Lecture 6: Convolutional Neural Networks

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Advanced Machine Learning 2021
Chennai Mathematical Institute

Deep Neural Networks for Recognizing Images

Last Lecture:

- We trained a Fully-Connected DNN to recognize handwritten digits from the MNIST Dataset
- It performed fairly well (97.5% Test Accuracy)
- Similar networks, but perhaps with larger number of neurons, can be built for more complex image classification tasks.
- However, we also saw that the Fully-Connected Network doesn't use *Visual Information*.
- We fixed an arbitrary permutation, and scrambled all training and test images using it. The resulting images were no longer recognizable as digits (by us humans).
- However the Fully-Connected network still managed a 97.5% Test accuracy. This means this network was not using visual information.
- More importantly, it seems difficult to improve the network performance without using some visual information in the images.

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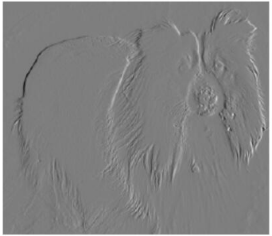

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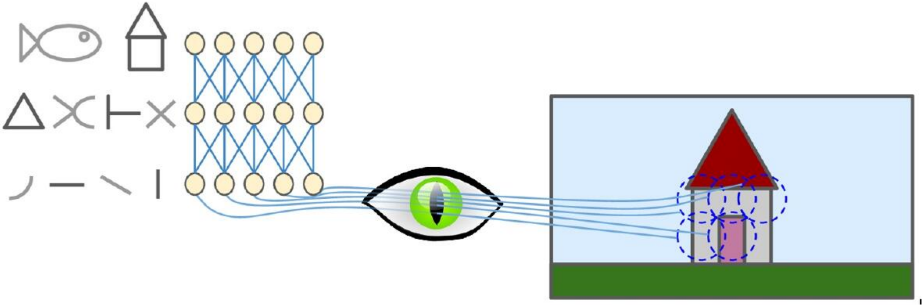
- Convolutional neural network (CNN) — layered network

Receptive field



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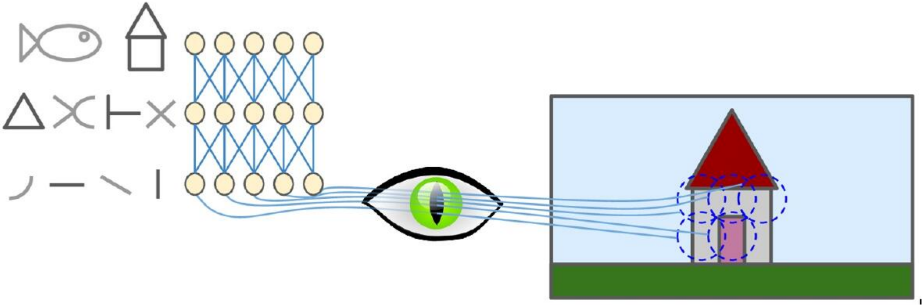

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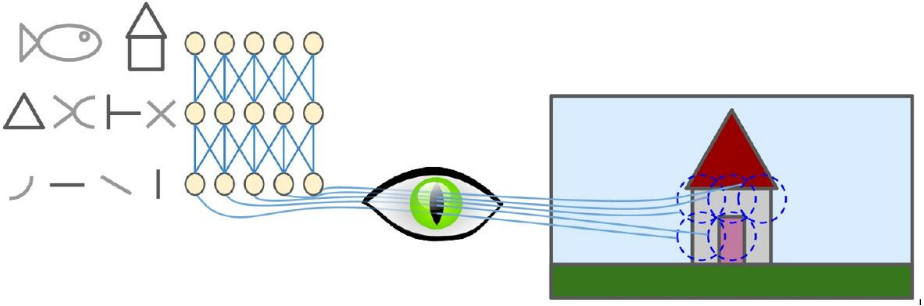

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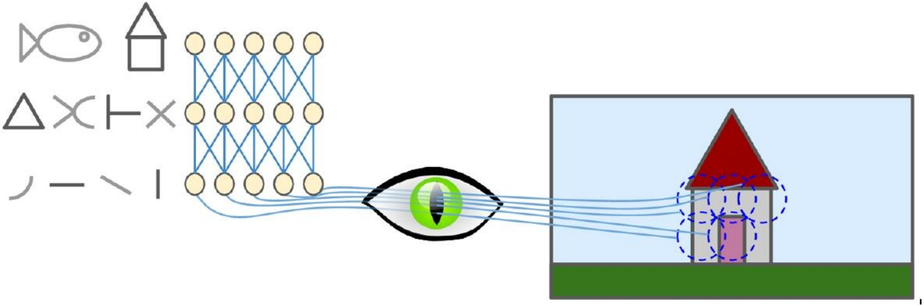

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  - Each neuron in first layer has $120,000$ input weights
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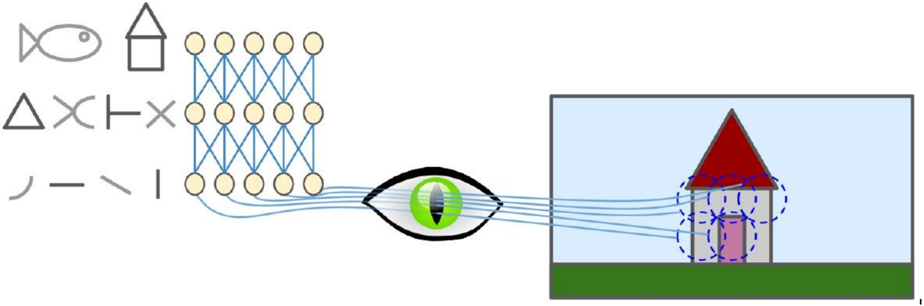

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- Parameter blowup, overfitting

Filters and convolution

- Aggregate values over a region
  - Smoothening — take average
  - Vertical lines — difference between adjacent columns
  - Horizontal lines — difference between adjacent rows
- Pass a filter f over the image
  - Convolution — $I * f$
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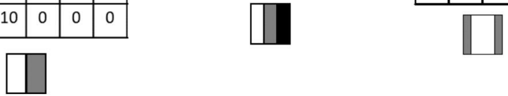

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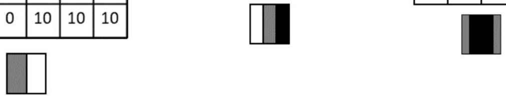

Feature maps

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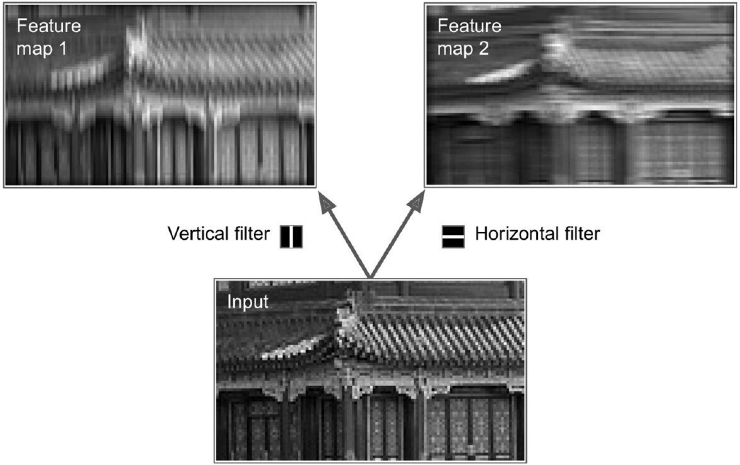
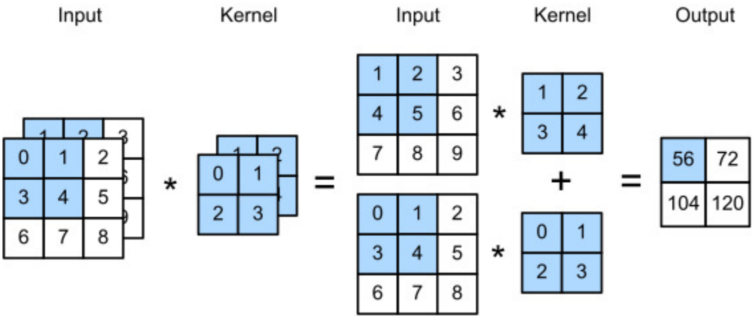


Multi-Channel Inputs

- Real World Images are in color (3-channels)
- Similarly convolution filters in higher layer will need to work with feature maps produced by multiple lower layer convolutions.
- So we need Multi-Channel Convolution



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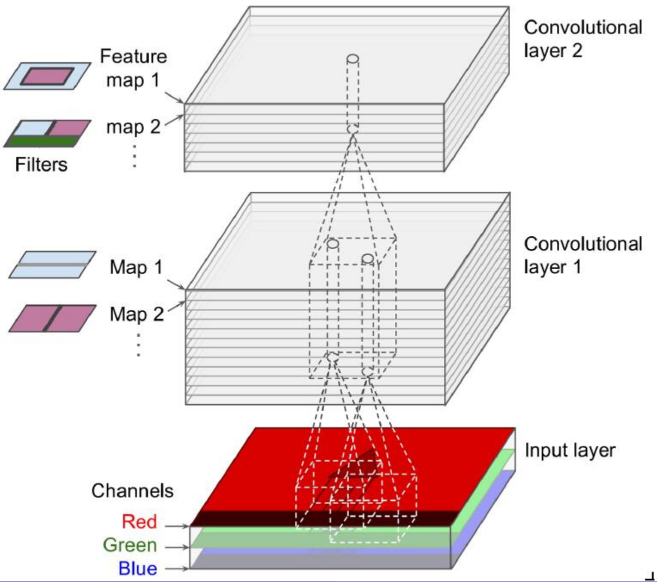

Feature maps

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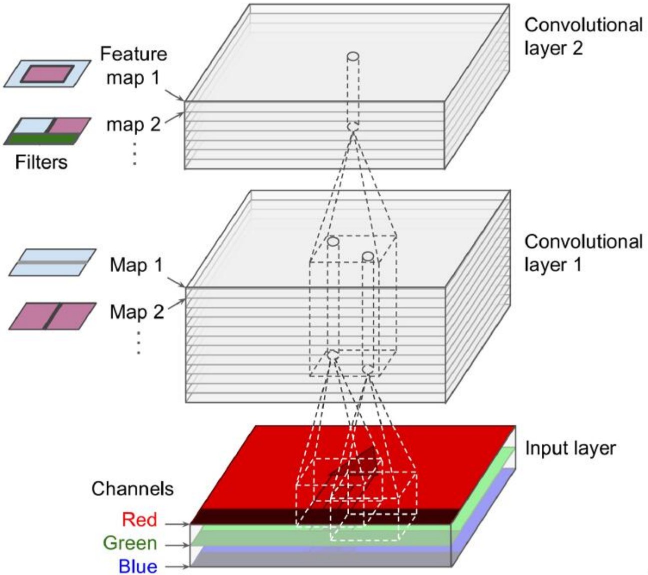
- Each layer has many feature maps
  - Array of filters, each connected to a different region



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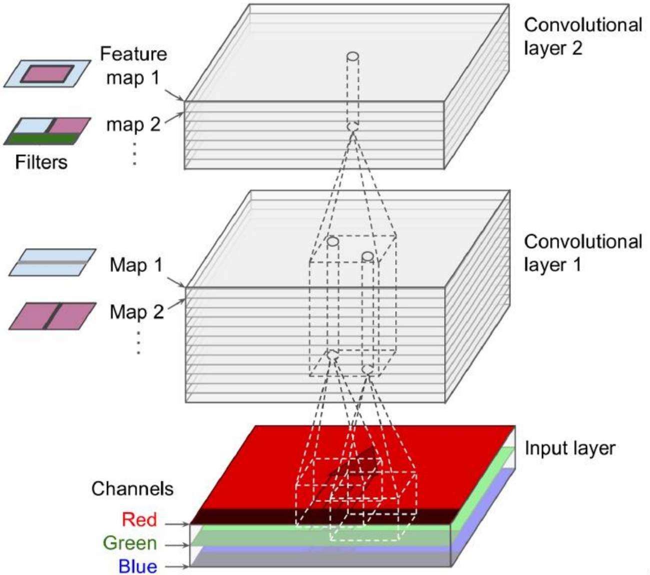
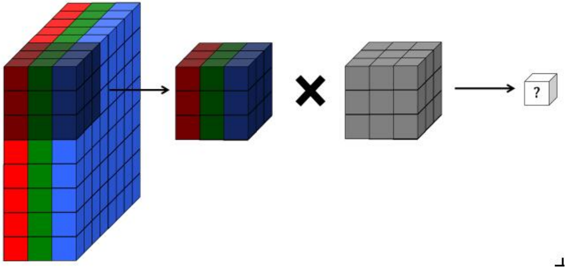


Volumetric view

- Each filter processes a volume of inputs



Volumetric view

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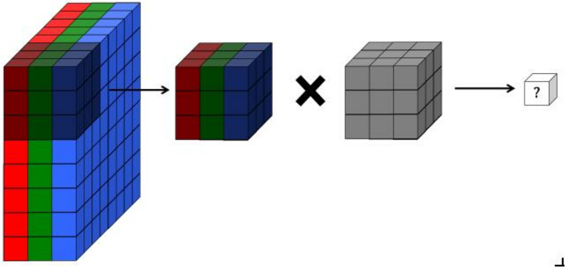

Volumetric view

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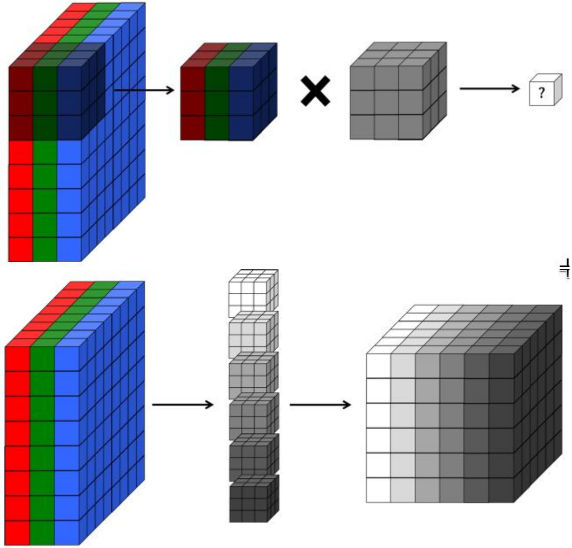
- Each layer produces a block of outputs



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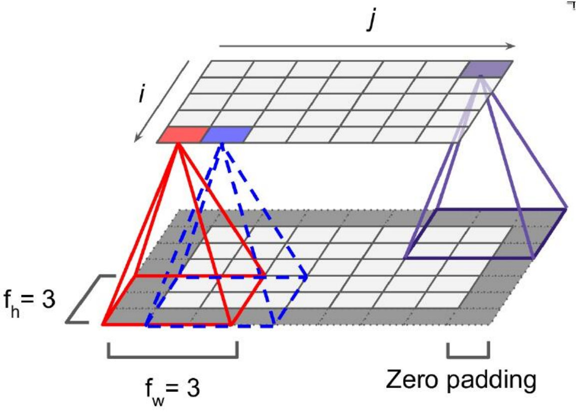

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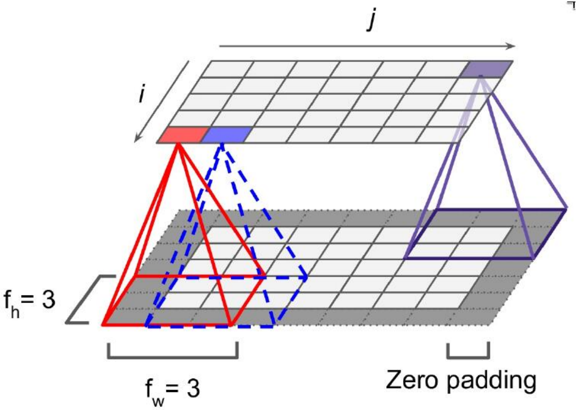

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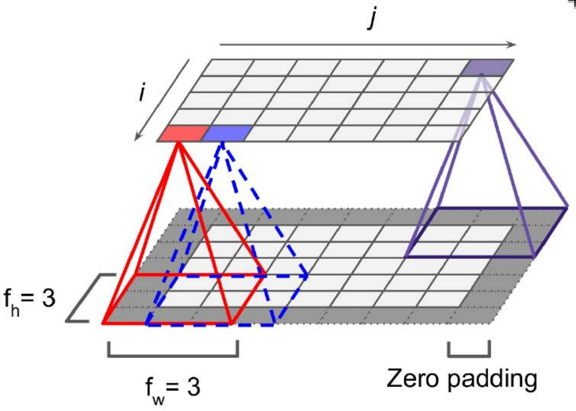
- Note: In an actual CNN, filters are not designed by hand
  - Fix f_h and f_w, but weights are learned from training data



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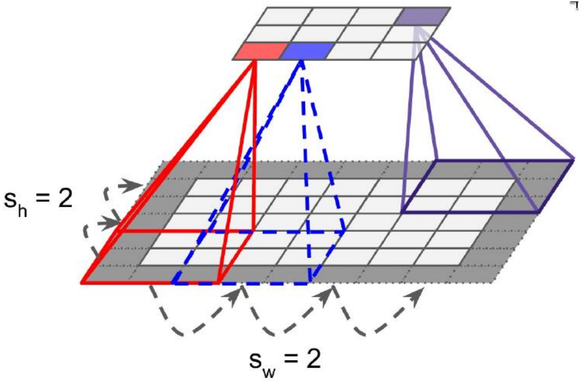
- To reduce dimension, we can space out the receptive fields
  - Horizontal and vertical stride



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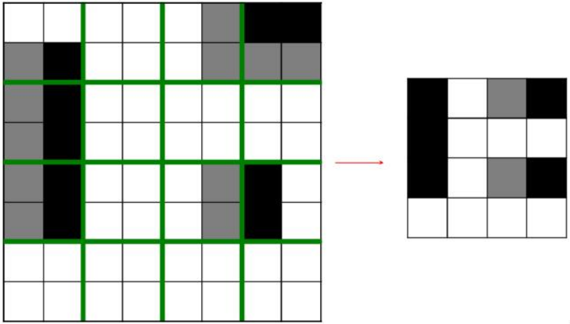

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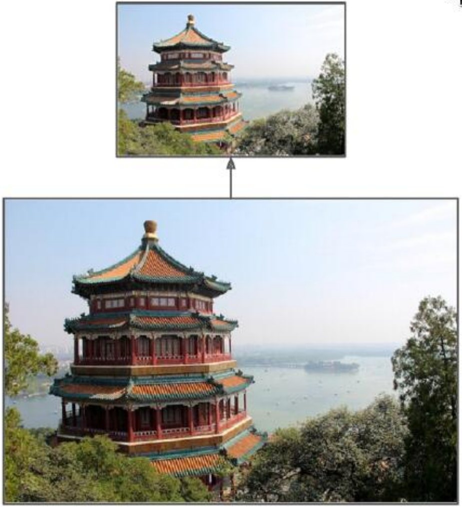

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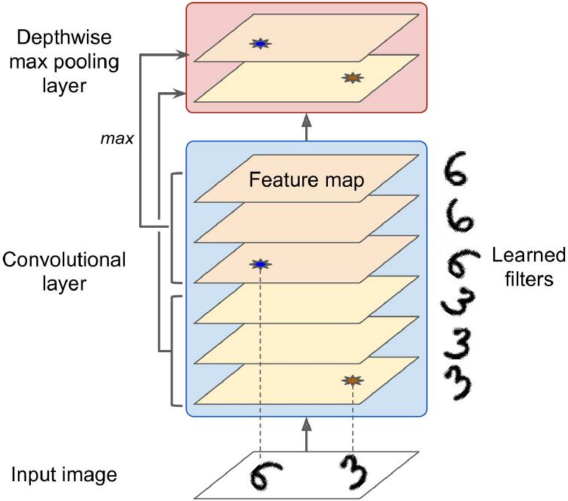
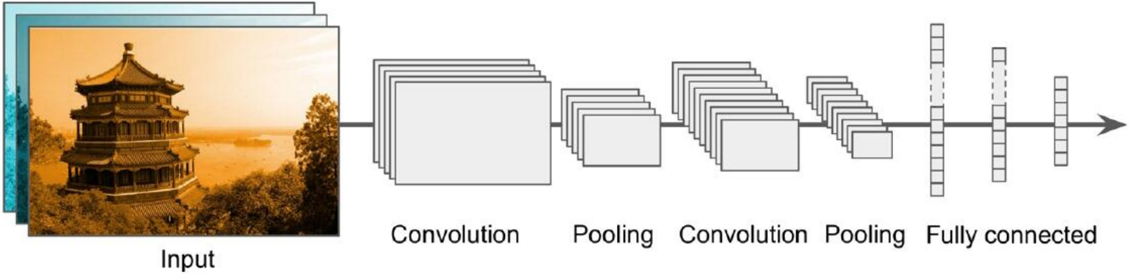
- Can also pool depthwise — for instance, to learn features invariant to rotation



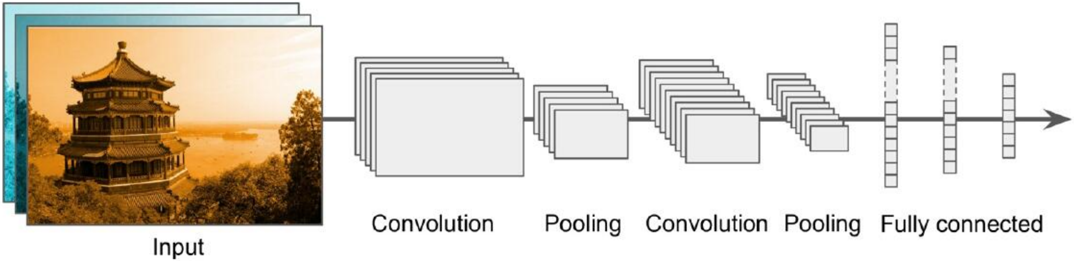
Typical CNN Architecture

- A typical CNN has multiple iterations of convolution followed by pooling
- After final pooling, conventional completely connected network



Parameter sharing

- A filter is a layer of identical nodes operating on different regions (receptive fields)
- All these nodes should behave the same
- While training, their weights are tied to each other — parameter sharing
- Thus, backward pass of backpropagation calculation is reduced
- Forward pass needs to compute individual outputs — still expensive



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  - Inception layer with 1×1 filters, operates in depth dimension, cross-channel features

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- Xception, Chollet, 2016
- SENet, Hu et al, 2017

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- CNNs continue to evolve with specialized hacks