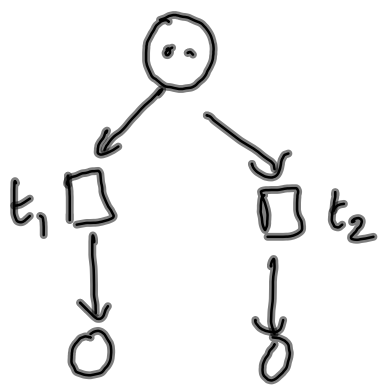
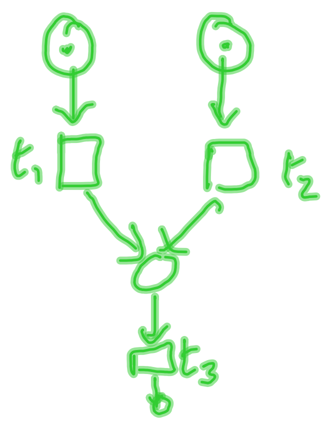
ENS

P/T Nets : $((P, T, F), M_{in})$ = "Petri" nets

Marking $M : P \longrightarrow \mathbb{N}_0$

Infinite state space (reachable markings)



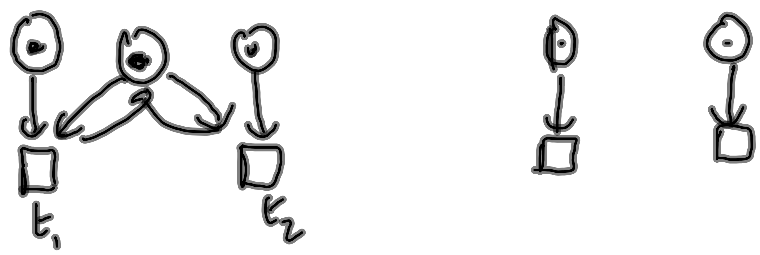


$PN = (P, T, F, M_{in})$

is k-bounded, or <u>k-safe</u>,

if $\forall M \in Reach(M_{in}),\ \forall p \in P,\ M(p) \le k$

1-safe — "like" ENS



Is a PN k-safe? (for some k)

Boundedness

Weighted nets: Replace F by

$$W : ((P \times T) \cup (T \times P)) \to \mathbb{N}_0$$

Reachability

Is $M \in Reach(M_{in})$?

Coverability

Does there exist $M' \in Reach(M_{in})$, $M' \geq M$

$M_1 \geq M_2$, $M_2 \xrightarrow{t} \Rightarrow M_1 \xrightarrow{t}$

$M_2 \xrightarrow[t_1 \cdots t_k]{\sigma} M_1$ $\quad M_1 > M_2$

$\quad \exists p.\ M_1(p) > M_2(p)$

$\quad \forall p.\ M_1(p) \geq M_2(p)$

$\big\} \sigma$

$M_1' > M_1$ $\quad M_1'(p) > M_1(p)$

Generalized marking : $M : P \to \mathbb{N}_0 \cup \{\omega\}$

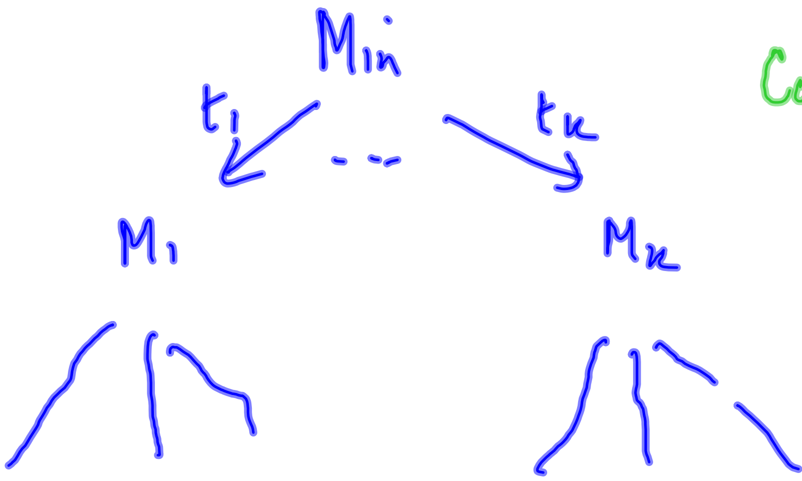
$$M_{in} \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \xrightarrow{t_3} \cdots \xrightarrow{t_k} M_k$$

Generalized firing sequence

Replace $M_j(p)$ by ω if

- $\exists i<j \quad M_i \leq M_j$
- $M_j(p) > M_i(p)$

Explore $Reach(M_{in})$ as a sequential transition system



Coverability Tree

Karp, Miller
1969

Truncate a path if ~~we generate an ω~~ ~~or~~ repeat a marking (wrt generalized firing rule, see later)

Claim : CT is finite

Suppose CT is not finite $\Rightarrow \exists$ infinite path in CT

König's lemma: Any infinite finitely-branching tree has an infinite path

$M_{in} \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \quad \cdots \quad \xrightarrow{t_i} M_i \longrightarrow \quad \cdots$

$\{M_{in}, M_1, M_2, \ldots\}$ does contain any M_i, M_j $i < j$ s.t. $M_i \leq M_j$

Dickson's Lemma:

Any infinite ~~set~~ sequence of vectors in $\mathbb{N}^k$ must contain an infinite non decreasing subsequence

By induction on k

Generalized markings.

Generalized firing rule

$$M \xrightarrow{t} M'$$

– . . .. $Gen(M) \xrightarrow{t}$ ~~two~~ manipulate finite places as usual

ω remains ω

upgrade to ω if req.

Vector Addition System

Initial vector v_0 : non negative

Change vectors $v_1, \ldots, v_k$

Each step: add a change vector to current vector

s.t. you stay nonnegative

Change wrt $p = 0$

Nets: $t \rightarrow$ column vector $p \mapsto F(t,p) - F(p,t)$ $\quad \vec{t}$

$m \xrightarrow{t} \quad \vec{m} + \vec{t} \cdot k$

$$\begin{array}{cc} & T \\ P & \left[\bar{t}_1 \; \bar{t}_2 \; \ldots \; \bar{t}_n \right] \end{array}$$

N = "adj" matrix of

$m \xrightarrow{\sigma} m_\sigma$ Parikh Vector(σ)

$= \left(\#_{t_1}(\sigma), \#_{t_2}(\sigma), \ldots, \#_{t_k}(\sigma)\right)$

$t_1 t_2 t_1$

$t_1 t_1 t_2$

$m_1, m_2, \ldots, m_k$

$$m_\sigma = m + N \cdot \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_k \end{bmatrix}$$

"Marking equation"

Reachability ~~Reach~~ Is $m \in Reach(m_{in})$?

If $m = m_{in} + Nx$

has no solution $x \geq 0$

then m is not reachable

Invariants

Place Invariant $i : P \to \mathbb{Z}$

$\forall \vec{t}$: $i \cdot t = 0$

$i N = (0, 0, \ldots 0)$

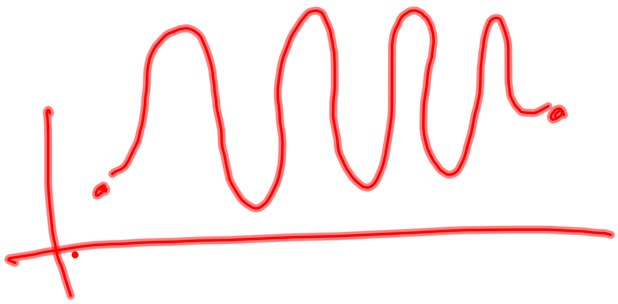
Find i by solving $y . N = (0, \ldots 0)$
(in integers)

$\forall m \in Reach(m_{in})$ $i.m = i.m_{in}$

$$m = m_{in} + Nk$$

$$i \cdot m = i \cdot m_{in} + \underbrace{iNk}_{0}$$

$\exists i : im \neq im_{in} \Rightarrow m$ not reachable



Transition invariant

$$j : T \to \mathbb{Z} \quad \text{s.t.} \quad N \cdot j = 0$$

Given firing sequence σ

$$PV(\sigma) : T \to \mathbb{N}_0$$

$PV(\sigma)$ is a T-invariant iff σ is a "loop"

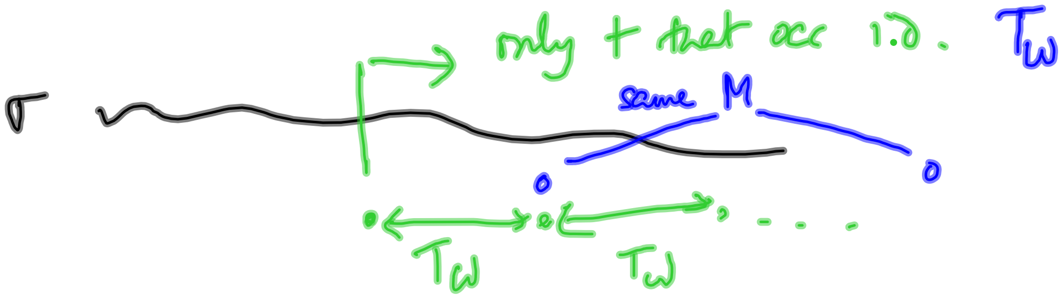
$$m_\sigma = m_{in} + Nk$$

Suppose $\overset{\text{bounded}}{\wedge}$ N has an infinite firing sequence σ

$\exists$ a T-invariant J s.t.

$J(t) \geq 1$ if t occurs $\overset{\text{inf. often}}{\text{i.o.}}$ in σ

$J(t) = 0$ if t occurs finitely (or zero times) in σ



1-safe net $\longrightarrow$ Unfold to a possibly infinite occurrence net

Unfolding is "complete"

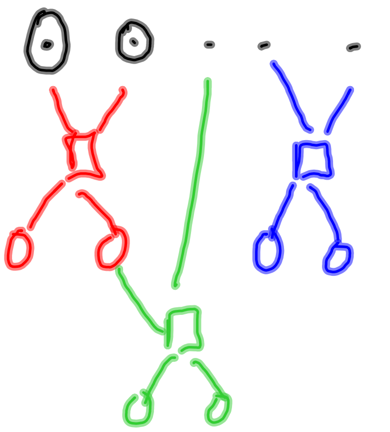
- Every reachable marking is represented
- Every transition that is not dead is fired in the unfolding

Identify a finite prefix of unfolding that remains complete

Construct unfolding

Add one (copy) of an event at each stage

Adding e generates a "local" configuration



$\downarrow e = [e]$

After $[e]$, future $\Uparrow e$

If $\Uparrow e_1 \cong \Uparrow e_2$ and e_1 is "smaller" or "earlier" than e_2, cut off exploration at e_2

$$M([e_1]) = M([e_2])$$

Configurations C_1 C_2

i.e. downward closed subsets of occ. net

McMillan

Cutoff at e_2

if $|C_1| < |C_2|$

Completeness

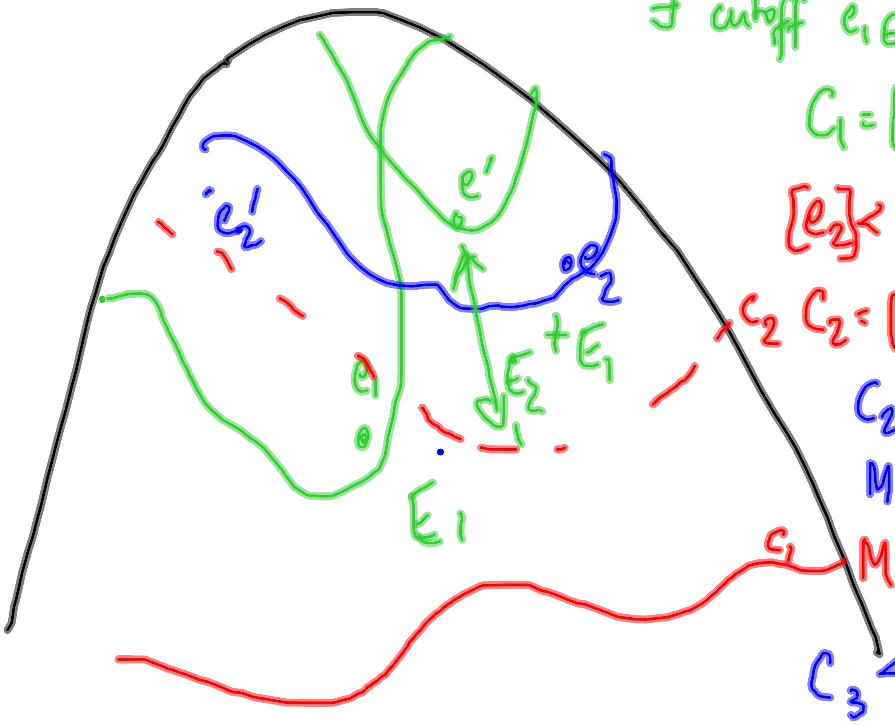
Suppose M is reachable but not represented

$\exists\, c_1$ that reaches M

$\exists$ cutoff $e_1 \in C_1$ wrt e_2

$C_1 = [e_1] \oplus E_1$

$[e_2] < [e_1]$, $M([e_1]) = M([e_2])$



$C_2 = [e_2] \oplus E_2$

$C_2 < C_1$

$M(C_2) = M(C_1)$

$C_3 < C_2 < C_1$

$\prec$, a p.o. on finite configurations of the unfolding is adequate if

Esparza, Romer, Vogler ≈ 2000

- $\prec$ is well founded
- $C_1 \subset C_2 \Rightarrow C_1 \prec C_2$
- $C_1 \prec C_2$, $M(C_1) = M(C_2)$ $\left(\therefore \Uparrow C_1 \cong \Uparrow C_2\right)$

  $\forall E \quad C_1 \oplus E \prec C_2 \oplus I(E)$

  $|C_1| \le |C_2| \Rightarrow \forall E \quad |C_1 \oplus E| \le |C_2 \oplus I(E)|$