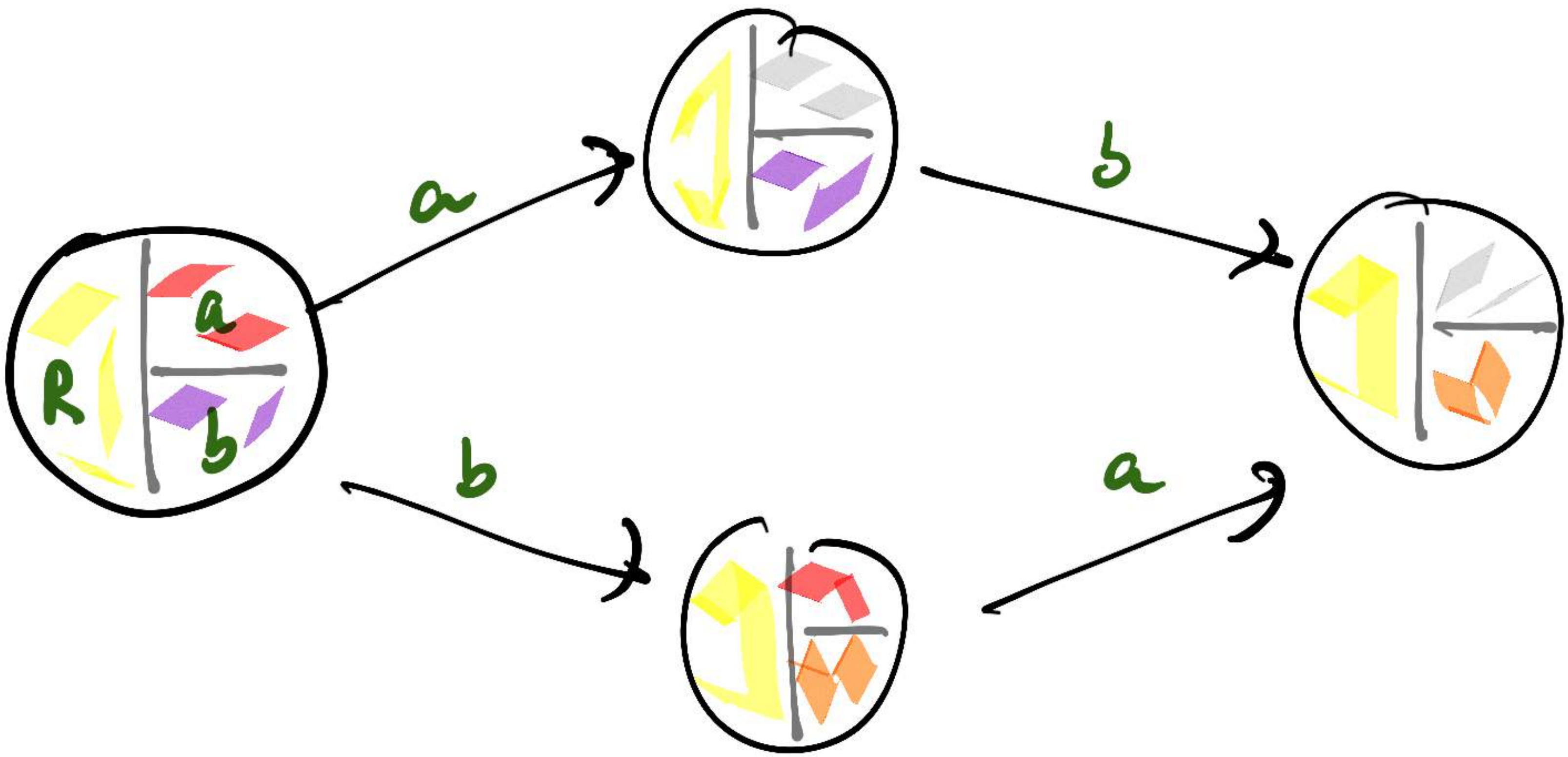
LHS $\neq$ RHS

1960 Carl Adam Petri

a & b happen independently at a state

$$\begin{tikzcd}
 & s_a \arrow[dr, "b"] & \\
s \arrow[ur, "a"] \arrow[dr, "b"'] & & s_{ab} \\
 & s_b \arrow[ur, "a"'] &
\end{tikzcd}$$

a, b affect disjoint parts of S



Decompose states into smaller units

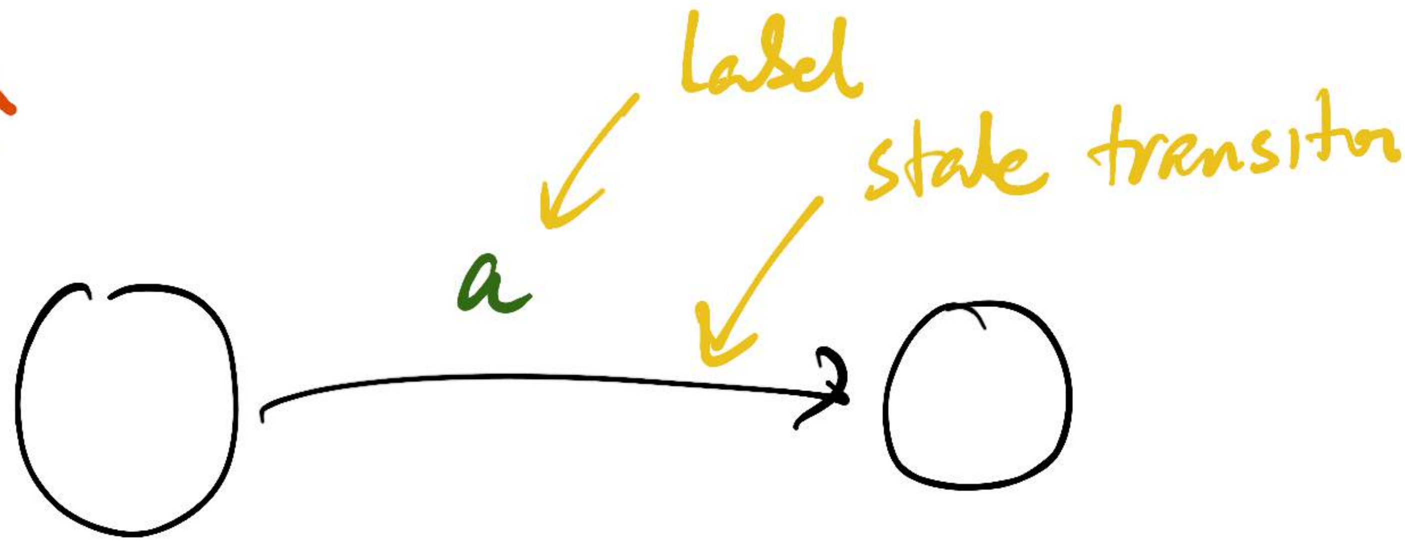
Each action updates some subset of units

Petri's Nets $\longrightarrow$ Petri Nets

"Local" states = "Places"

"Distributed" actions = "Transitions"

Automaton



Net = (P, T, F)

P - Places $\longrightarrow$ ○

T - Transitions $\longrightarrow$ □

F = Flow relation

$$(P \times T) \cup (T \times P)$$

Bipartite Graph

State – Which places are "active"

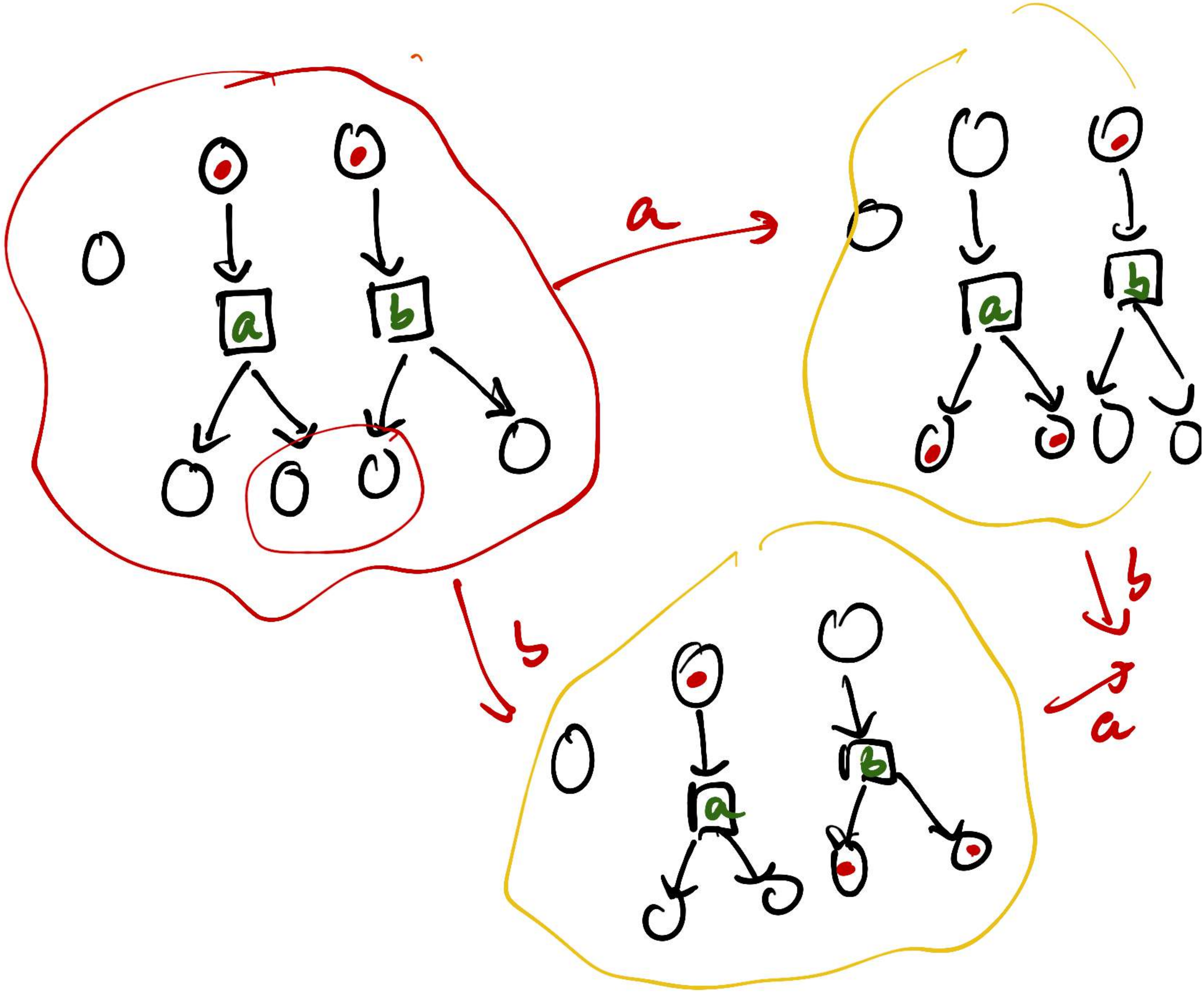
"Mark" them with a "token"

Marking: $M : P \longrightarrow \mathbb{N}_0$

Represent by dots

Transitions consume input tokens

and produce output tokens


a
b
a
b
b
a
a
b
a
b

$P, T, \quad F \subseteq (P \times T) \cup (T \times P)$

$M : P \to \mathbb{N}_0$

When is $t \in T$ <u>enabled</u> at M ?

All input places are marked

$^\bullet t = \{p \mid (p, t) \in F\}$ Input Places

$\forall p \in {}^\bullet t, \; M(p) \geq 1$

When t occurs at M

- Input tokens are consumed
- Output tokens are created

$$t^{\bullet} = \{ p \mid (t, p) \in F \}$$

$$M \xrightarrow{t} M'$$

$$\forall p \in {}^{\bullet}t \quad M'(p) = M(p) - 1$$

$$\forall p \in t^{\bullet} \quad M'(p) = M(p) + 1$$

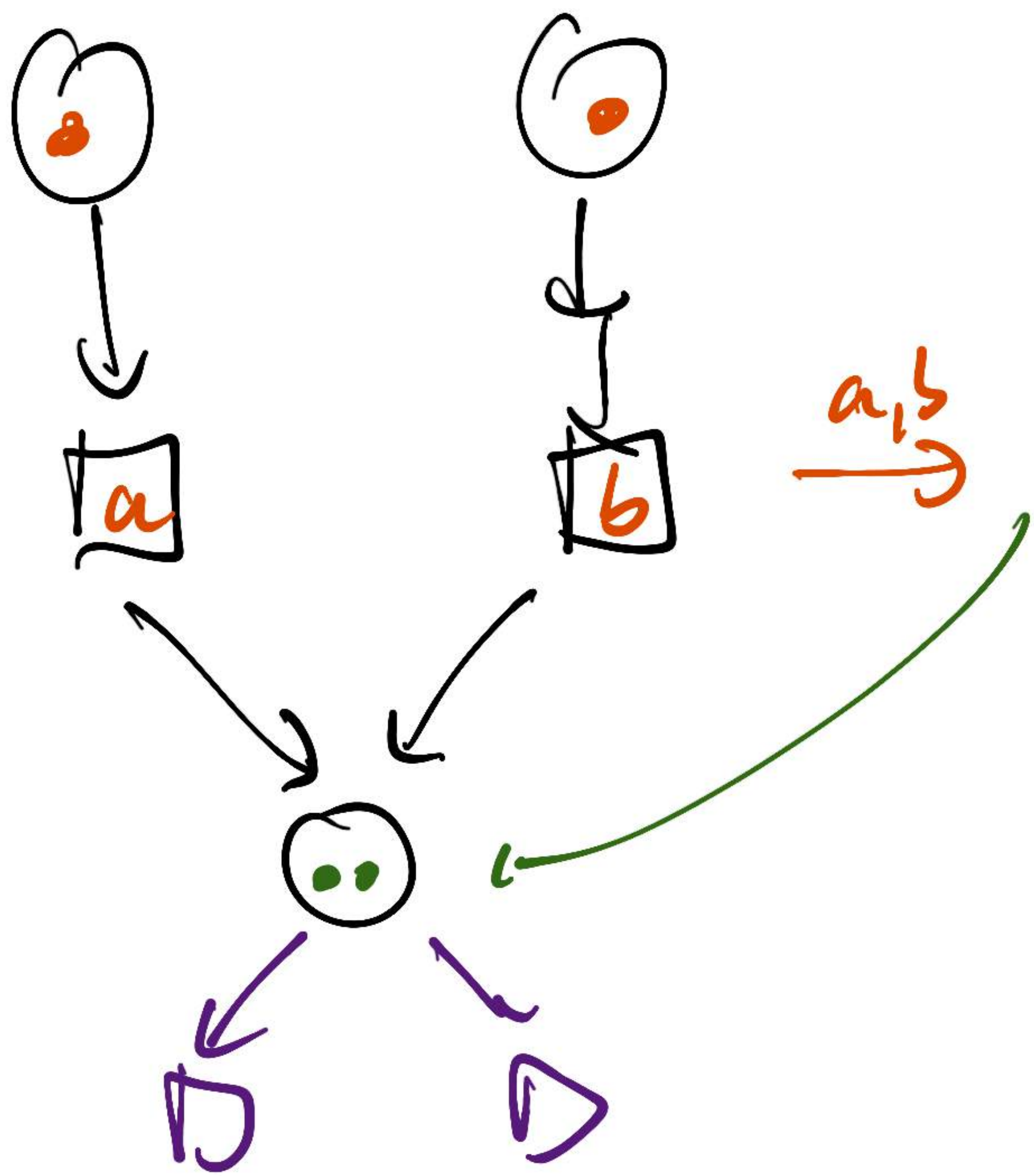
$$\forall p \notin {}^{\bullet}t \cup t^{\bullet} \quad M'(p) = M(p)$$

Old papers

$$M\ [t\rangle\ M'$$

$M \xrightarrow{t}$ t is enabled at M

$M \not\xrightarrow{t}$ t is not enabled at M


a
b
a, b