- Computing the evalues of $G = C_n$.

- Max eigen value $\leq$ max degree;
- Spectrum of bipartite graphs.

- $\alpha(G) \leq \dfrac{n}{1 - \frac{d}{\lambda_n}}$ G is d regular.

- Kneser graph. Erdos Ko-Rado theorem;

$$\lambda_{min} = -\binom{n-k-1}{k-1}.$$

Adj matrix.

THM 1 G is bipartite iff the eigen values of G are symmetric;

$$\begin{bmatrix} 0 & B \\ B^T & 0 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} = \lambda \begin{bmatrix} v \\ w \end{bmatrix}$$

$$\lambda \begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} Bw \\ B^T v \end{bmatrix}$$

$$\begin{bmatrix} 0 & B \\ B^T & 0 \end{bmatrix} \begin{bmatrix} v \\ -w \end{bmatrix} = \begin{bmatrix} -Bw \\ B^T v \end{bmatrix}$$

$$= -\lambda \begin{bmatrix} v \\ -w \end{bmatrix}$$

λ eigen value $\Leftrightarrow$ $-\lambda$ e.v.

= e.v are symm $\Rightarrow$ G is bipa.

$A^k(i,i)$ = # closed walks of length k.

$$A^k = \begin{bmatrix} & & \\ & \cdot & \\ & & \end{bmatrix}$$

$\mathrm{Tr}\left[A^k\right] = \#$ closed walks of length k in A.

A - diagonalizable.

$$S^{-1} A S \qquad \begin{bmatrix} \lambda_1 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix}$$

$$(S^{-1} A S)^k = (S^{-1} A^k S) = \begin{bmatrix} \lambda_1^k & & & \\ & \lambda_2^k & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix}$$

for k odd λ^k & $(-\lambda)^k$ cancel. $\forall \lambda$;

$\therefore$ No

G - a d-regular graph; (connected).

• Then

$$\alpha(G) \le \frac{n}{1 - \frac{d}{\lambda_n}},$$

Let S be an indep set,

$$x_S^T \, A \, x_S = 0$$

char vector of S

$\boxed{S \text{ is independent if } \forall\, i, j \in S,\ a_{ij} = 0. \quad \subseteq V \ \therefore \text{ all } \times \text{ above are zero.}}$

$$x_S \quad \boxed{0\ 1\ 0\ 0\ 1\ 0\ 0\ 1}$$

$\in S \qquad \in S \qquad \in S$

λ_n = smallest e.v

The matrix $A - \lambda_n I$ has e.v ≥ 0;

$$\boxed{x_s^T A x_s = 0}$$

Consider the matrix

all 1's matrix

$$\boxed{M = \left[A - \lambda_n I\right] - \left(\frac{d - \lambda_n}{n}\right) J}$$

$$\therefore (A - \lambda_n I) \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} d - \lambda_n \\ d - \lambda_n \\ \vdots \\ \vdots \end{bmatrix}$$

$$M \left[\vec{j}\right] = \left[(d - \lambda_n) - \left(\frac{d - \lambda_n}{n}\right) n\right] \vec{j} = 0;$$

$$\boxed{\text{for } M = A - \lambda_n I - \left(\frac{d - \lambda_n}{n}\right) J.}$$

j is an e.v of M of e.v $\underline{\underline{0}}$.

Let w be an e.v of M.

Then:

$$\left[A - \lambda_n I - \left(\frac{d - \lambda_n}{n} \right) J \right] w$$

$Mw = \mu w = Aw - \lambda_n I w =$

$= (A - \lambda_n) w \quad (\because w \perp j, \ Jw = 0)$

Any ev of M is an e.vector of $(A - \lambda_n)$ $\therefore$ $\underline{\mu \geq 0}$. $\therefore$ M is + semidefinite

$\therefore$ $\forall S \subseteq V$, independent & x_S characteristic vector

$$\underbrace{0 \leq x_S^T M x_S}_{} = x_S^T A x_S - x_S^T \lambda_n x_S - x_S^T \left(\frac{d - \lambda_n}{n} \right) J (x_S)$$

$\forall S$

$$0 \le 0 - |S|\lambda_n - |S|^2\left(\frac{d-\lambda_n}{n}\right)$$

$$|S|^{\cancel{2}}\left(\frac{d-\lambda_n}{n}\right) \le -\cancel{|S|}\lambda_n$$

$$|S| \le \frac{+\lambda_n \, n}{-d+\lambda_n}$$

$$\boxed{|S| \le \frac{n}{1-d/\lambda_n}} \rightarrow \text{True } \forall \ S \text{ independent}$$

$$\therefore \ \alpha(G) \le \frac{n}{1-d/\lambda_n}.$$

For

Kernel graph $\lambda_n = -\binom{n-k-1}{k-1}$

Kesner: Vertex set: $\{S \mid |S| = k,\ S \subseteq [n]\}$;

Edges: S_1, S_2 connected iff $S_1 \cap S_2 = \phi$;

· degree of a vertex S — it is connected to all vertices, corresponding subsets of which are disjoint from S. ∴ degree = $\binom{n-k}{k}$

$$\alpha(G) \le \frac{\binom{n}{k}}{1 + \binom{n-k}{k}}$$

$$\frac{}{\binom{n-k-1}{k-1}}$$

$$\alpha(G) = \frac{k}{n}\binom{n}{k} = \binom{n-1}{k-1}$$

A subset of vertices in the Kesner graph is independent iff every pair of vertices (i.e.) subsets, intersect.

∴ Independent set in Kesner gives an intersecting family.

∴ We've shown that the largest size of an intersecting family of k-element subsets of $[n]$ is $\leq \binom{n-1}{k-1}$

Erdos Ko Rado theorem.
