Elementary Net Systems

Net $N = (P, T, F)$ (B, E, F) — event

Modelling

State space is bounded

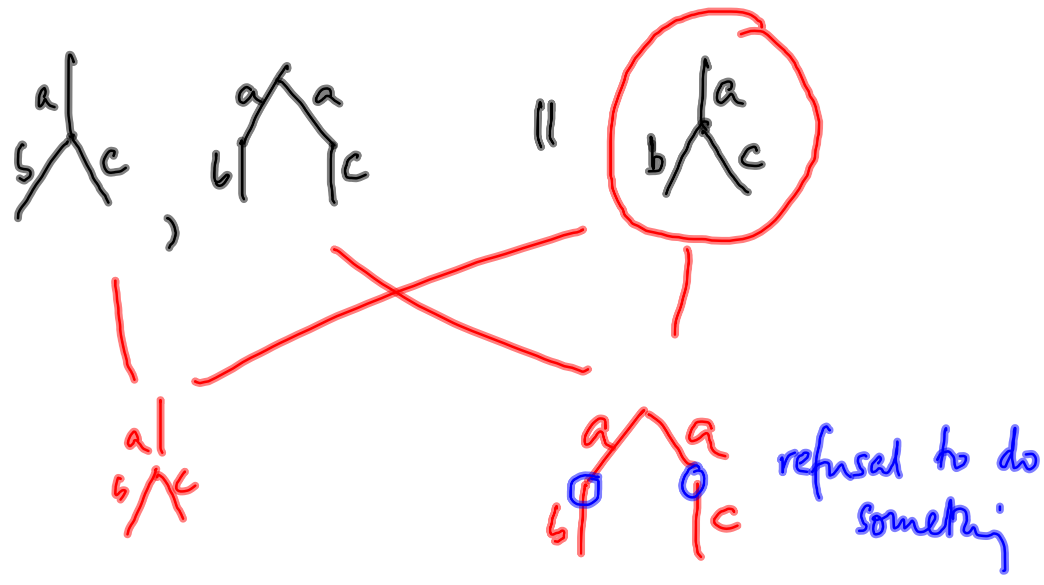
↳ for Petri nets, reachability is hard

Today: Behaviour of ENS

Languages vs trees

FSA's: languages abstract the behaviour

With interaction:



$TS = (S, \rightarrow, s_{in})$

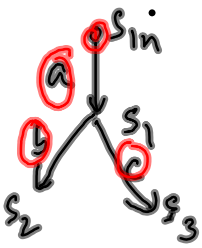
$L(TS) = \{ w \mid \exists \text{ run } s_{in} \xrightarrow{a_1} s_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} s_n, \; w = a_1 a_2 \ldots a_n \}$

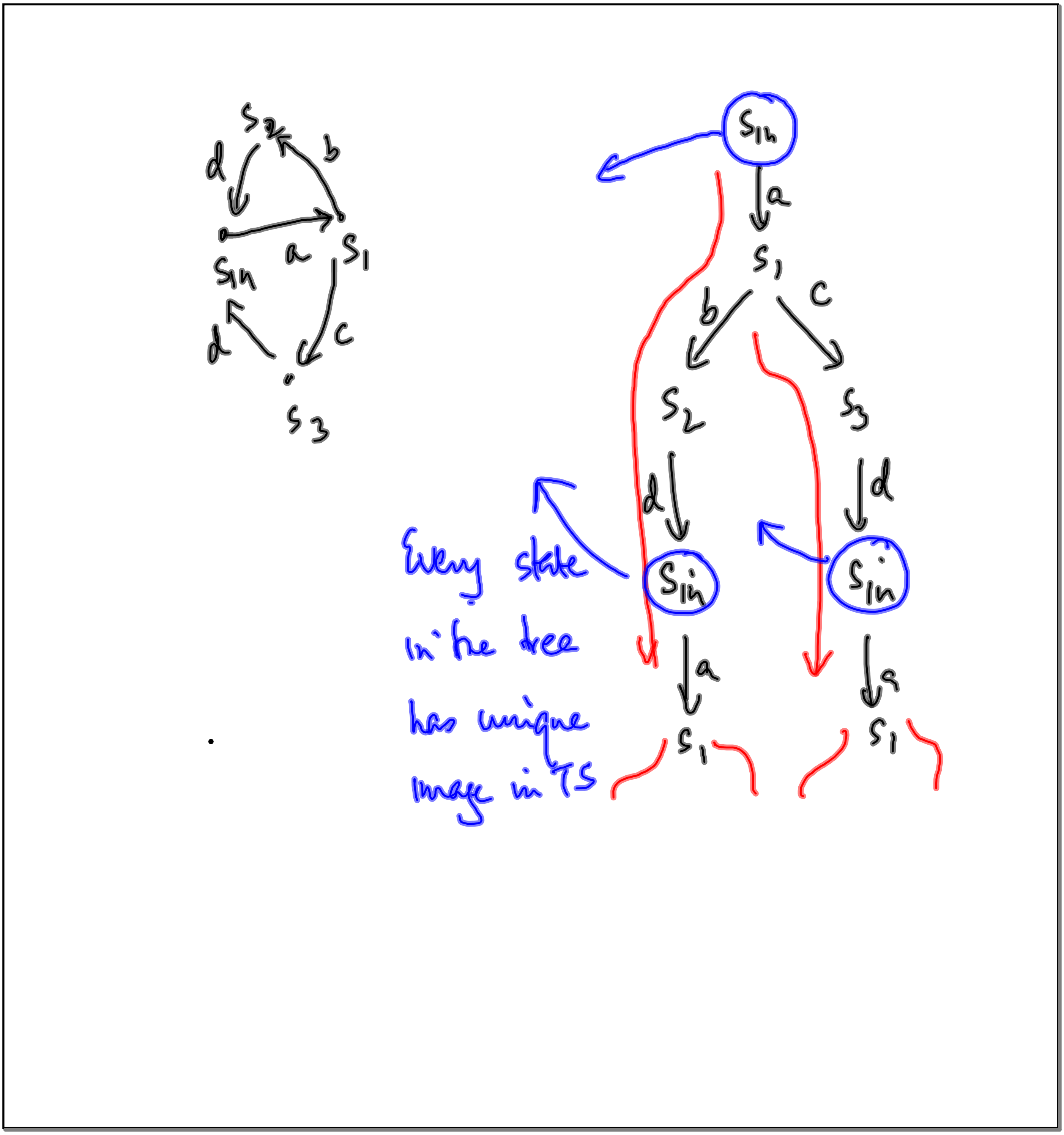
Prefix closed

Glue together runs on common prefixes

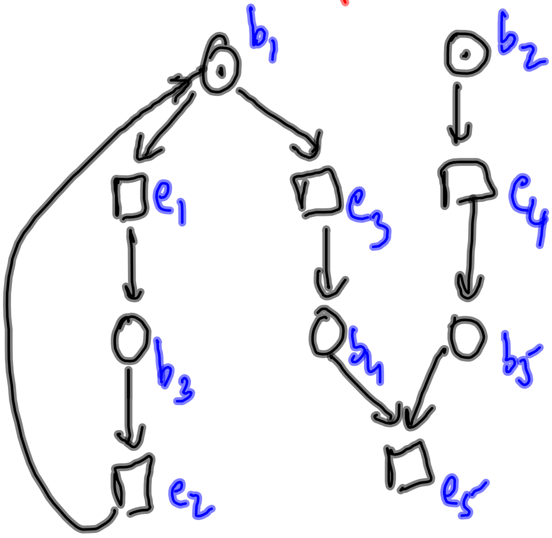
$s_{in} \xrightarrow{a} s_1 \xrightarrow{b} s_2$

$s_{in} \xrightarrow{a} s_1 \xrightarrow{c} s_3$




Every state
in the tree
has unique
image in TS

Assume "contact free":



$e_1 e_2 \; e_3 e_4 e_5$

$e_4 e_1 e_2 e_3 e_5$

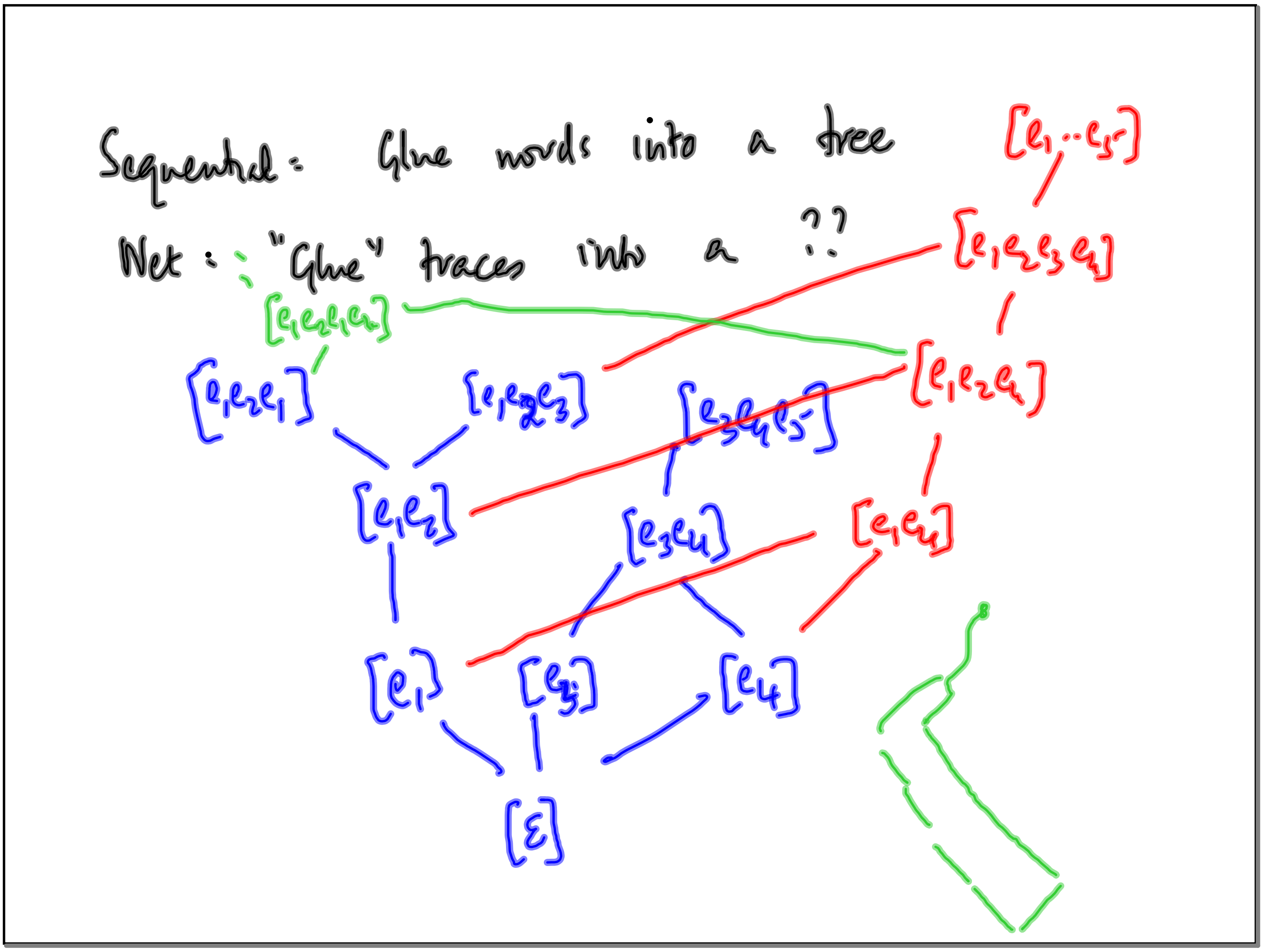
$N = (B, E, F)$

$ENS = (N, c_{in})$

Language is not a set of words, but a set of (Maz) traces

L(ENS) is a prefix closed trace lang over (Σ, I) $\quad \Sigma = E$

$I = \{ (e_1, e_2) \mid {}^\bullet e_1^\bullet \cap {}^\bullet e_2^\bullet = \phi \}$


Sequential: Glue words into a tree
Net: "Glue" traces into a ??
[e1 .. e5]
[e1e2e3e4]
[e1e2e4]
[e1e2e1e2]
[e1e2e1]
[e1e2e3]
[e3e4e5]
[e1e2]
[e3e4]
[e1e4]
[e1]
[e3]
[e4]
[ε]

Define partial order on traces:

$\rho \sqsubseteq \rho'$ if ρ' is an extension of ρ

$\rho \uparrow \rho'$ (compatibility) $\exists \rho''$ s.t. $\rho \sqsubseteq \rho''$, $\rho' \sqsubseteq \rho''$

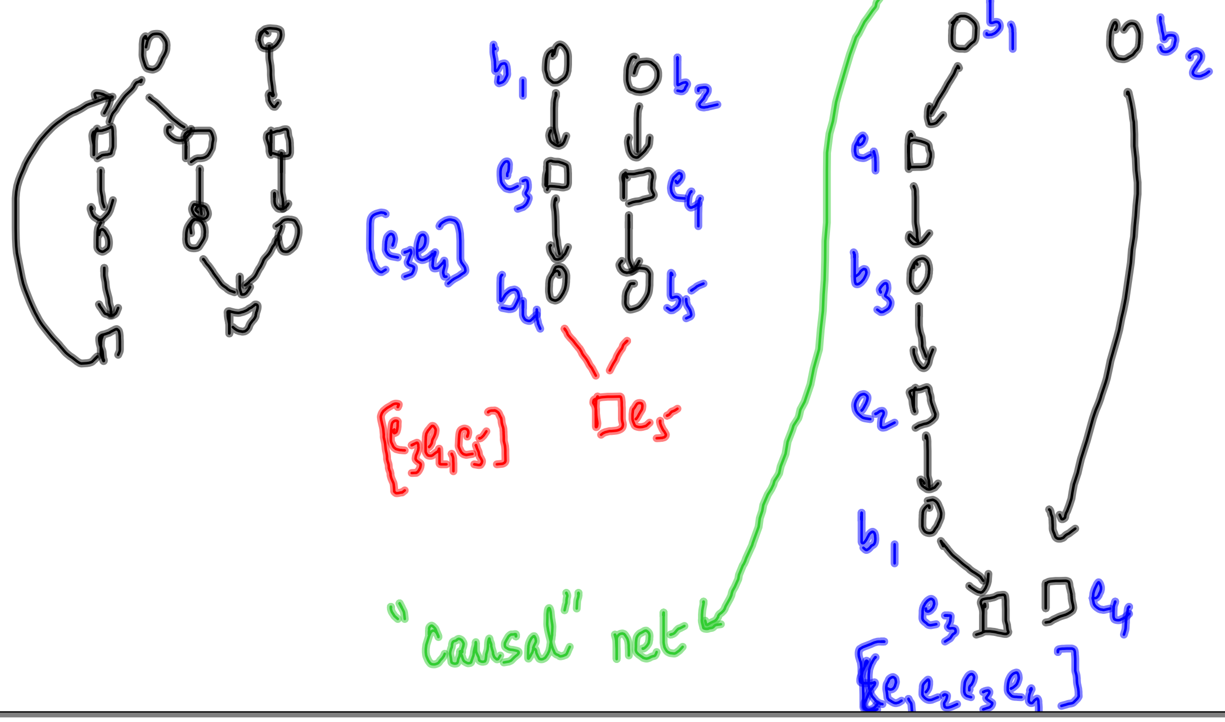
$\rho \not\uparrow \rho'$ <u>not</u> compatible $\nexists \rho''$ . . .

<u>Claim</u> $\rho \not\uparrow \rho' \iff \exists \rho_1, e, e'$, $\rho_1 e \sqsubseteq \rho$, $\rho_1 e' \sqsubseteq \rho'$

At c_{ρ_1} (i.e. $c_{in} \xrightarrow{\rho_1} c_{\rho_1}$) e, e' are in conflict

Traditionally: net theory described "non sequential" behaviour using nets!

"Processes" : all choices resolved $\forall b \; |{}^\bullet b| = |b^\bullet| \leq 1$



Firing sequence ρ $\longrightarrow$ Process (causal net) N_ρ

$\rho \sqsubseteq \rho e$, $N_\rho \sqsubseteq N_{\rho e}$

$N_\rho = (B_\rho, E_\rho, F_\rho, \varphi_\rho)$ — φ_ρ: labelling of $B_\rho \cup E_\rho = X_\rho$ in terms of $B \cup E = X$

$N_\rho \subseteq N_{\rho'}$ if $B_\rho \subseteq B_{\rho'}$, $E_\rho \subseteq E_{\rho'}$, $F_\rho \subseteq F_{\rho'}$, $\varphi_\rho = \varphi_{\rho'} \restriction X_\rho$

$\rho \approx \rho' \Rightarrow N_\rho = N_{\rho'}$

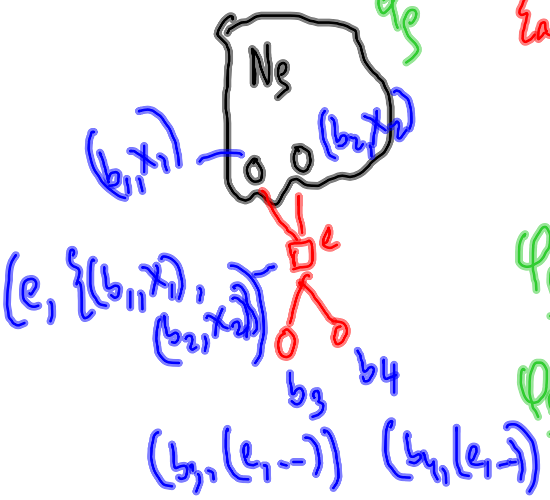
Lem: $\rho \sqsubseteq \rho'$ iff $N_\rho \subseteq N_{\rho'}$

To construct $N_\xi = (B_\xi, E_\xi, F_\xi)$, φ_ξ Aux: c_ξ

$\{(b,\phi) \mid b \in B\}$??

$N_\varepsilon = (\phi, \phi, \phi)$ $c_\varepsilon = \{(b,\phi) \mid b \in c_{in}\}$ $\varphi_\varepsilon = \phi$

$N_\xi = (B_\xi, E_\xi, F_\xi)$, φ_ξ e enabled at c_ξ in N_ξ



Each element of N_ξ is of the form (b,X) or (e,Y) where $b \in B$, $e \in E$

$\varphi_\xi(b,X) = b$

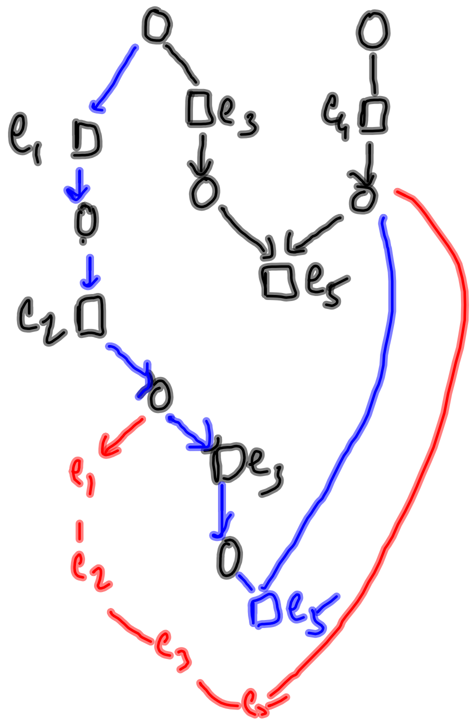
$\varphi_\xi(e,Y) = e$

Update $c_{\xi e}$ appropriately

Glue individual processes together into single structure

Branching processes — Nielsen, Plotkin, Winskel 1981

Construct $\bigcup N_\rho$, ρ an execution of N



$\forall b \; |{}^\bullet b| \leq 1$

... "Occurrence net"

NPW: Throw away the places in the branching process (unfolding) of a net

to get event structure (labelled)

$ES = (E, \leq, \#, \varphi)$

- E set of events
- $\leq$ p.o. on E
- $\#$ irreflexive, symmetric conflict relation inherited via $\leq$ $\quad e_1 \# e_2, e_2 \leq e_3 \Rightarrow e_1 \# e_3$
- $\varphi : E \to \Sigma$

Event structure explicitly records causality, conflict

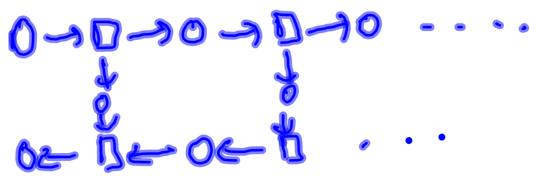
Concurrency is a derived relation

$e \ co \ e'$ if not ($e \leq e'$ or $e' \leq e$ or $e \# e'$)

$\forall e \in E$, $\downarrow e$ is finite

Don't want $\cdots e_1 \rightarrow e_2 \rightarrow e_3$

Also

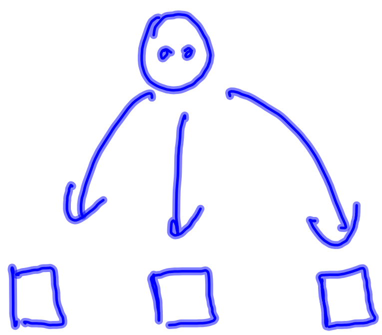


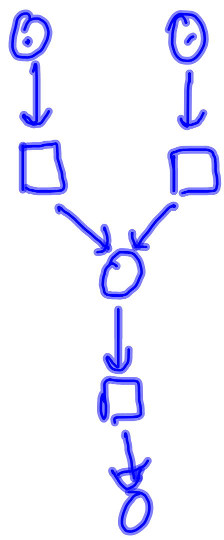
Minimal conflict

$e \#_\mu e'$ if $e \# e'$ and no $e_1 < e$, $e_1 \# e'$

no $e_2 < e'$, $e \# e_2$

2 out of 3 !





Configuration = set of events seen so far

$C \subseteq E$ s.t. $C = \downarrow C$, $C \times C \cap \# = \emptyset$

$(C_{ES}, \subseteq)$

$(Traces(N), \sqsubseteq)$ isomorphic to $(C_{ES}, \subseteq)$

Representation result: $(E, \leq, \#)$ can be recovered from $(C_{ES}, \subseteq)$

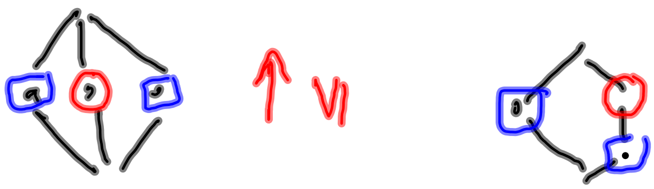
Prime event structures

$(C_{ES}, \subseteq)$: prime algebraic, coherent p.o.

$(X, \sqsubseteq)$ be a p.o.

$x \in X$ is a prime if $\forall Y \subseteq X$, s.t. $\sqcup Y$ exists,

$x \leq \sqcup Y \Rightarrow x \leq y$ for some $y \in Y$



Prime algebraic: Every $x \in X = \bigsqcup \{p \le x,\ p \text{ prime}\}$

$(C_{ES}, \subseteq)$ Some configurations are 'prime'

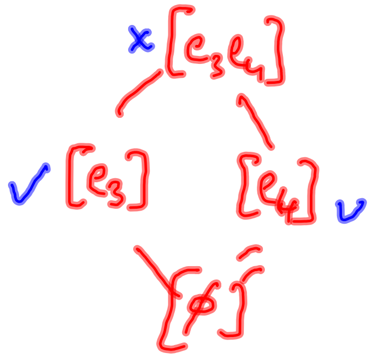
Prime configuration

Minimal way of adding an event

c'

$\uparrow$

c



Recover $(E, \le, \#)$ from $(C_{ES}, \subseteq)$

E = prime configurations

$\le$ = $\subseteq$

$\not\uparrow$ = $\#$

$(E, \le, \#) \rightleftarrows (C_{ES}, \subseteq)$

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