Net : $N = (P, T, F)$

P : places

T : transitions - unlabelled

F : Flow relation

$$F \subseteq (P \times T) \cup (T \times P) \text{ - bipartite}$$

$x \in P \cup T$ $\quad {}^\bullet x = \{ y \mid (y, x) \in F \}$

$x^\bullet = \{ y \mid (x, y) \in F \}$

$X \subseteq P \cup T$ $\quad {}^\circ X, X^\circ$

Labelled transitions

(s, s') $\quad (s_1, s_1')$

$t_1 \downarrow a$ $\quad t_2 \downarrow a$

(t, t') $\quad (t_1, t_1')$

Dynamics of a net

Marking $M : P \to \mathbb{N}_0$

"Case" $M : P \to \{0,1\}$ $M \subseteq P$ } Distributed State

$C \subseteq P$ or $C \subseteq P$

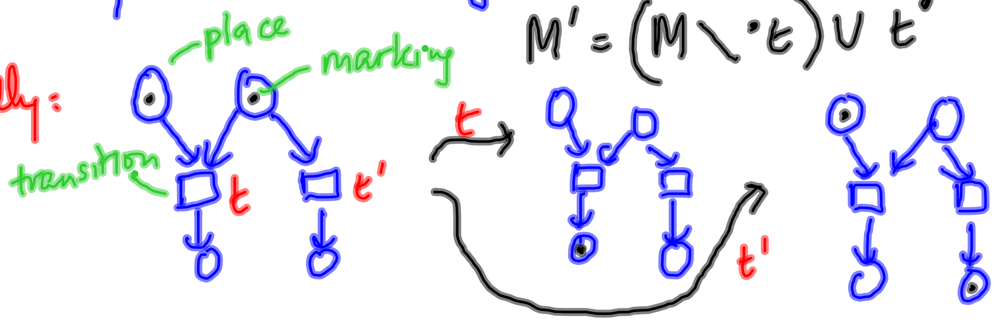
Initial marking : M_{in}

$t \in T$ being enabled at a marking M ${}^\bullet t \subseteq M$

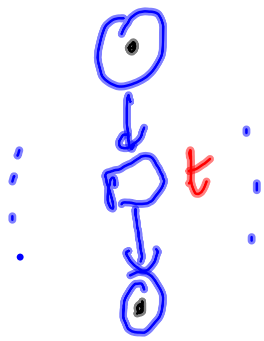
t occurs / fires and changes M to M'

$M' = (M \setminus {}^\bullet t) \cup t^\bullet$

Graphically:



Dynamics:



Is t enabled?

Elementary Net Systems

Disable t

t enabled at M

$^\bullet t \subseteq M \wedge t^\bullet \cap M = \phi$

effect: $M' = (M \setminus {}^\bullet t) \cup t^\bullet$

Place/Transition net "Petri nets"

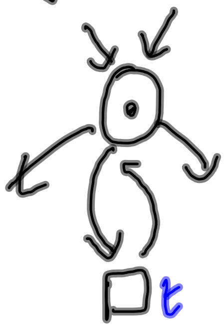
Enable t, allow arbitrary markings

$\forall p \in {}^\bullet t \quad M(p) > 0$

Effect: $\forall p \in {}^\bullet t$, $M'(p) = M(p) - 1$

Then $\forall p \in t^\bullet$, $M''(p) = M'(p) + 1$

Elementary net systems : $ENS = (N, M_{in})$, $N = (P, T, F)$



t is "dead"

$M \xrightarrow{t} M'$

t enabled at M & produces M'

$$M_{in} \xrightarrow{t_1} M_1$$
$$M_{in} \xrightarrow{t_2} M_2 \;\cdots$$

Reachable markings from M_{in}

Reach(M_{in}) :

$M_{in} \in Reach(M_{in})$

$M \in Reach(M_{in})$, $M \xrightarrow{t} M'$ $\Rightarrow M' \in Reach(M_{in})$

$M \xrightarrow{t} M'$ is written as $M[t\rangle M'$

$M \xrightarrow{t}$ t is enabled at M $M[t\rangle$

t con M "t has concession at M"

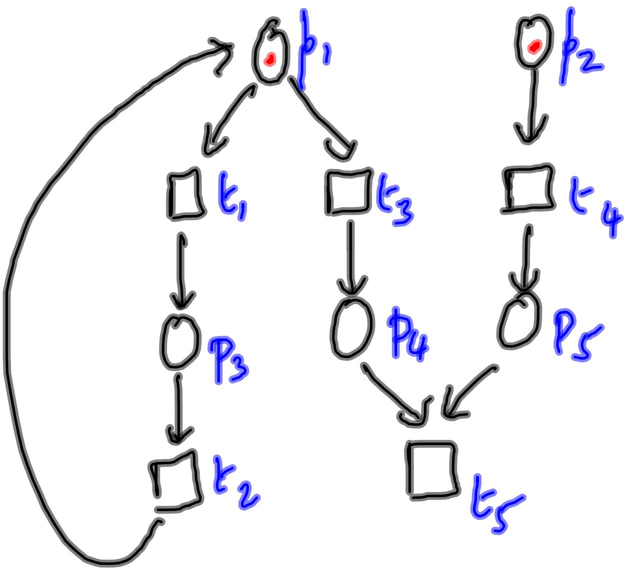
t is "useful" if $\exists M \in Reach(M_{in})$, $M[t\rangle$

Assume all transitions are useful

M is a "deadlocked" marking if $\nexists t$ $M[t\rangle$

t is live if from every $M \in Reach(M_{in})$
t is useful in (N, M)

(N,M) is live if every $t \in T$ is live



$M_{in}[t_1\rangle$

$M_{in}[t_3\rangle$

$M_{in}[t_4\rangle$

t_1 & t_4 are "independent"

$$^\bullet t_1^\bullet \cap {}^\bullet t_4^\bullet = \emptyset$$

t_1 & t_3 are "in conflict"

$M_{in}[t_1\rangle M_1$, $M_1 \cancel{[t_3\rangle}$

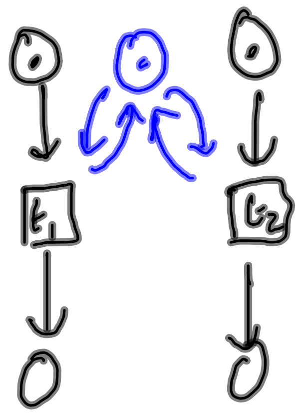
$M_{in}[t_3\rangle M_3$, $\cancel{M_3 [t_1\rangle}$

$M[t_1\rangle M_1$ & $M_2[t_1\rangle$ & $M_1[t_2\rangle$

$M[t_2\rangle M_2$ $\iff$ t_1 & t_2 are independent

$$\begin{tikzcd} & M_1 \arrow[dr, "t_2"] & \\ M \arrow[ur, "t_1"] \arrow[dr, "t_2"'] & & M' \\ & M_2 \arrow[ur, "t_1"'] & \end{tikzcd}$$

Not true for P/T nets



Marking graph (or case graph) of (N, M_{in}) $\quad N = (P, T, F)$

$TS = (S, \rightarrow, s_{in})$ $\quad S = Reach(M_{in})$

$s_{in} = M_{in}$

$\rightarrow \subseteq S \times T \times S$ $\quad M \xrightarrow{t} M'$ if net permits it

Extended marking graph

$\rightarrow \subseteq S \times 2^T \times S$

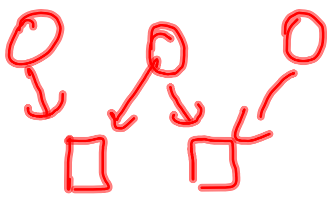
$$\begin{tikzcd}
& s_1 \arrow[dr, "t_2"] & \\
s \arrow[ur, "t_1"] \arrow[dr, "t_2"'] \arrow[rr, dashed, "\{t_1, t_2\}"] & & s' \\
& s_2 \arrow[ur, "t_1"'] &
\end{tikzcd}$$

"Fundamental" situations

t_1 & t_2 can occur concurrently

independence

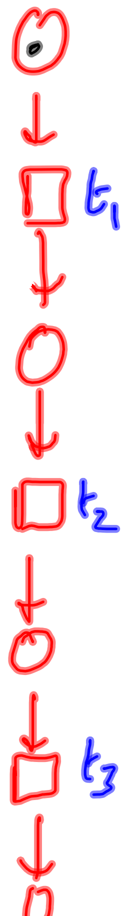
t_1 & t_2 are in conflict



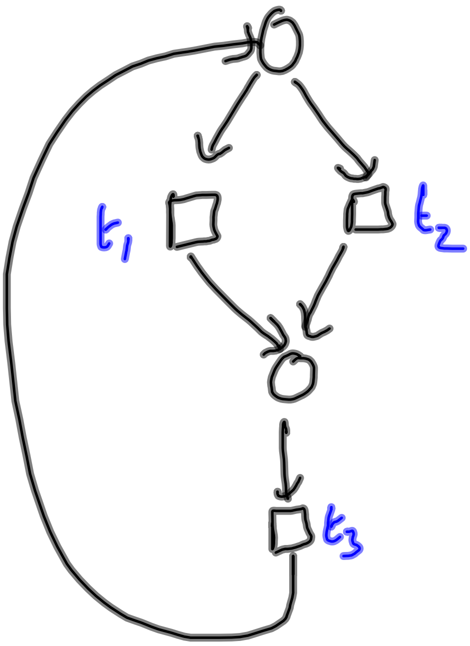
t_2 causally depends on t_1

define wrt a sequence of executed transitions

firing sequence



Difficult to define causality abstractly based on structure of net



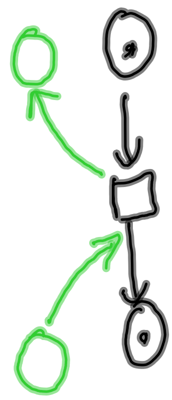
Is t_3 causally dependent on t_1 or t_2?

Or wrt marking (cf. Engelfriet, Rozenberg)

t_2 is causally dep on t_j at M if

$M \overset{t_2}{\not\to}$ but $M \xrightarrow{t} M_1 \xrightarrow{t_2}$

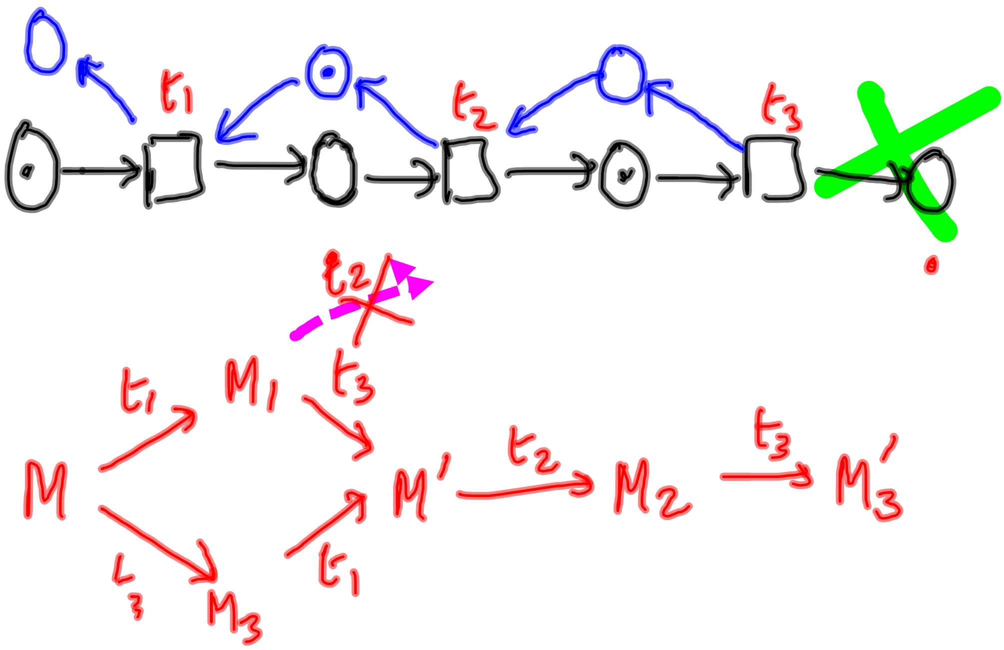
t is disabled at M only because $t^\bullet \cap M \neq \emptyset$



"Contact"

Can make all ENS contact free by adding complementary places

(equivalent nets have isomorphic marking graphs)



Proof ? Ex for Pretend

Language of net = set of firing sequences

$FS(ENS)$ prefix closed $w \in FS(ENS)$, $v \leq w \Rightarrow v \in FS(ENS)$

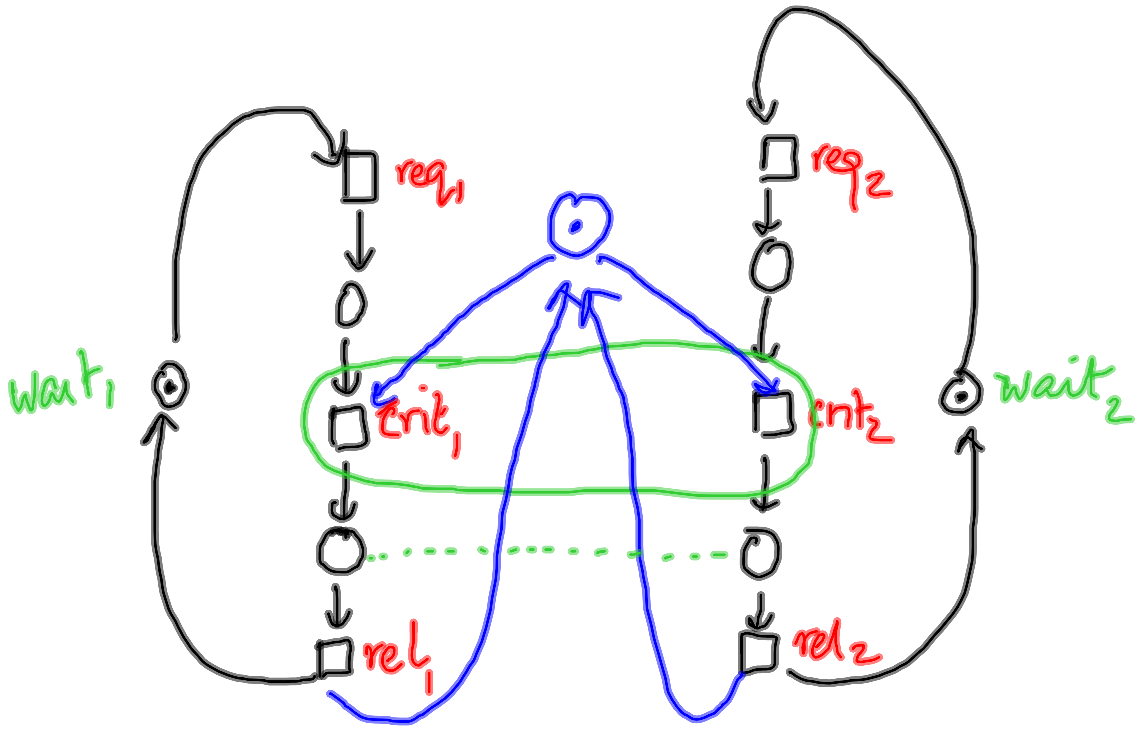
regular (because $Reach(M_{in})$ is always finite)

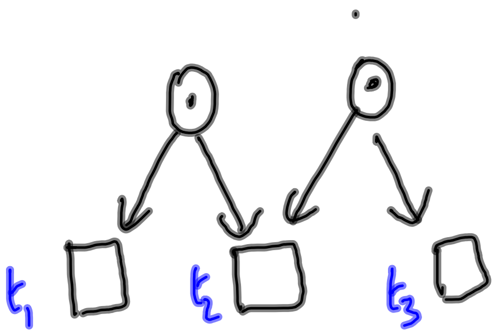
Recovering info from $FS(ENS)$

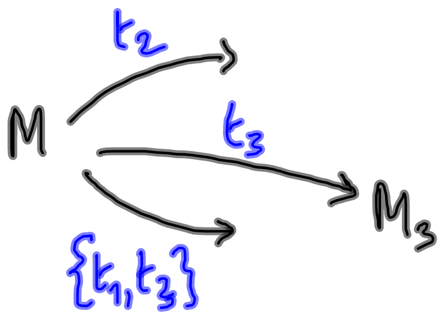
$w t_1 t_2 \in FS$, $w t_2 t_1 \in FS$ > independent!

Note: Unlabelled nets are deterministic !

Example: Mutual Exclusion





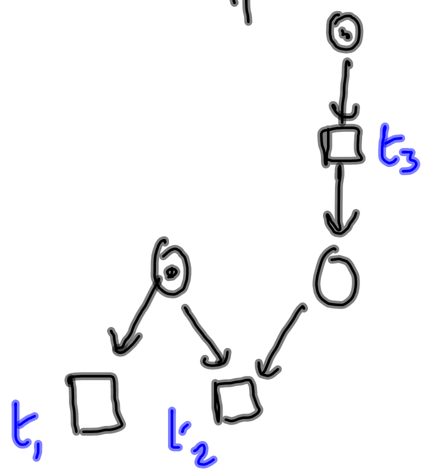


At M: t_1 & t_2 in conflict
t_2 & t_3 in conflict

At M_3 t_1 & t_2 are no longer in conflict

"Confusion" (M, t_1, t_2) is a confusion if $M \xrightarrow{t_1}$, $M \xrightarrow{t_2}$ and firing t_2 changes conflict set of t_1

Symmetric ~~conflict~~ confusion: (M, t_1, t_2) & (M, t_2, t_1) are both confusions

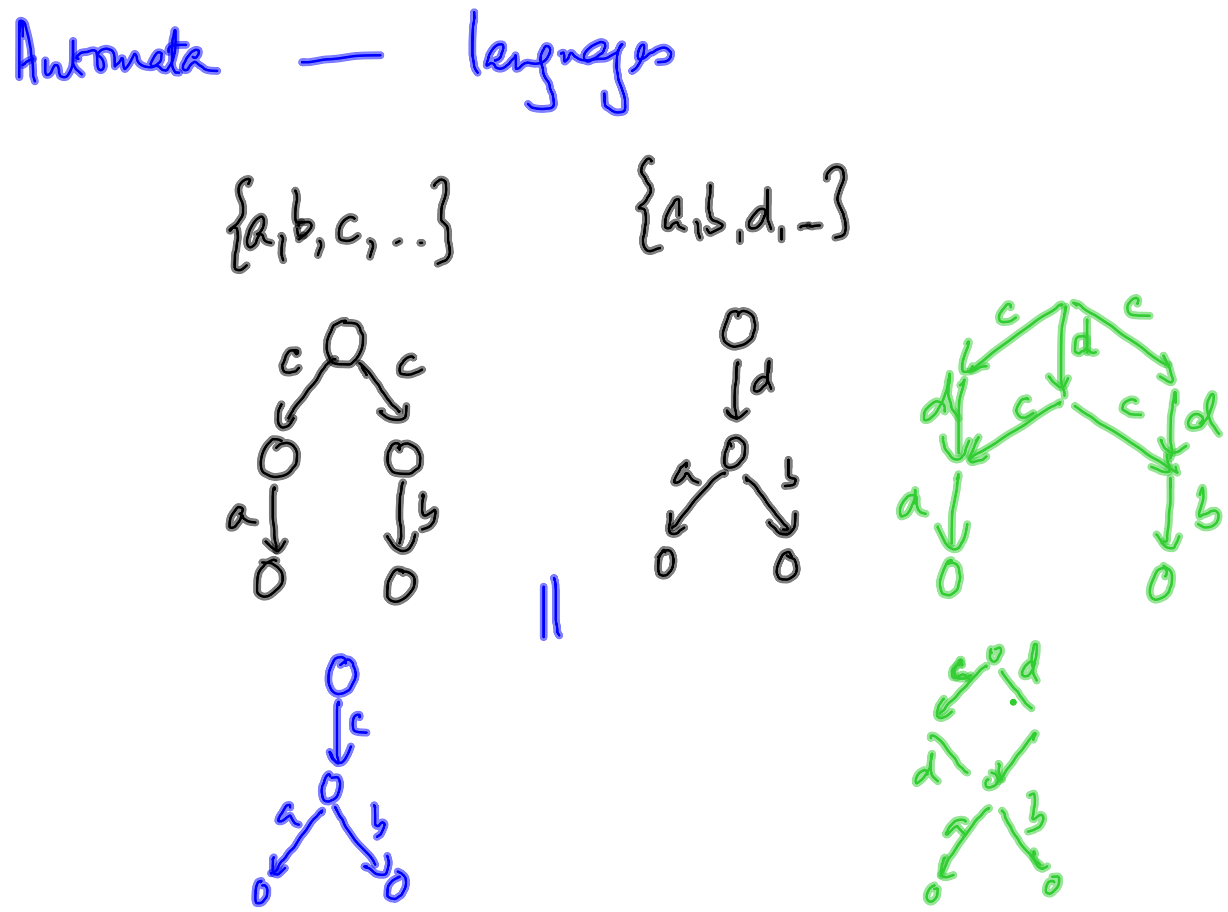


(M, t_1, t_3) is a confusion

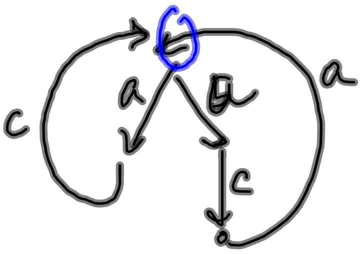
(M, t_3, t_1) is <u>not</u> a confusion

Asymmetric

Confusion-free systems are "easier" to analyse


Automata — languages
{a, b, c, ..}
{a, b, d, ..}
c
c
a
b
d
a
b
c
d
c
d
c
c
d
d
a
b
||
c
a
b
b
d
d
d
a
b

Automaton → unfolding



Tree