• Tutte's theorem $\qquad$ 4/4/20.

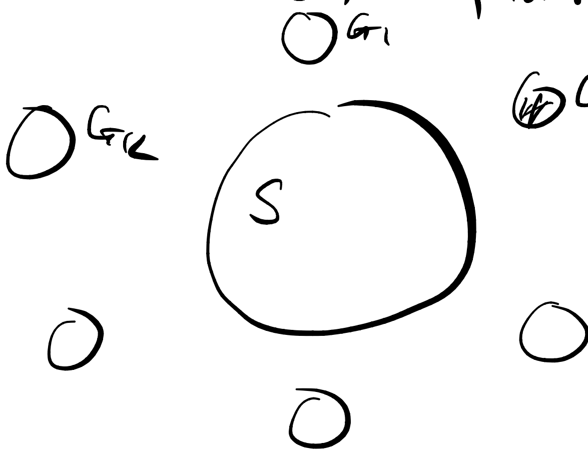
G - connected; G has a pm iff $\forall S \subseteq V$

$$odd(G-S) \le |S|;$$

• Saturated, Nonfactorizable:

$\Leftarrow \quad G \rightsquigarrow G'$. $\underline{odd(G'-S)} \le \underline{odd(G-S) \le |S|}$

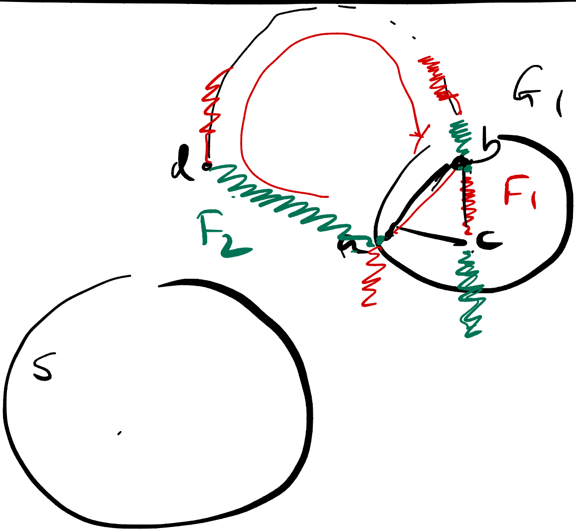
Thm: For a saturated, nonfactorizable graph



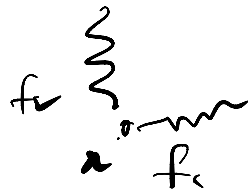
G_i Complete graph odd # vertices;

$K = |S| + 2$, S connected to all vertices

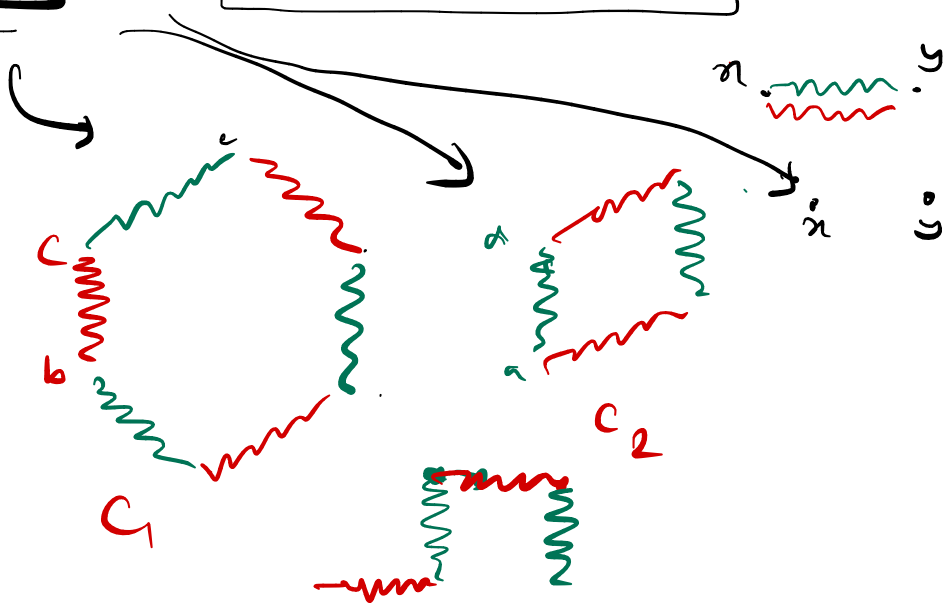
• Each G_i is a complete graph:



$\boxed{F_2 \oplus (P_v + ac)}$



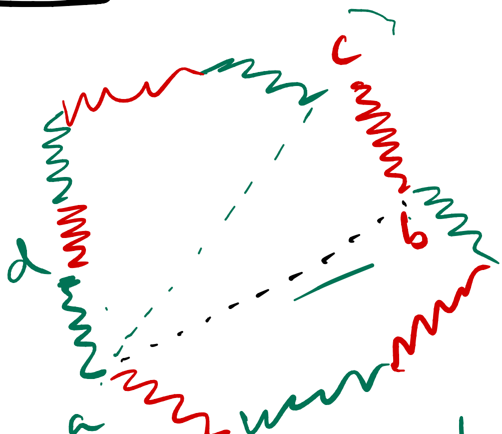
$F_1 \oplus F_2 := \underbrace{F_1 \setminus F_2 \cup F_2 \setminus F_1}$



$G_1 \neq G_2$:

$F_1 \oplus G_1$: p. Matching ✗.

$\boxed{G_1 = G_2}$:



b first.

$F_2 \oplus (P \cup ab)$ ← P.M.

or $F_2 \oplus (P \cup ac)$. ← p.m.

×

c first.

t.s.t: G_i is odd $\to$ complete;

components $= k+2$, $k = |S|$.

- THM: $\boxed{\text{Let } G \text{ be s.t. non factorizable}}$

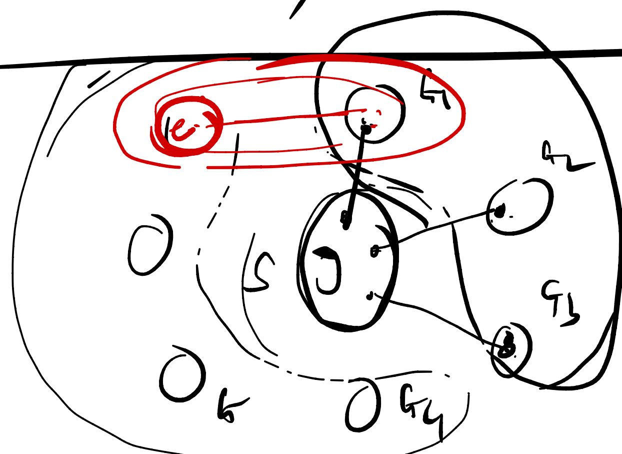
(a) If $|G|$ is odd then G is complete;

(b) If $|G|$ is even, then it has a structure $S, G_1, G_2, \ldots, G_k$

S — connected to all vertices;

G_i — odd & complete;

$k = |S| + 2$;



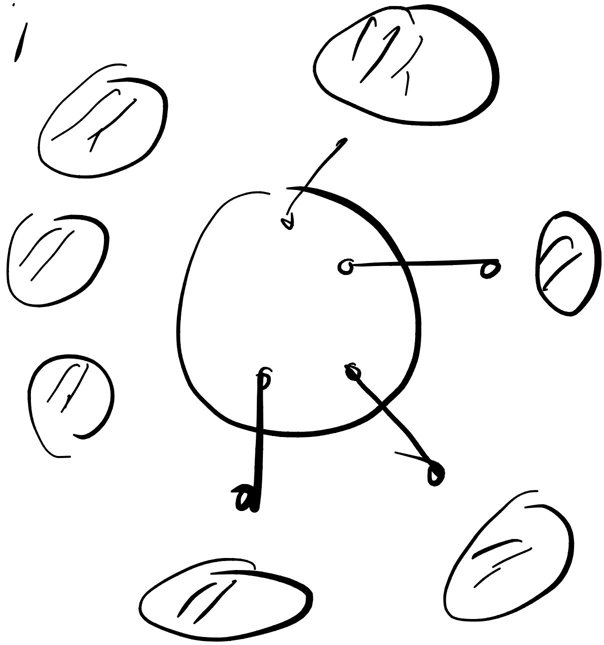
G has no p.m.

$|S| > 3$.

odd $\geq |S| + 2$

• If # odd components G_i is $\le |S|$,

we have a p.m.;



• # odd $\ge |S|+1$

$\underbrace{\neq |S|+1}_{\text{\# vertices}}$

vertices

$2|S|+1$ + even

×.

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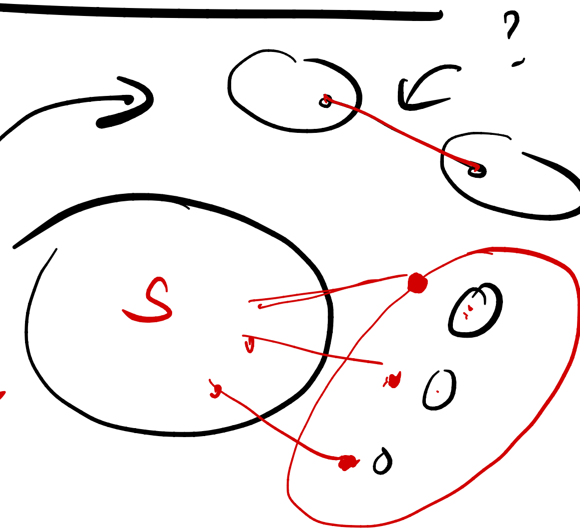
odd $\ge |S|+2$

$\ge |S|+3$

$|S|+1$



- Can't have more than $\geq |S|+3$ odd components.

Else: Connect two odd; by an edge;

$\#$ odd components $\geq |S|+1$

$\therefore$ No pm — Contradicts Saturation ×.

- Can't have even components,

o.w.: Connect an edge b/w the even comp & one odd comp.

— $\#$ odd components $= |S|+2$

No pm $\leftarrow$ Contradicts Saturat.