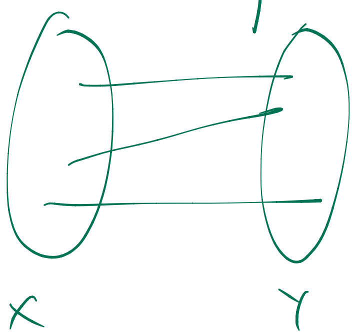
• Bipartite graphs:

$G = (V, E)$ is bipartite if $V = X \cup Y$ and $E \subseteq (X \times Y)$

(i.e.) there is a bipartition of vertices



and edges only go from X to Y.

• Characterization of bipartite graphs:

Thm

G is bipartite iff G has no odd cycle.

Pf:

• Can do this using BFS.

. HAMILTONIAN CYCLES:

. In general hard to find algorithmically in polynomial time.

Thm: G be connected. If $\deg(v_i) \geq |V|/2$ $\forall i$ then G has a H. Cycle;

Pf: Take a maximal graph (maximal in terms of edges) satisfying the hypothesis and not the conclusion

Show this is a contradiction;

$\therefore$ No graph exists satisfying the hypothesis & not the conclusion (WHY?)

$\therefore$ Theorem is true;

In the maximal graph:

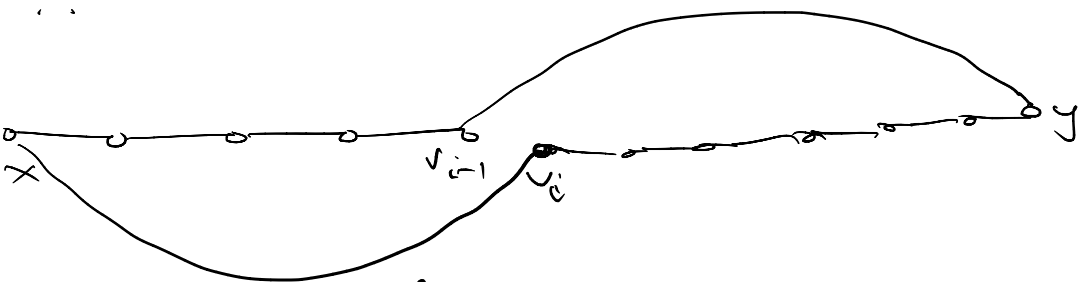
H-Path is present.

Not present

on the H-Path $\exists$ i, s.t

$x - v_i$ is an edge and $v_{i-1} - y$ is an edge; WHY?

$\therefore$



Contradiction?

- Directed graphs.

- Tournaments.