Thiagarajan's Conjecture:

Regular event structures $\leftrightarrow$ 1-safe finite Petri nets

$((E, \leq, \#), \lambda) \longleftarrow N = (P, T, F, M_{in})$

$\lambda : E \to T$

implicitly labelled ES $\longleftarrow$ unlabelled net

explicitly labelled ES

Add : $\varphi : T \to \Sigma$

$(((E, \leq, \#, \lambda), \varphi)$ $\varphi(e) = \varphi(\lambda(e))$

Conjecture:

Start with (explicitly) labelled $ES = (E, \leq, \#, \varphi)$

No implicit labelling

Goal: Construct implicit labelling

if ES is "regular"

Regularity for ES?

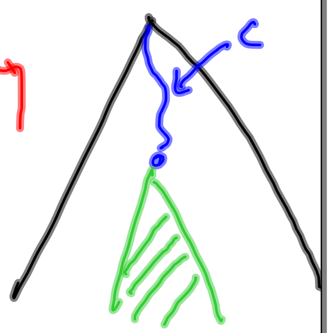
$ES = (E, \leq, \#)$

$c \subseteq E$ is a configuration $\begin{cases} (c \times c) \cap \# = \phi \\ c = \downarrow c \end{cases}$

$\#(c) = \{ e' \mid \exists e \in c,\ e \# e' \}$

Define the "future" of ES after c

like a subtree in sequential unfolding



"ES after c" $= (E', \leq', \#') = ES \setminus c$

$E' = E \setminus \{ c \cup \#(c) \}$

$\leq' = \leq \cap (E' \times E')$

$\#' = \# \cap (E' \times E')$

labelled ES\c

retain $\varphi(e)$ for $e \in E'$

Isomorphism of two labelled ES

$(E_1, \leq_1, \#_1, \varphi_1)$ $(E_2, \leq_2, \#_2, \varphi_2)$

Bijection $f: E_1 \rightarrow E_2$

$e_1 \leq_1 e_2$ iff $f(e_1) \leq_2 f(e_2)$

$e_1 \#_1 e_2$ iff $f(e_1) \#_2 f(e_2)$

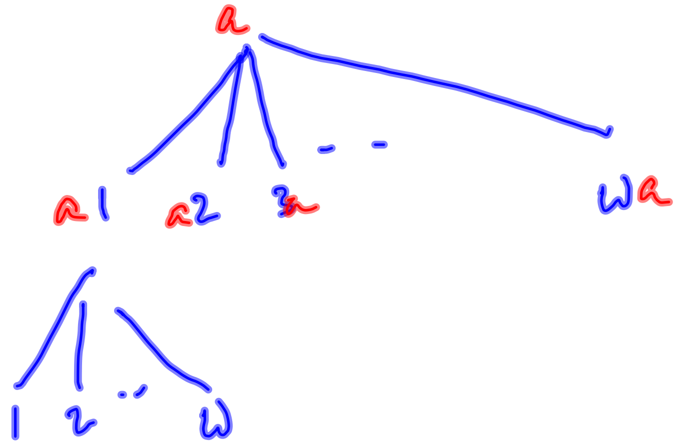
$\varphi(e_1) = \varphi_2(f(e_2))$

$c \approx c'$ iff $ES \backslash c$ is isomorphic to $ES \backslash c'$

$LES = (E, \leq, \#, \ell)$ is regular if $\approx$ is of finite index and $|enabled(c)|$ is bounded for every c

Thiagarajan's Conjecture:

Every regular labelled event structure is "generated" by a finite labelled 1-safe net


a
a1
a2
3a
. . .
ωa
1
2
. .
ω

Trace-labelled event structure

Implicit labelling has a constraint

Finite 1-safe net

$N = (P, T, F, M_{in})$

$I_N \subseteq T \times T$

$t_1 I_N t_2$ if $\,^\circ t_1^\circ \cap \,^\circ t_2^\circ = \phi$

Corresponding ES

$ES = (E, \leq, \#, \lambda)$

$\lambda : E \to T$

$e_1 \; co \; e_2 \Rightarrow \lambda(e_1) I_N \lambda(e_2)$

$\lambda : E \to (T, I_N)$

$\downarrow e$ is finite $\forall e$

Trace labelled $ES = (E, \leq, \#, \lambda)$, $\lambda : E \to (\Sigma, I)$

- $\#_\mu \subseteq \# \Rightarrow$ minimal conflict relation

  $e \#_\mu e'$ if $e \# e'$, $(\downarrow e \times \downarrow e') \cap \# = \{(e,e')\}$

- $<\cdot$ immediate successor : $e <\cdot f$; $e < f$

  $\nexists g \; e < g < f$

$e \#_\mu e' \Rightarrow \lambda(e) \neq \lambda(e')$

$e <\cdot e'$, or $e' <\cdot e \Rightarrow (\lambda(e), \lambda(e')) \in D$

$(\lambda(e), \lambda(e')) \in D \Rightarrow e \leq e'$ or $e' \leq e$ or $e \# e'$

Trace structure

Forward closed & prefix closed trace language

$t \in L,\ t' \in L$
& $t \uparrow t'$ exists
$\Rightarrow t \uparrow t' \in L$

$t \sqsubseteq t'$ & $t' \in L \Rightarrow t \in L$

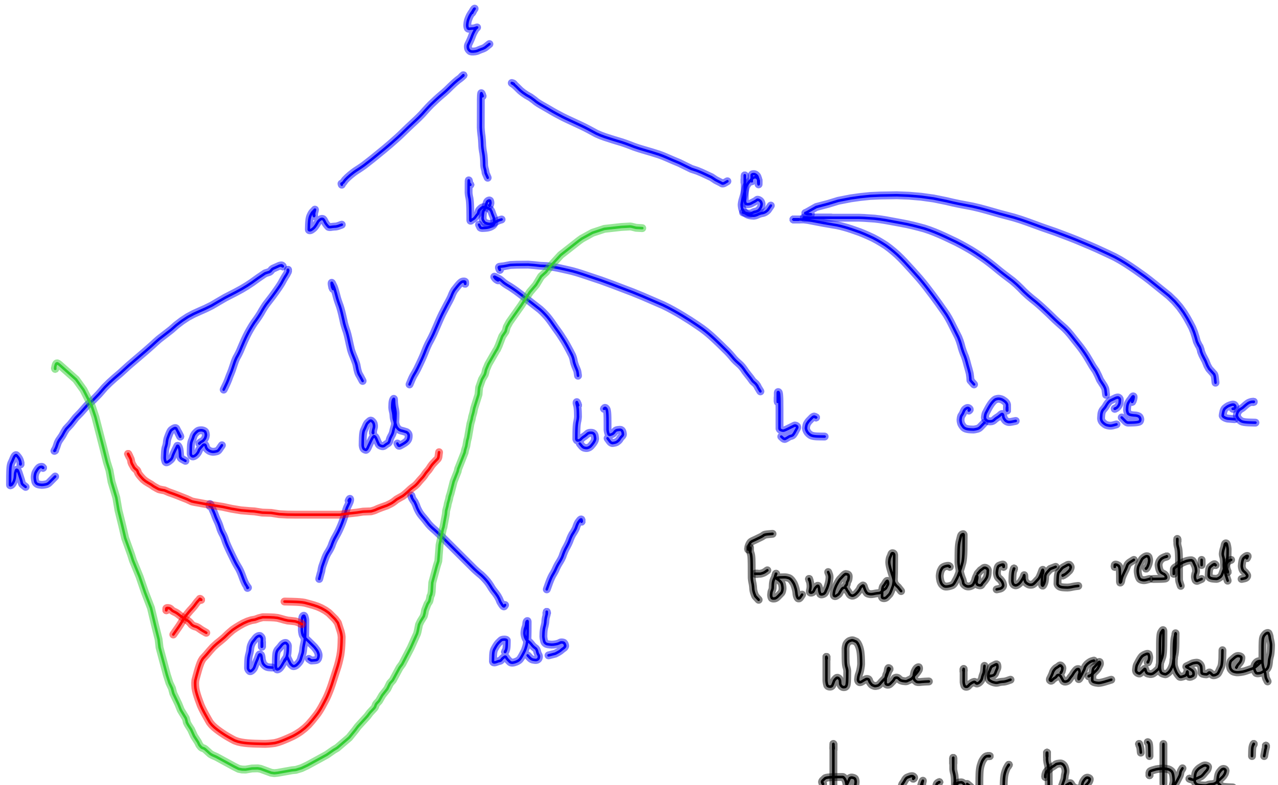
$\Sigma = \{a, b, c\}$

$I = \{(a,b), (b,a)\}$

$L:$ ε

$ac \rightarrow a$
$bc \rightarrow b$ } pref closure

$a, b \rightarrow ab$ } fwd clos.



Forward closure restricts where we are allowed to cutoff the "tree" of traces

Let B be a trace structure

B is regular

$\Downarrow$

$\sigma \in B$

$B_\sigma \triangleq \{\sigma' \mid \sigma\sigma' \in B\}$

$\sigma_1 \approx \sigma_2$ iff $B_{\sigma_1} = B_{\sigma_2}$

B regular iff $\approx$ is of finite index

$\mathrm{lin}(B)$ is a recognizable string lang over Σ

$\therefore$

B has a natural p.o. structure

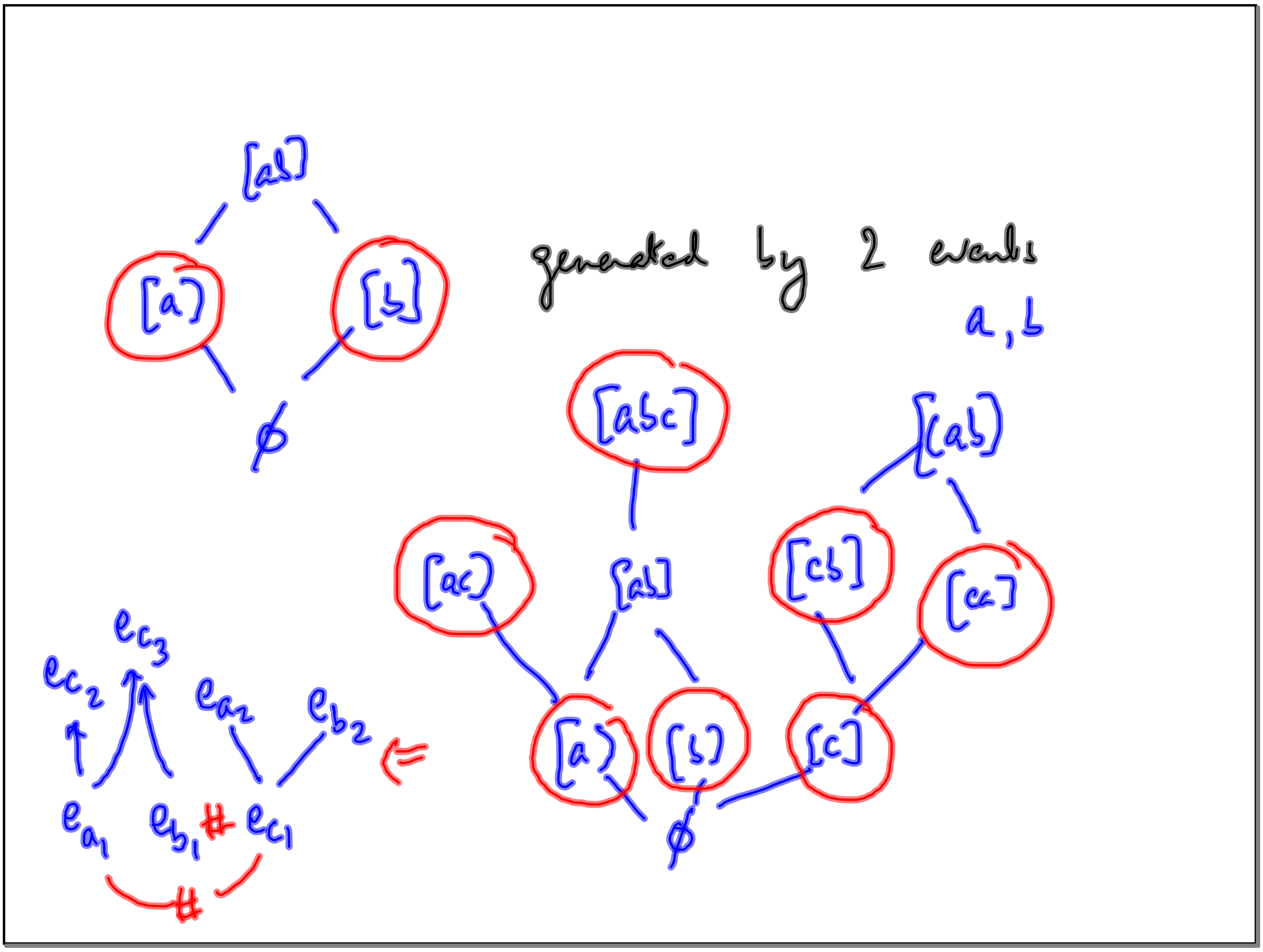
$\sigma \sqsubseteq \sigma'$ if σ' is an extension of σ

$[a] \sqsubseteq [ab]$ $\quad\quad \sqsubseteq \; : \; \leq$

$[b] \sqsubseteq [ab]$ $\quad\quad \cancel{\#} \; : \; \#$

This is how we went from Nets to ES

$N \longrightarrow TL(N) \longrightarrow$ identified events $\rightarrow ES(N)$


[ab]
[a]
[b]
ϕ
generated by 2 events
a, b
[abc]
[ab]
[ac]
[ab]
[cb]
[ca]
[a]
[b]
[c]
ϕ
e_{c_3}
e_{c_2}
e_{a_2}
e_{b_2}
e_{a_1}
e_{b_1}
#
e_{c_1}
#

Given $w \in \Sigma^*$, $last(w)$ is the last letter in w

$\sigma \in \Sigma^*/\sim$ $\quad last(\sigma) = \{ last(w) \mid w \in \sigma \}$

σ is a prime trace if $|last(\sigma)| = 1$

$B \longrightarrow B_{pr}$ is the set of prime traces in B

$lin(B)$ is regular $\longrightarrow$ $lin(B_p$

Let $\mathcal{A}$ be an fsm for lin(B)

$\downarrow$

(Q, q_{in}, δ, F)

$\mathcal{A}_{pr}$: $Q_{pr} = Q \times 2^{\Sigma}$

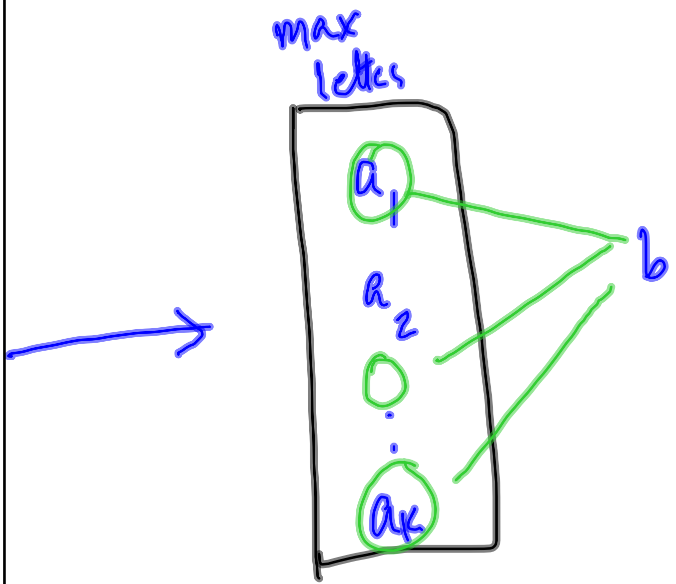
labels on the max. events in the trace seen so far

$\Sigma = \{a, b, c\}, \quad I = \{(a,b), (b,a)\}$

$A: \quad q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{c} q_3$

a   a b   c a b

$(q_0, \phi) \quad (q_1, \{a\}) \quad (q_2, \{a,b\}) \quad (q_3, \{c\})$



Add edge (a_i, b)
for each a_i s.t.
$a_i \, D \, b$

$q \xrightarrow{b} q'$ in A

$(q, X) \xrightarrow{b} (q', X')$

$X' = X \setminus \{a \mid a \in X, a \, D \, b\}$
$\cup \{b\}$

A_{pr} : $F_{pr} = F \times \Sigma$

($\text{Recall } Q_{pr} = Q \times 2^{\Sigma}$)

$\therefore$ $\mathrm{Lin}(B)$ regular $\Rightarrow$ $\mathrm{lm}(B_{pr})$ regular

Conversely $\mathrm{lin}(B_{pr})$ regular $\Rightarrow$ $\mathrm{lm}(B)$ regular & so B is regular

$\mathcal{A}_{pr}$ for $Lin(B_{pr})$

(Σ, I) Fix some distribution $(\Sigma_1, .., \Sigma_k)$ that corresponds to I

$B = \{ \sigma \mid \text{If } \sigma' \sqsubseteq \sigma \text{ is prime}, \sigma' \in B_{pr} \}$

B accepts σ if each maximal prime trace in σ belongs to B_{pr} Simulate Zielonka's construction

$(B, \sqsubseteq) \implies (E, \leq, \#, \lambda)$

B_{pr} $\lambda : E \to \Sigma$

trace labelled

$\Longleftarrow$

$\lambda(c)$ is a trace $(B, \sqsubseteq)$ $\longleftarrow$ $c \subseteq E$

$\longleftarrow$ $(C_{ES}, \subseteq)$

$(B, \sqsubseteq) \xleftarrow{\lambda(c) \,\leftarrow\!\mapsto\, c \subseteq E} (E, \leq, \#, \lambda)$ trace labelled

regular $\longleftrightarrow$ regular

fin. many B_σ — $ES \setminus c$ has fin. many $\approx$ classes

$\sigma \longrightarrow c_\sigma$

$B_\sigma = \{\sigma' \mid \sigma\sigma' \in B\} \longrightarrow ES \setminus c_\sigma$

Regular trace labelled ES $(E, \leq, \#, \lambda)$ ---> 1 safe net

$\downarrow$

Regular trace structure $(B, \sqsubseteq)$ $\longrightarrow$ $lin(B_{pr})$ is reg.

$\mathcal{A}_{pr}$ fsm

$\downarrow$

$\mathcal{A}_B$

$\mathcal{A}$: $S = \{S_p\}_{p \in P}$

$\rightarrow_a \subseteq S_{loc(a)} \times S_{loc(a)}$

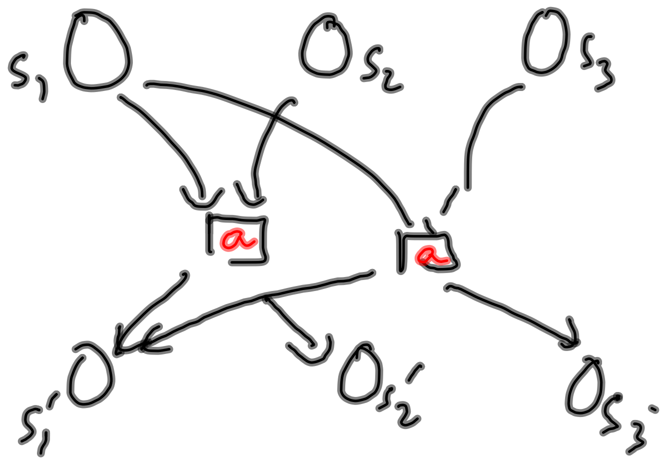
$\mathcal{A}_B \Rightarrow$ Net , labelled

$\bigcup_{p \in P} S_p \longrightarrow P$

$(s_1, s_2) \xrightarrow{a} (s_1', s_2')$

$(s_1, s_3) \xrightarrow{a} (s_1', s_3')$

Each entry in δ_a generates an a-labelled $t \in T$



$(E, \leq, \#, \lambda)$ trace labelled over (Σ, I)

$((E, \leq, \#, \lambda), \varphi)$ where $\varphi : E \to \Delta$

labelled (trace labelled) event structure

lift (Σ, I) to $(\Sigma \times \Delta, I)$

$(a_1, d_1) \, I \, (a_2, d_2)$ iff $a_1 I a_2$

For ES, $a \in \Sigma$

$$L_a = \{ tr(\downarrow e) \mid \lambda(e) = a \}$$

$$= \{ \sigma \in B_{pr} \mid last(\sigma) = a \}$$

$(E, \leq, \#, \varphi) \xrightarrow{\text{construct}} (P, T, F, M_{in}, \lambda)$

$\varphi : E \to \Sigma$ $\qquad$ $\lambda : T \to \Sigma$

$\varphi \to \varphi_1, \varphi_2$

isomorphic

unf

φ_1 is a trace labelling

$(E_N, \leq_N, \#_N, \lambda_N)$

$\varphi_1 \quad \varphi_2$

Problem in general case:

$$\underbrace{ES\backslash c_1 \approx ES\backslash c_2 \approx ES\backslash c_3}_{f_{13}}$$

Two min events f_{12} f_{23}