

Lecture 20: Deep Reinforcement Learning

Madhavan Mukund and Pranabendu Misra

Advanced Machine Learning 2021

We use lecture slides by Fei-Fei Li & Justin Johnson & Serena Yeung

http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture14.pdf

Today: Reinforcement Learning

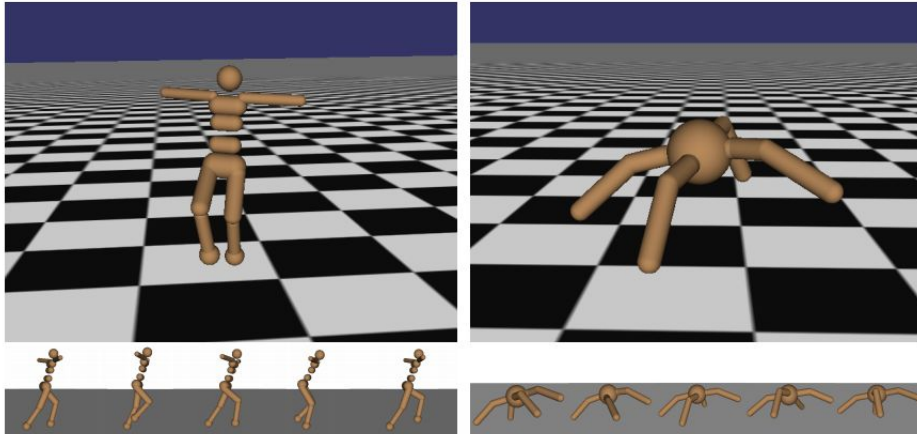
Problems involving an **agent** interacting with an **environment**, which provides numeric **reward** signals

Goal: Learn how to take actions in order to maximize reward



Atari games figure copyright Volodymyr Mnih et al., 2013. Reproduced with permission.

Robot Locomotion



Objective: Make the robot move forward

State: Angle and position of the joints

Action: Torques applied on joints

Reward: 1 at each time step upright + forward movement

Figures copyright John Schulman et al., 2016. Reproduced with permission.

Atari Games



Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state

Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step

Figures copyright Volodymyr Mnih et al., 2013. Reproduced with permission.

Markov Decision Process

- At time step $t=0$, environment samples initial state $s_0 \sim p(s_0)$
- Then, for $t=0$ until done:
 - Agent selects action a_t
 - Environment samples reward $r_t \sim R(\cdot | s_t, a_t)$
 - Environment samples next state $s_{t+1} \sim P(\cdot | s_t, a_t)$
 - Agent receives reward r_t and next state s_{t+1}
- A policy π is a function from S to A that specifies what action to take in each state
- **Objective:** find policy π^* that maximizes cumulative discounted reward: $\sum_{t \geq 0} \gamma^t r_t$


A simple MDP: Grid World

actions = {

1. right 

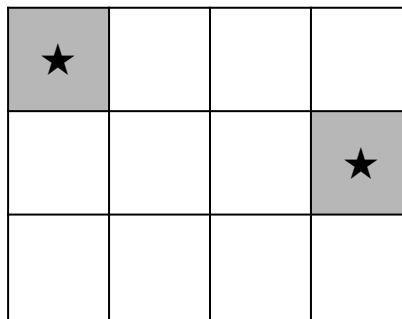
2. left 

3. up 

4. down 

}

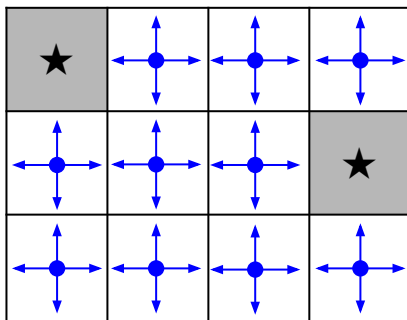
states



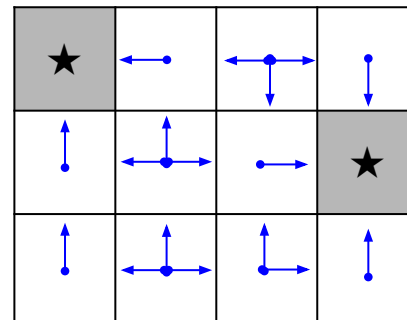
Set a negative “reward”
for each transition
(e.g. $r = -1$)

Objective: reach one of terminal states (greyed out) in
least number of actions

A simple MDP: Grid World



Random Policy



Optimal Policy

The optimal policy π^*

We want to find optimal policy π^* that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability...)?

The optimal policy π^*

We want to find optimal policy π^* that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability...)?

Maximize the **expected sum of rewards!**

$$\text{Formally: } \pi^* = \arg \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t | \pi \right] \quad \text{with } s_0 \sim p(s_0), a_t \sim \pi(\cdot | s_t), s_{t+1} \sim p(\cdot | s_t, a_t)$$

Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

How good is a state?

The **value function** at state s , is the expected cumulative reward from following the policy from state s :

$$V^\pi(s) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, \pi \right]$$

Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

How good is a state?

The **value function** at state s , is the expected cumulative reward from following the policy from state s :

$$V^\pi(s) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, \pi \right]$$

How good is a state-action pair?

The **Q-value function** at state s and action a , is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^\pi(s, a) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Bellman equation

The optimal Q-value function Q^* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s, a) = \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Bellman equation

The optimal Q-value function Q^* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s, a) = \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Q^* satisfies the following **Bellman equation**:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') \mid s, a \right]$$

Intuition: if the optimal state-action values for the next time-step $Q^*(s', a')$ are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s', a')$

Bellman equation

The optimal Q-value function Q^* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s, a) = \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

Q^* satisfies the following **Bellman equation**:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') \mid s, a \right]$$

Intuition: if the optimal state-action values for the next time-step $Q^*(s', a')$ are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s', a')$

The optimal policy π^* corresponds to taking the best action in any state as specified by Q^*

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E} \left[r + \gamma \max_{a'} Q_i(s', a') | s, a \right]$$

Q_i will converge to Q^* as $i \rightarrow \text{infinity}$

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E} \left[r + \gamma \max_{a'} Q_i(s', a') | s, a \right]$$

Q_i will converge to Q^* as $i \rightarrow \text{infinity}$

What's the problem with this?

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E} \left[r + \gamma \max_{a'} Q_i(s', a') \mid s, a \right]$$

Q_i will converge to Q^* as $i \rightarrow \text{infinity}$

What's the problem with this?

Not scalable. Must compute $Q(s, a)$ for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E} \left[r + \gamma \max_{a'} Q_i(s', a') | s, a \right]$$

Q_i will converge to Q^* as $i \rightarrow \text{infinity}$

What's the problem with this?

Not scalable. Must compute $Q(s, a)$ for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

Solution: use a function approximator to estimate $Q(s, a)$. E.g. a neural network!

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

If the function approximator is a deep neural network => **deep q-learning!**

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning!**

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} [(y_i - Q(s, a; \theta_i))^2]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} [(y_i - Q(s, a; \theta_i))^2]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i) \right] \nabla_{\theta_i} Q(s, a; \theta_i)$$

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} \left[(y_i - Q(s, a; \theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

Iteratively try to make the Q-value close to the target value (y_i) it should have, if Q-function corresponds to optimal Q^* (and optimal policy π^*)

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i) \right] \nabla_{\theta_i} Q(s, a; \theta_i)$$

Case Study: Playing Atari Games



Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state

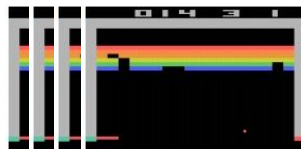
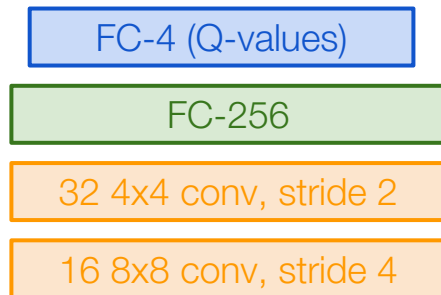
Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step

Figures copyright Volodymyr Mnih et al., 2013. Reproduced with permission.

Q-network Architecture

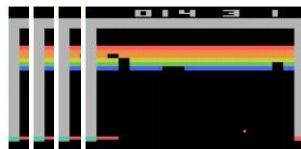
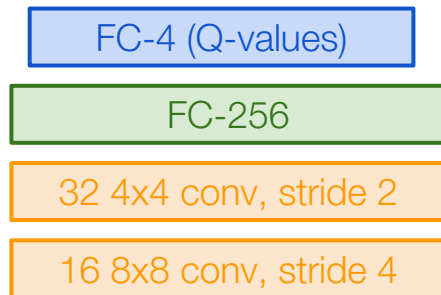
$Q(s, a; \theta)$:
neural network
with weights θ



Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ

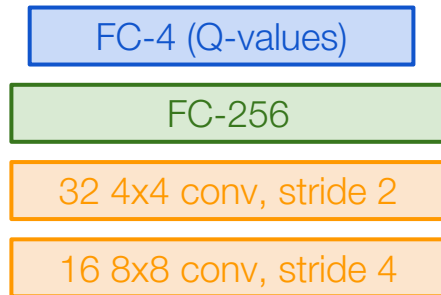


← Input: state s_t

Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ



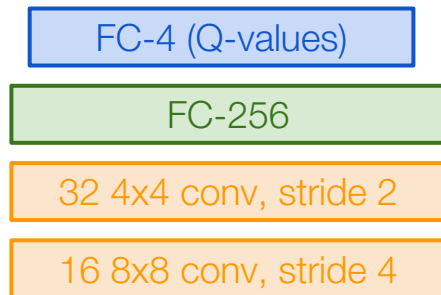
← Familiar conv layers,
FC layer



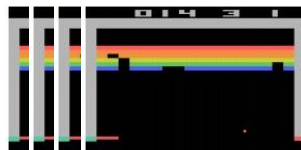
Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ



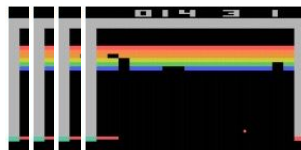
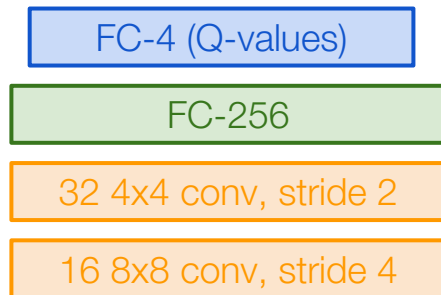
← Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t, a_4)$



Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ



Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

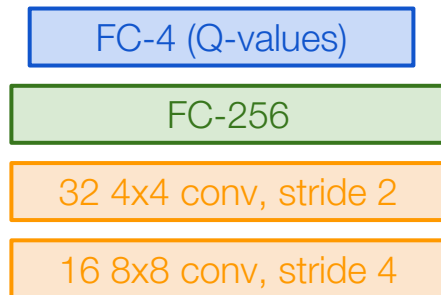
← Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t, a_4)$

Number of actions between 4-18 depending on Atari game

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ

A single feedforward pass
to compute Q-values for all
actions from the current
state => efficient!



Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

← Last FC layer has 4-d
output (if 4 actions),
corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$,
 $Q(s_t, a_4)$

Number of actions between 4-18
depending on Atari game

Training the Q-network: Loss function (from before)

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} \left[(y_i - Q(s, a; \theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

Iteratively try to make the Q-value close to the target value (y_i) it should have, if Q-function corresponds to optimal Q^* (and optimal policy π^*)

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[(r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i)) \nabla_{\theta_i} Q(s, a; \theta_i) \right]$$

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

Address these problems using **experience replay**

- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

Address these problems using **experience replay**

- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

Each transition can also contribute to multiple weight updates
=> greater data efficiency

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

← Initialize replay memory, Q-network

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

← Play M episodes (full games)

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$ ←

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

Initialize state
(starting game
screen pixels) at the
beginning of each
episode

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for



For each timestep t
of the game

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

← With small probability, select a random action (explore), otherwise select greedy action from current policy

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

← Take the action (a_t), and observe the reward r_t and next state s_{t+1}

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

← Store transition in
replay memory

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory \mathcal{D} to capacity N

Initialize action-value function Q with random weights

for episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$

for $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

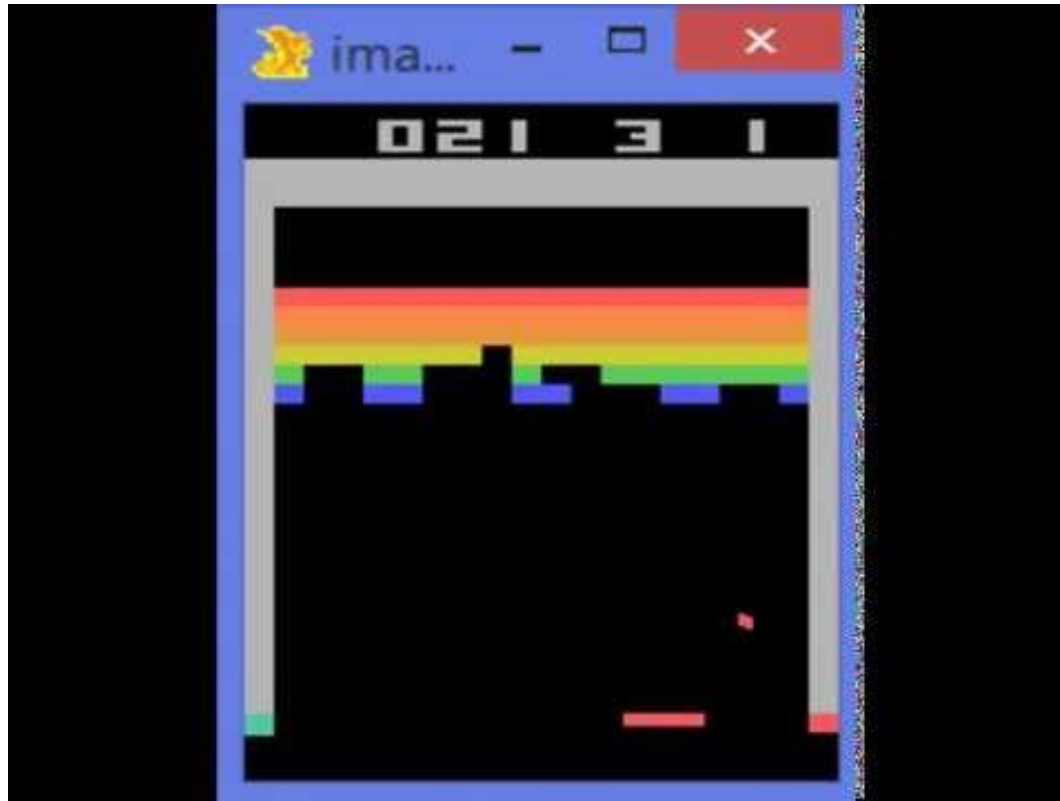
 Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

← Experience Replay:
Sample a random minibatch of transitions from replay memory and perform a gradient descent step



<https://www.youtube.com/watch?v=V1eYniJ0Rnk>

Video by Károly Zsolnai-Fehér. Reproduced with permission.

References:

These slides are based on:

- http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture14.pdf
- <https://www.youtube.com/watch?v=lvoHnicueoE>