The Theory of Message Sequence Charts – II

K Narayan Kumar

Chennai Mathematical Institute
http://www.cmi.ac.in/~kumar

TIFR, Mumbai, 01 May, 2009
MSCs describe runs or behaviours of message passing systems.
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MSGs, a visual formalism to describe languages of MSCs.
Summary

- MSCs describe runs or behaviours of message passing systems.
- MSGs, a visual formalism to describe languages of MSCs.
- MSGs: regularity is not decidable, but boundedness is.
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MPAs: An operational model, distributed, ...
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Verifying implementability for MSGs is undecidable.
The Model-checking problem

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- **Negative Model-checking** Given a specification language $S$ and an implementation $L$ decide whether $S \cap L = \emptyset$.
  
  Are all the negative instances avoided?
The Model-checking problem ...
If $S$ and $L$ are given as locally synchronized MSGs, both the model checking problems are decidable.
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- These results can be generalized further ...
Model-checking ...

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Sufficient conditions for the decidability of model-checking:

- The system \( L \) has a regular set of representatives. A regular language \( R \) such that the set of MSCs generated by the words in \( R \) is \( L \).

- Given \( B \), we can effectively construct \( \text{Lin}^B(S) \) consisting of all the \( B \) bounded linearizations of MSCs in \( S \).
Globally Cooperative MSGs
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[One of the many results best proved via a translation to Mazurkiewicz traces.]
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**Corollary:** Systems given as MSGs can be model-checked w.r.t. specifications presented as GC-MSGs.
Implementing Regular MSC languages

- Allow global accepting states.
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\[ M_1 \]
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By tagging auxiliary information to $m$, $p$ informs $s$ whether it has sent a message to $q$.

This rules out the implied scenario $M$. 
Distributed Synthesis
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- Explicitely construct a deterministic MPA from a FA accepting the linearizations of $L$. 

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1. Process 1 keeps the effect of the MSC in its past.
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A sophisticated local timestamping algorithm is needed to make all this work.
Monadic Second Order Logic

The Monadic Second Order logic over MSCs.

- $x, y, \ldots$ an infinite collection of first-order variables.
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**Quantification and boolean connectives.**

- The first order variables take values over the events in the given MSC.
- The second order variables take subsets of events as values.
- \( \leq \) is interpreted by the ordering on the MSC.
- \( <_m \) denotes the message ordering and cannot be defined using \( \leq \)
∀x. ∀y. (p!q(x) ∧ p!q(y) ∧ (x < y)) ⇒ ∃z. (x <_m z) ∧ (z ≤ y).
∀x. ∀y. (p!q(x) ∧ p!q(y) ∧ (x < y)) \implies \exists z. (x <_m z) ∧ (z \leq y).

- p sends to q only after receiving an (indirect) acknowledgement for the previous send.
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- One can express \( B \)-boundedness for any \( B \).  

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- One can express B-boundedness for any B.

**Theorem:** An MSC language $L$ is regular if and only if there is a formula $\varphi$ in MSO and a constant $B$ such that

$L = L(\varphi) \cap \{M \mid M \text{ is universally B-bounded}\}$
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2. The MSC $M_w$ satisfies $\varphi$. This involves showing that $\leq, <m$ are definable over $w$. 


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Observe that 1, 2 ensure that the set of $B$-bounded linearizations of $L(\varphi)$ is always a regular language.
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**Theorem:** Model-checking MSGs w.r.t. MSO is decidable.
The same observation also leads to the decidability of satisfiability for MSO.

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Further, MSO is strictly more expressive than MPAS w.r.t. general MSCs.
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**Theorem:** Satisfiability is decidable for MSO over the class of universally (existentially) $B$-bounded models.

Further, MSO is strictly more expressive than MPAS w.r.t. general MSCs.

**Theorem:** The quantifier alternation hierarchy for MSO over MSCs is strict. In particular EMSO is strictly weaker than MSO.
Existentially bounded languages
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3. $L$ is the MSO definable.

However, deterministic MPAs do not suffice.
Adding time to MSCs
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- Time constrained MSCs
  - MSCs with timing constraints between events
Adding time to MSCs

- Time constrained MSCs
  - MSCs with timing constraints between events
- Time constrained Message Sequence Graphs
  - Generate infinite families of time constrained MSCs
MSCs with time constraints

User  ATM  Server

$u_1$  $a_1$  $s_1$

$u_2$  $a_2$  $s_2$

$u_1$  $a_3$  $a_4$

$u_2$  $a_2$  $a_3$

$card$  $card-data$  $card$  $card-OK$  $pin-request$

$(0,4]$  $(0,2]$
Time Constrained MSCs

- Associate time interval constraints with pairs of events
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- If \((e, e') \mapsto [l, u]\), then the time between occurrence of \(e\) and \(e'\) must be between \(l\) and \(u\)
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  - . . . or across a single message
    - \(e\) is \(p!q(m)\) and \(e'\) is corresponding receive \(q?p(m)\)
A timed behaviour
Timed MSCs

- Add timestamps to events on MSC, $\tau : E \rightarrow \mathbb{R}_{\geq 0}$
Timed MSCs

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- All timestamps refer to same global time
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- All timestamps refer to same global time
- Order of timestamps respects partial order on events
Timed MSCs

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  - Replace $\leq$ by $<$, as appropriate, for open, half-open intervals
- TC-MSC $T \Rightarrow L(T)$, set of timed MSCs that cover $T$
TC-MSCs and Timed MSCs
The set of timed MSCs covering a TC-MSC may be empty.
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A TC-MSC is said to be realizable if it is covered by at least one timed MSC.
Time Constrained Message Sequence Graphs

- States labelled by time constrained MSCs
- Local constraints for each process along edges
- Legal paths in the automaton generate time constrained MSCs

![Diagram of a time-constrained message sequence graph with labelled edges and constraints.]

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\[
\begin{align*}
(r, m_1, s) &\quad [0,3] \\
(r, m_2, s) &\quad ([0,2], [1,1]) \\
(r, m_2, s) &\quad (2,3), [1,1]) \\
(r, m_3, s) &
\end{align*}
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Reachability
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Given a TC-MSG $G$ and a state $q$ in $G$, does there exist a path $q_0q_1 \ldots q_k = q$ from an initial state $q_0$ such that the TC-MSG generated by this path is realizable?
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(The control state reachability problem for TC-MSGs.)
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(The control state reachability problem for TC-MSGs.)

This problem is trivial for ordinary MSGs.
Reachability ...
Reachability ...
Reachability ...

\[
\begin{align*}
p &\quad q \\
0 &\quad 1, 1 &\quad 1, 4 &\quad 3, 1 &\quad 1, 1 &\quad 1 \\
q &\quad p \\
\end{align*}
\]
Reachability ...

\[
\begin{array}{c}
p \quad 0 \quad q \\
1,1 \\
p \quad q \\
1,1 \\
p \quad q \\
3,1 \\
p \quad q \\
1,1 \\
p \quad q \\
1 \\
q \\
1 \\
p \\
1
\end{array}
\]
Reachability ...
Reachability ...
Reachability ...
Reachability ...
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The first loop is to be executed $k$ times and the second one $l$ times such that $a.k - b.l = 1$. 

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Simple paths may not be realizable while those with loops may be.
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Boundedness for Timed MSCs

- A timed MSC is **universally $B$ bounded** if all its timed linearizations are $B$ bounded.

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- A TC-MSG is (universally/existentially) bounded if there is a \( B \) such that all the TC-MSCs realizing it are (universally/existentially) \( B \) bounded.
Time constraints may ensure boundedness.
Boundedness ...

Time constraints may ensure boundedness.

\[
\begin{array}{c}
P \\
\hline
[2, 5]
\end{array}
\quad 
\begin{array}{c}
C \\
\hline
[2, 3]
\end{array}
\]
Boundedness ...

Time constraints may ensure boundedness.

\[ P \rightarrow [2, 5] \rightarrow [2, 3] \]

\[ q \rightarrow p \]
Boundedness ...

Time constraints may ensure boundedness.
Boundedness ...

Time constraints may ensure boundedness.
Time constraints may ensure boundedness.
Time constraints may ensure boundedness.
Time constraints may rule out existential boundedness.
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\[
\begin{array}{c}
\text{P} \\
[0, \infty] \\
\text{C} \\
([0, 2], [3, 4])
\end{array}
\]

q

p
Time constraints may rule out existential boundedness.

\[ P \rightarrow ([0, \infty], ([0, 2], [3, 4])) \]

\[ q \rightarrow p \]
The Results

**Theorem:** The control state reachability problem for TC-MSGs is undecidable. The problem is undecidable even when there are no timing constraints on messages.
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Thank you.
Consider TC-MSGs where there are no time constraints associated with transitions between nodes.
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- The control state reachability problem is decidable. A path is realizable if and only if each node in the path is realizable.
Edge Constraint free TC-MSGs

Consider TC-MSGs where there are no time constraints associated with transitions between nodes.

- The control state reachability problem is decidable. A path is realizable if and only if each node in the path is realizable.
- The boundedness problem is still open. Time constraints can enforce boundedness.