Temporal Difference Learning

Madhavan Mukund

https://www.cmi.ac.in/~madhavan

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Adding bootstrapping to Monte Carlo methods

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  - Bootstrap from an initial estimate through incremental updates
  - Need to know the model

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- Temporal Difference (TD) learning
  - Apply bootstrapping to Monte Carlo methods

From Monte Carlo to TD

- Monte Carlo update for non-stationary environments
  - $V(S_t) \leftarrow V(S_t) + \alpha[G_t - V(S_t)]$, $\alpha \in (0, 1]$ is a constant
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- Instead
  - Expand G_t as $R_{t+1} + \gamma V(S_{t+1})$
  - Revised update rule: $V(S_t) \leftarrow V(S_t) + \alpha[\underbrace{R_{t+1} + \gamma V(S_{t+1})}_{} - V(S_t)]$

    G_t new
  - R_{t+1} is available after choosing A_t
  - Use current estimate for $V(S_{t+1})$
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  - Update $V(S_t)$ on the fly, as the episode evolves
- Also called TD(0), because it has zero lookahead
  - More generally, can look ahead n steps to update, TD(n)
  - Most general version is called TD(λ), we only consider TD(0)

TD(0) algorithm for policy evaluation

Tabular TD(0) for estimating v_π

Input: the policy π to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(\mathit{terminal}) = 0$

Loop for each episode:
    Initialize S
    Loop for each step of episode:
        $A \leftarrow$ action given by π for S
        Take action A, observe R, S'
        $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
        $S \leftarrow S'$
    until S is terminal

TD(0) example: Driving home from work

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State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

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- Rewards: elapsed time on each leg
- No discounting: $\gamma = 1$, return at a state is actual time remaining

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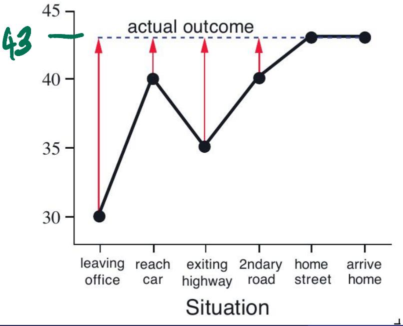
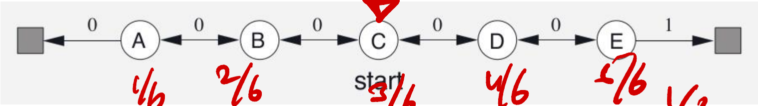


Comparing MC and TD(0)

All actions have probability 0.5

- Markov Reward Process



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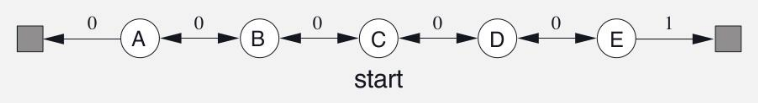

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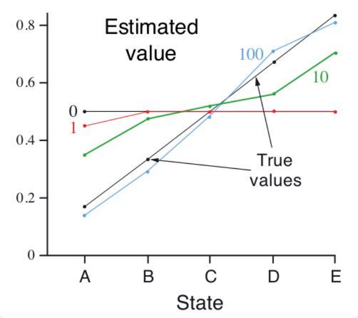

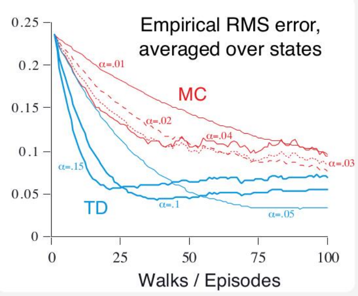

Comparing MC and TD(0) . . .

Predict the values of states A and B

Episodes

1. A, 0, B, 0 ✗
2. B, 1
3. B, 1
4. B, 1
5. B, 1
6. B, 1
7. B, 1
8. B, 0 ✗

Comparing MC and TD(0) ...

Predict the values of states A and B

$\mathrm{A}, 0, \mathrm{B}, 0$	$\mathrm{B}, 1$
$\mathrm{B}, 1$	$\mathrm{B}, 1$
$\mathrm{B}, 1$	$\mathrm{B}, 1$
$\mathrm{B}, 1$	$\mathrm{B}, 0$

- $V(B) = 6/8 = 0.75$

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A, 0, B, 0	B, 1
B, 1	B, 1
B, 1	B, 1
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- $V(B) = 6/8 = 0.75$
- What about $V(A)$?

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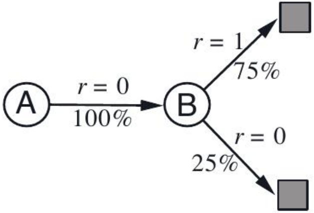
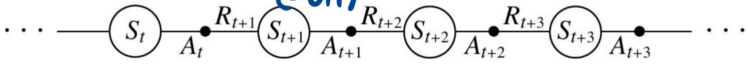
- MC — one episode, $V(A) = 0$
- TD(0) – $V(A) = 0.75$



SARSA: On policy TD control, estimating π_*

- For π_*, better to estimate q_π rather than v_π
- Structure of an episode

$V(S_{t+1})$



- Use the following update rule

  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)]$
- Update uses $(S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1})$, hence the name SARSA
- As with Monte Carlo estimation, use ε-soft policies to balance exploration and exploitation

SARSA algorithm on-policy TD control

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
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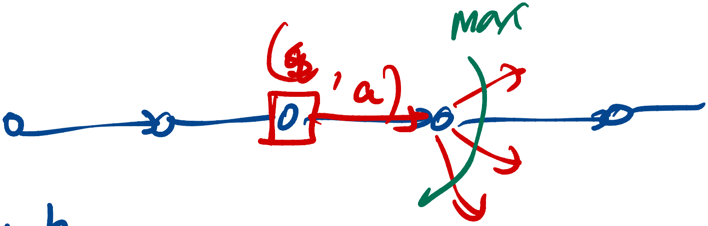
not necessarily A_{t+1}

- Underlying policy still needs to be designed to visit all state-action pairs
- With suitable assumptions, Q-learning provably converges to q_*

Episode dictated by some policy b



Q-learning algorithm, off-policy TD control

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

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Summary

- Temporal difference methods combine bootstrapping with Monte Carlo exploration of state space
- SARSA is a TD(0) algorithm for on-policy control — estimating π_*
- Q-learning is an off-policy algorithm that provably converges to q_*
- TD-based approaches apply beyond reinforcement learning
  - General methods to make long term predictions about dynamical systems
- Theoretical properties such as convergence still an area of research