Concurrency Theory, August–November 2019

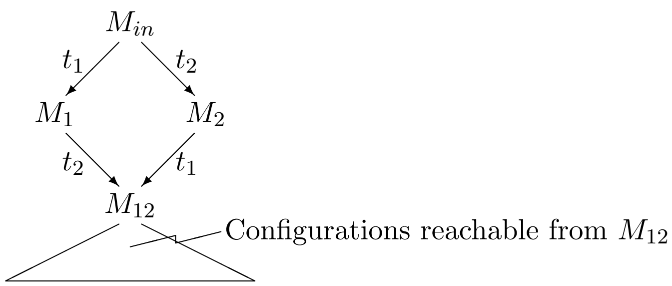
Assignment 1, 18 September, 2019
Due: 29 September, 2019

Note: Only electronic submissions accepted, via Moodle.

Definitions

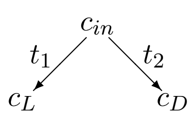
- A net system $(N = (P, T, F), M_{in})$ is *k-safe* if $M(p) \leq k$ for each place $p \in P$ and every reachable marking M. We usually abbreviate 1-safe as just *safe*. A net is *bounded* if it is k-safe for some k.
- A safe net system $(N = (P, T, F), M_{in})$ is *sequential* if it exhibits no concurrency. That is, for every reachable marking M, if $M \xrightarrow{t_1}$ and $M \xrightarrow{t_2}$, then ${}^\bullet t_1 \cap {}^\bullet t_2 \neq \emptyset$.
- A net system $(N = (P, T, F), M_{in})$ is *live* if for each transition $t \in T$ and every reachable marking M, there is a marking M' reachable from M where t becomes enabled.
- A net system $(N = (P, T, F), M_{in})$ is said to be *determinate* if for every reachable configuration M and for every pair of transitions $t, t' \in T$, if $t \neq t'$ and $M \xrightarrow{t}$ and $M \xrightarrow{t'}$ then $({}^\bullet t \cup t^\bullet) \cap ({}^\bullet t' \cap t'^\bullet) = \emptyset$. In other words, in a determinate system, whenever two distinct transitions are simultaneously enabled, they are independent.

1. Construct an **elementary net system** $\mathcal{N} = (P, T, F, M_{in})$ whose configuration graph is structured as follows



such that $\mathcal{N}_{12} = (P, T, F, M_{12})$, the elementary net system with initial configuration M_{12}, is determinate, sequential and live. In general, there will be transitions other than t_1 and t_2 in T, but remember that the liveness of $\mathcal{N}_{12}$ includes t_1 and t_2. Also, as the diagram above shows, the configurations M_{in}, M_1 and M_2 are not reachable from M_{12}. Moreover, there are no other transitions other than the ones shown leading out of these three configurations.

2. Construct an **elementary net system** $\mathcal{N} = (P, T, F, c_{in})$ whose configuration graph is structured as follows



such that $\mathcal{N}_L = (P, T, F, c_L)$ is live, sequential and determinate and c_D is a *dead* configuration, with no enabled transitions.

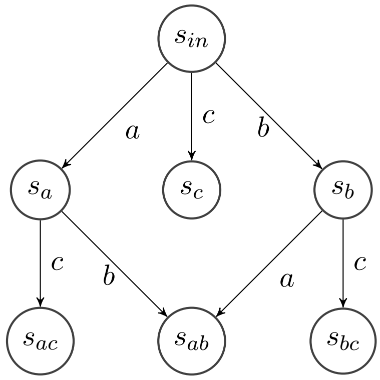
3. A Petri net (P, T, F) is *weakly connected* if the underlying undirected graph (that is, ignore the directions of the arrows in F) is a connected graph.

   Let (P, T, F) be a weakly connected net such that the net system $((P, T, F), M_{in})$ is *live* and *bounded*. Show that the net is, in fact, *strongly connected*. In other words, for every pair $(x, y) \in (P \cup T) \times (P \cup T)$, there is a directed path via F from x to y.

   Note: This is Theorem 11 (Section 3.5) in the survey paper by Desel and Reisig, but the proof in the paper is wrong.

4. Is the transition system on the right isomorphic to the marking graph of an unlabelled elementary net system? Justify your answer using regions.

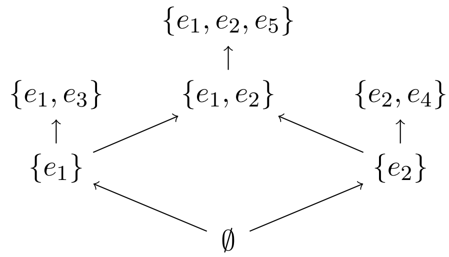
   Recall that a region of a transition system is a subset of states that has a consistent crossing behaviour with respect to each transition label. A transition system is isomorphic to the marking graph of an elementary net system if it satisfies the state-state and event-state separation conditions.



5. The figure on the right below shows the configurations of an event structure ordered under inclusion.

   (a) Draw the corresponding event structure. Use $\longrightarrow$ to indicate causality and # to indicate conflict.

   (b) Draw an unlabelled 1-safe net that generates this behaviour.

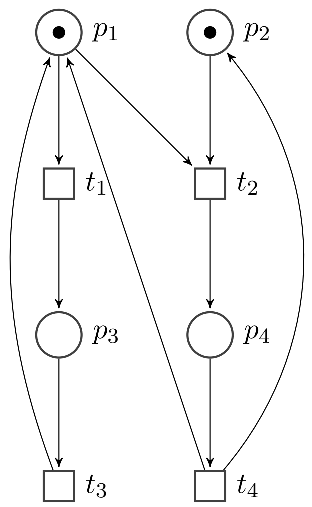


6. (a) Construct the unfolding upto depth 3 of the net on the right. Recall that the depth of a transition t is the maximum length k among sequences of the form $t_1 < t_2 < \cdots < t_k$ where $t_k = t$.

   (b) Identify all cutoff events at this depth using the following definition of an adequate order for configurations.

   $C \prec C'$ iff

   $\quad\quad \text{Mark}(C) = \text{Mark}(C')$ and $|C| < |C'|$



7. Let (Σ, I) be a trace alphabet with $\Sigma = \{a, b, c, d, e\}$. We are told that the trace $[abcacdbea]$ consists of the sequences $\{abcacdbea, abcadcbea, bacacdbea, bacadcbea\}$.

   (a) Draw the trace $[abcacdbea]$ as a labelled partial order $(E, \leq, \ell)$.

   (b) What can you say about the independence relation I?