Lower bound for determinization

$2^{k^{n^3}}$

k states per process

k^n global states

At least one process needs

N processes $\quad 2^{K^N/N}$ $\quad \sqrt[N]{2^{K^N}}$

K states per process

$\exists$ nondet AA states of each proc poly(K)

global state space $\approx K^{O(N)}$

Smallest det AA for L has $\geq 2^{K^N}$ global states

$\Rightarrow$ local state space of at least one process $\geq \sqrt[N]{2^{K^N}}$

m^{th} last letter is b over $\Sigma = \{a,b\}$

NFA $O(m)$ $m+1$ Guess last m moves

DFA 2^m right-equivalence classes

2^m states

$\underbrace{a \; . \; . \quad b}_{m}$

Generalize : for some i, the $i \cdot m^{th}$ last letter is b

Distributed version of this

i.e. $K^{N\text{th}}$ last letter is b

a, b

$P_N \quad \ldots \quad P_2 \quad P_1$

c_1

c_2

c_{N-1}

c_N

Carry letters

"Carry extended words"

After every K $\{a,b\}$'s, c_1

After every K c_1's, $\exists c_2$

$\vdots$

$w \in \{a,b\}^* \longrightarrow C(w)$ carry extension of w

Focus on words of the form $C(w)$

L_m - modulo m^{th} last letter is b (over $\{a,b\}$)

L_k^N $\quad$ L'_{k^N} = trace lang generated by

$$\{C(w) \mid w \in L_k^N\}$$

Suppose u, v of length k^N over $\{a, b\}$

$C(u) \not\equiv_R C(v)$ wrt L'_{k^N}

$\exists w \quad uw \in L_{k^N} \quad vw \notin L_{k^N}$

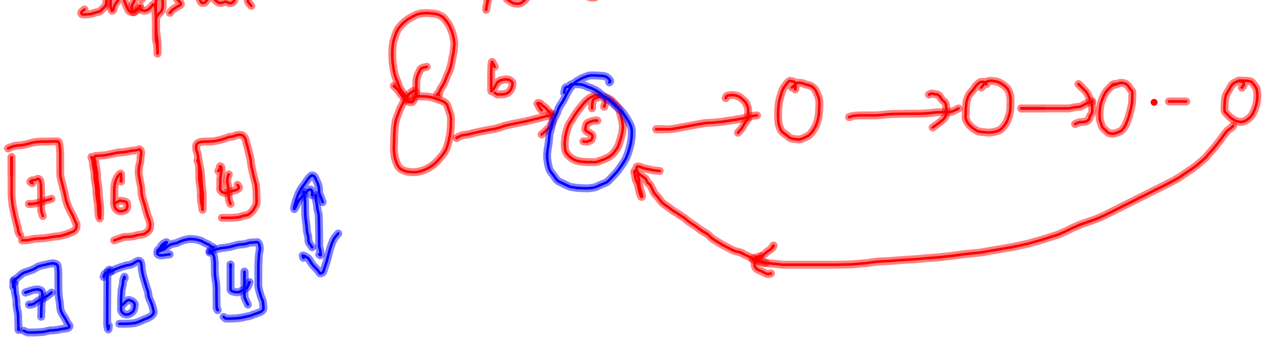
$\therefore$ smallest DFA for L'_{k^N} has $\geq 2^{k^N}$ global states

Smaller NAA for L'_{KN}

States of each P_i are $\{0, \ldots, K-1\}$

P_1 : $i < K-1$ $\quad i \xrightarrow{a,b} i+1$

Snapshot → copy your current counter value



NAA ~ $O(K^2)$ local states per process

$2 \cdot K \cdot K$

Trace closure

(Σ, I) $\quad L \subseteq \Sigma^*$ regular, but not trace closed

$$\mathrm{TraceClosure}(L) = \{ w' \mid \exists w \in L,\ w \sim w' \}$$

$L = (ab)^*$, $\quad a I b$

$TC(L) = \{ w \mid w$ has equal # of a's, b's $\}$

$$TC(L) \cap a^* b^* = \{ a^n b^n \mid n \geq 0 \}$$

$L \subseteq \Sigma^*$ regular, is Closure(L) regular (wrt I)

Undecidable

Post's Correspondence Problem:

Σ $\quad u_1, u_2, \ldots, u_k \in \Sigma^*$

$\quad v_1, v_2, \quad v_k \in \Sigma^*$

Find a sequence $i_1 i_2 \ldots i_n$ s.t $u_{i_1} u_{i_2} \ldots u_{i_n} = v_{i_1} v_{i_2} \ldots v_{i_n}$

	u	v
1	a	ab
2	bbb	bb
3	ac	a
4	b	cb

$u_1u_2u_3u_4 = a|bbb|ac|b$

$v_1v_2v_3v_4 = ab|bb|a|cb$

$1, 2, 3, \ldots, k$

alphabet $\Delta = \{b_1, b_2, \ldots, b_k\}$

List 1: $g : \Delta \to \Sigma^*$ wlog $\Sigma = \{a, b\}$

2. $h : \Delta \to \Sigma^*$

PCP : Find $w \in \Delta^*$ s.t. $g(w) = h(w)$

Choose a new letter $c \notin \Delta \cup \Sigma$ — $I = \{(x, c), (c, x) \mid x \in \Delta \cup \Sigma\}$

$$W_g = \{ w \cdot g(w) \cdot c^{|g(w)|} \mid w \in \Delta^+ \}$$

$$W_h = \{ w \cdot h(w) \cdot c^{|h(w)|} \mid w \in \Delta^+ \}$$

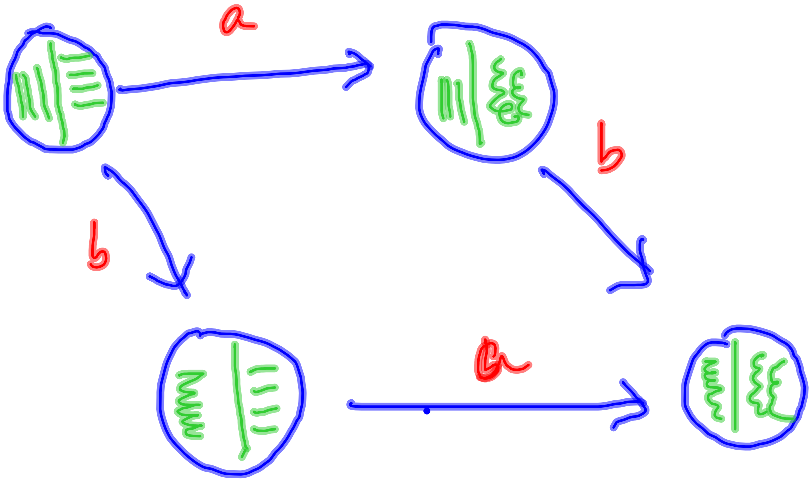
Identify L_g, L_h s.t. $[L_g] = \overline{W_g}$, $[L_h] = \overline{W_h}$.

No solution $\Rightarrow [L_g] \cup [L_h] = (\Delta \cup \Sigma \cup \{c\})^*$

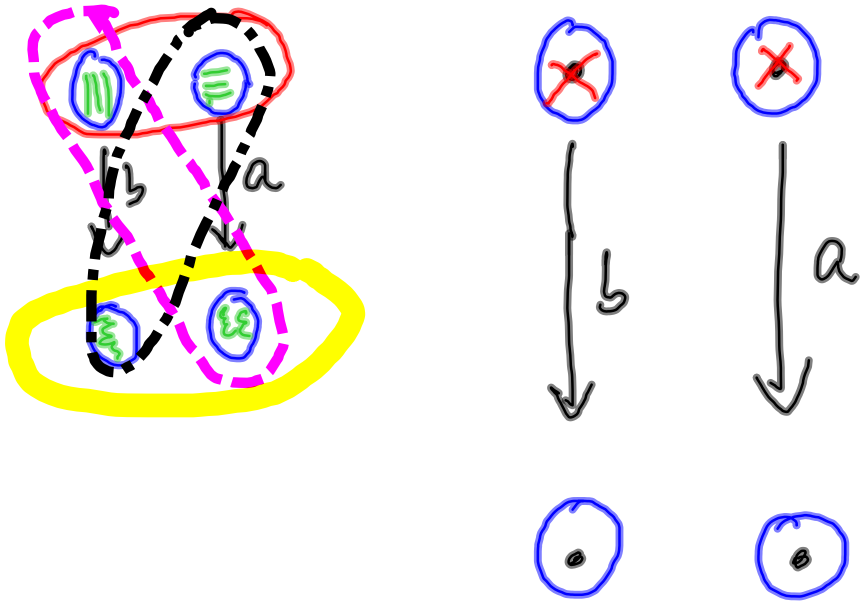
$\exists$ solution $\Rightarrow \neq (\Delta \cup \Sigma \cup \{c\})^*$

not regular

Petri nets



Distribute states into smaller units
Transitions affect portions of state


b
a
b
a

Petri net

Net = $N = (P, T, F)$

P: Places $\equiv$ "local" state

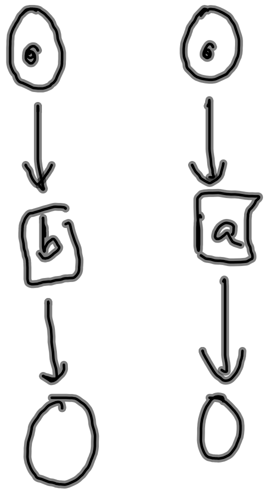
T: Transition $\equiv$ "change" of local state

F: "Flow relation" $F \subseteq (P \times T) \cup (T \times P)$

Places

Transitions

Flow



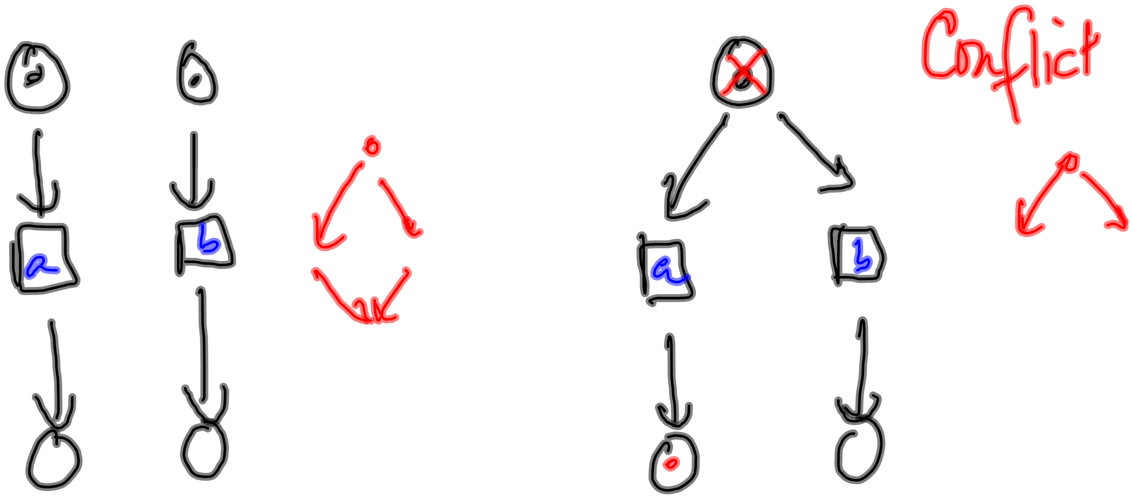
Concurrency

Global state = Marking = $M : P \to \{0,1\}$

Token game

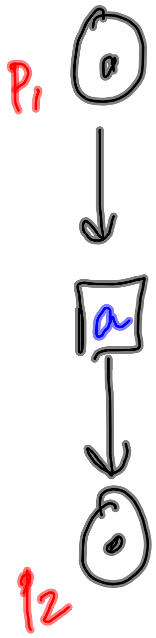
t occurs : Requires all incoming p to be marked

Fires – causes outputs to be marked, consumes input



a & b consume disjoint resources

a & b compete for input token



What happens?

Marking on p_2 is "idempotent"

p_2 counts } Petri Net = Place/Transition Net

p_2 disables a !

Elementary Net Systems