- Pigeon hole:

We are given a sequence of $mn+1$ distinct real numbers. Either there is an increasing subsequence of length $n+1$ or a decreasing subsequence of length $m+1$.

- Let α be an irrational number.
Let N be a positive integer. One of $\alpha, 2\alpha, \ldots, N\alpha$ differs at most $1/N$ from an integer.

- Solution : By pigeon hole.

for each number x_j in the given list of numbers associate a tuple

$$(d_j, i_j)$$

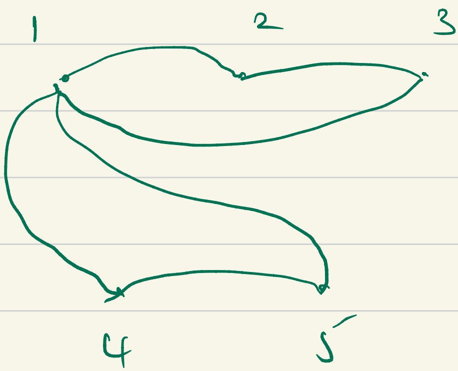
i_j: length of longest increasing subsequence ending in x_j

d_j: length of the longest decreasing subsequence ending in x_j

Claim: If $j \neq k$, $(d_j, i_j) \neq (d_k, i_k)$.

- Permutation graph

$\pi = [3, 2, 5, 4, 1]$



decreasing subsequence 3, 2, 1 is a clique

increasing subsequence 3, 5 is an indep set.

= On these graphs dynamic programming gives polytime solutions to finding largest independent set and largest cliques.

In general graphs - these are hard problems

- Permutations
- Permutations of multisets.
- Strings over a finite alphabet.

- Method of bijections:

<u>Example</u>: a) # Subsets of an n element set.

Subsets with an odd number of elements?

b) A city has 10 intersections. Some will get traffic lights, some with a traffic light may get a gas station. In how many ways can this happen?

$= \overset{k}{\#}$ subsets of $[n]$
$\wedge$

- $\binom{n}{k} = \binom{n}{n-k}$

- Picking 5 elements from $[1, \dots, 30]$ so that there are no consecutive integers;

- Picking a multiset of 5 from [90]
- A salesman has to visit 4 cities 5 times.

  What if he cannot start and end with same city?

- # k element multisets from $[n]$.

- Ex:

  # 5 digit numbers $- - 6 - -$ divisible by 3?

- # 5 digit numbers divisible by 3 and containing a 9.

- 49 Countries taking part in a tournament. Each has a flag with 3 stripes of different colours among RBGY. [3 Countries have same flag]

- Placing 8 rooks on a 8×8 chess board

  Bijection with $\longleftrightarrow$ $\left\{ f : \begin{matrix} 1 \\ 2 \\ \vdots \\ 8 \end{matrix} \xrightarrow{\text{bij}} \begin{matrix} 1 \\ 2 \\ \vdots \\ 8 \end{matrix} \right\} \{ f \text{ bijective} \}$

  $\therefore$ $8!$

- Picking 5 from $[1, \dots, 30]$, so that we have no consecutive integers;

Let $a_1 \lneq a_2 \lneq a_3 \lneq a_4 \lneq a_5$ one such valid subset;

Then $a_1, a_2 - 1, a_3 - 2, a_4 - 3, a_5 - 4$ are all distinct integers ($\because a_2 \geq a_1 + 2, \; a_3 \geq a_2 + 2, \dots$)

$\therefore$ We get a 5 element subset from $[1, \dots, 26]$

This is a bijection $(b_1\, b_2\, b_3\, b_4\, b_5)$

$$\downarrow$$

$$(b_1, b_2 + 1, b_3 + 2, b_4 + 3, b_5 + 4)$$

k size
- Multisets from $\underrightarrow{n}$: $[1, \dots, n]$;

$$\binom{n+k-1}{k}$$