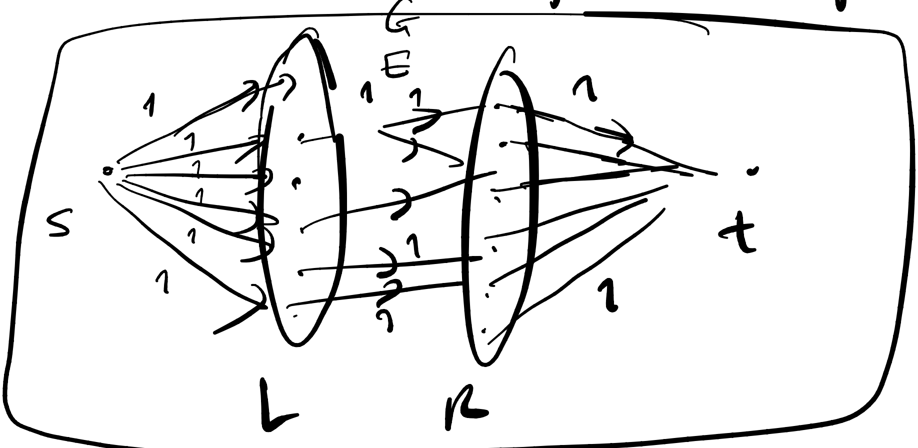
· 21/4/20 :

· Applications of Max flow : —

→ Ford Fulkerson, with integral capacities, yields an integral flow;

- $G = (\overset{L \cup R}{\overset{\shortparallel}{V}}, E)$ be a bipartite graph



$\underbrace{G' = (V', E')}$, $V' = L \cup R \cup \{s, t\}$

Given G, construct G' as above; $\underbrace{c' : E' \to \mathbb{N}}$

$|E'| = O(|E|)$

A max flow from s to t in G'.

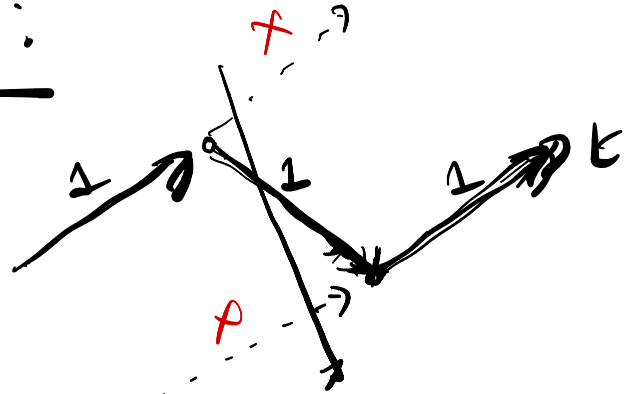
=

- If M is a matching in G then we get a flow of size $|M|$; f
- If there is an integral flow in G' then there is a matching in G of size $|f|$;

Given M get f
$|f| = |M|$;

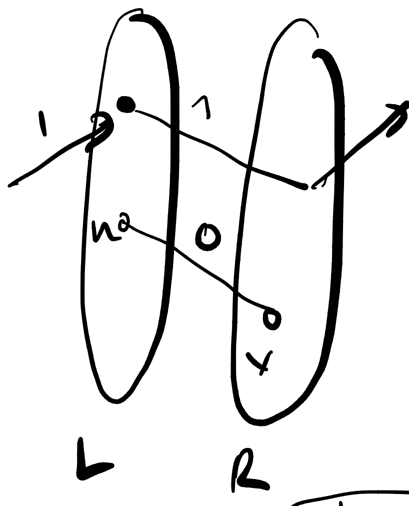
=

G':



The edges in E carrying a nonzero flow must be a matching;

$|M| = |f|$;



$$\boxed{f(L,R) = |M|}$$

$$= \underbrace{f(L,V')}_{} - \overset{0}{f(L,L)} \qquad (V' = L \cup R \cup \{s,t\}.$$

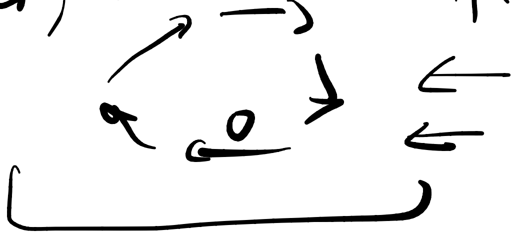
$$\underbrace{- f(L,s)} - \underbrace{f(L,t)}.$$

$$= 0 + f(s,L) - 0 = f(s,L) = |f|.$$

- Applications of Max flow min cut:

• Let f be a flow;

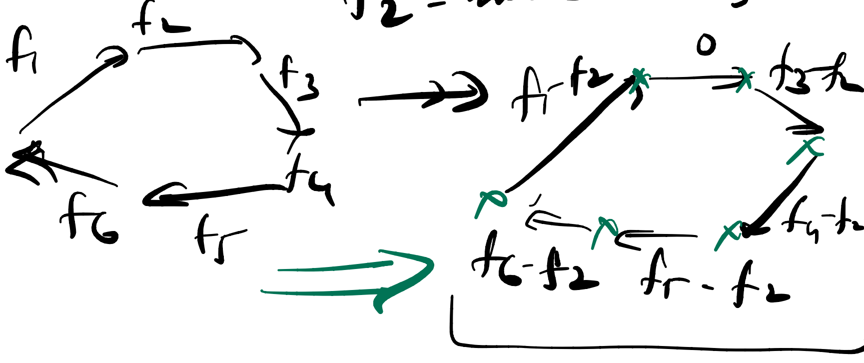
We say f is acyclic if $\forall$ directed cycles in G, $\exists e \in C$ st $f(e) = 0$.



carries no flow.

• **Lemma:** If f is a flow in G, there is an acyclic flow in G with the same value;

$$f_2 = \min\{f_1, \dots, f_6\}$$



$$\sum_{e \in E} f(e)$$

- Let f be an acyclic flow;

then there is a collection of s,t paths,

$P_1, \ldots \ldots P_{|f|}$, st $\forall e \in E$,

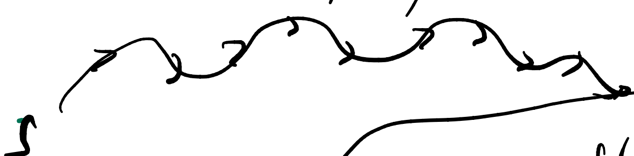
$$f(e) = \# \{ i \mid e \in P_i \}.$$

A flow is partitionable into $|f|$, path flows;

$$\mathcal{P} = \{ P_1, P_2, \ldots P_{|f|} \}$$

$$f = f_{P_1} + f_{P_2} + \cdots + f_{P_{|f|}}. \qquad |f|$$

Path;



$$f(e) = f'(e) + p(e) \quad \forall e.$$

$$f = f' + p$$

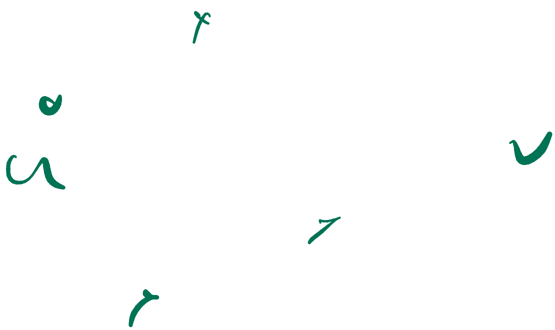
$$|f| = |f'| + |p|$$

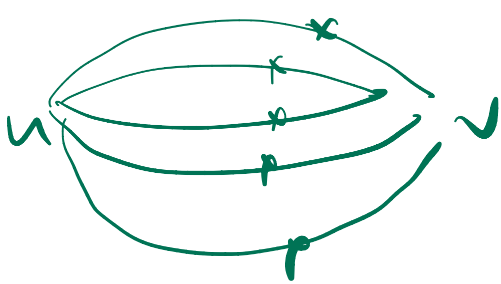
directed / undirected

- let G be a graph, u, v two vertices in G;

- Vertex version (Menger's thm); (there is no edge from u to v);

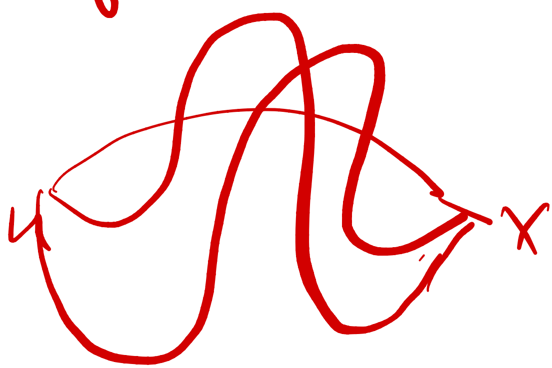
the max number of vertex-disjoint u, v paths in G is equal to the min cardinality an an u, v. vertex cut;





• Edge disjoint version (Menger's)

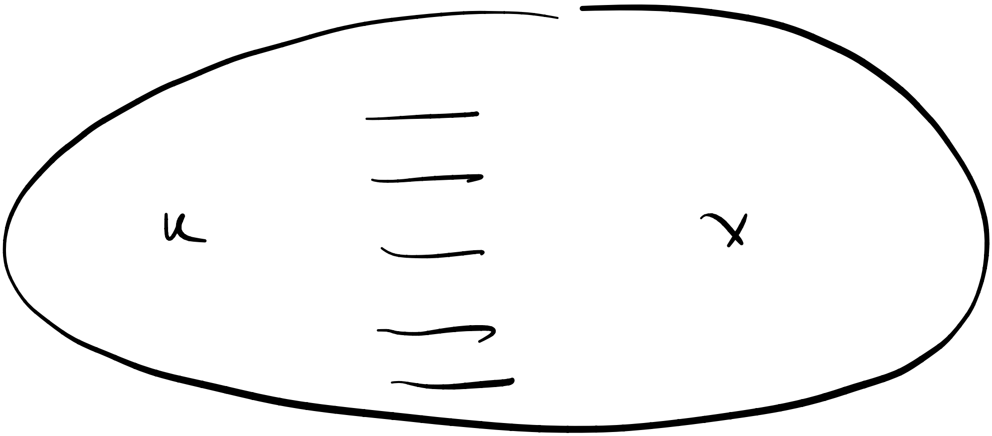
min # of edges to be removed to disconnect u from v = max cardinality of edge disjoint family of u, v; paths.

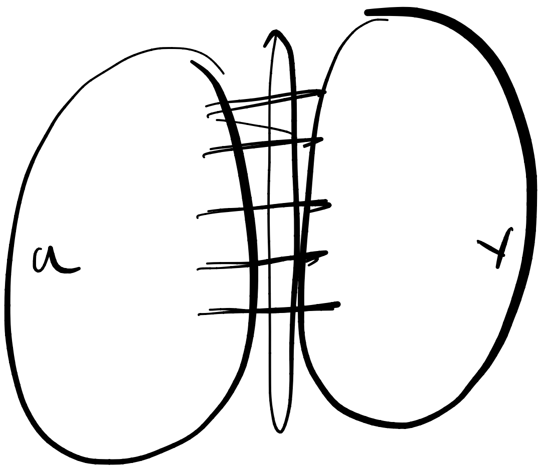


Undirected reduces to directed

= **Proof:**

• Edge version – easy using Max flow;





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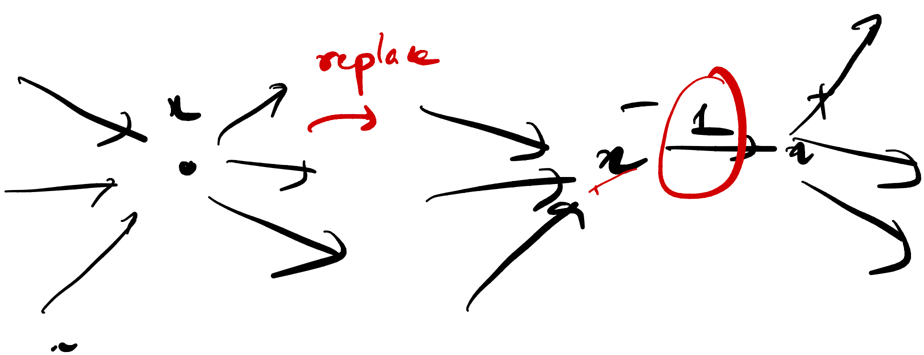
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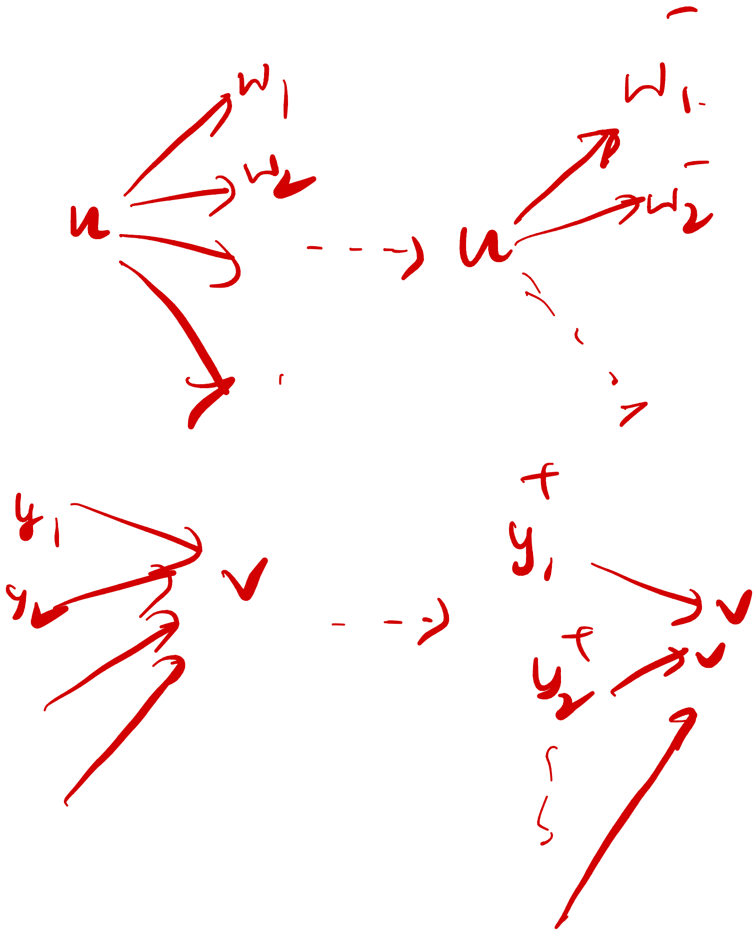

= Put ∞ capacity on edges.

$$f = p_1 + p_2 + \cdots + p_{n+1}$$

all of them are flat; value of each is 1.