Topology, Problem Set 13

(1) Let $p : E \to B$ be a covering map and suppose that B is simply connected and E is path connected. Then show that p is a homeomorphism.

(2) Show that if A is a strong deformation retract of X, and B is a strong deformation retract of A, then B is a strong deformation retract of X.

(3) For each of the following spaces, the fundamental group is either trivial, infinite cyclic, or isomorphic to the fundamental group of the figure eight. Determine for each space which of the three alternatives holds:

(a) The "solid torus" $B^2 \times S^1$.

(b) The torus T with a point removed.

(c) The cylinder $S^1 \times I$.

(d) The infinite cylinder $S^1 \times \mathbb{R}$.

(e) $\mathbb{R}^3$ with nonnegative x, y, z axes removed.

The following subsets of $\mathbb{R}^2$:

(f) $\{x| \ ||x|| > 1\}$

(g) $\{x| \ ||x|| \geq 1\}$

(h) $\{x| \ ||x|| < 1\}$

(i) $S^1 \cup (\mathbb{R}_+ \times 0)$

(j) $S^1 \cup (\mathbb{R}_+ \times \mathbb{R})$

(k) $S^1 \cup (\mathbb{R} \times 0)$

(l) $\mathbb{R}^2 - (\mathbb{R}_+ \times 0)$

(4) Show that there are no retractions $r : X \to A$ in the following cases:

(a) $X = \mathbb{R}^3$ with A any subspace homeomorphic to S^1.

(b) $X = S^1 \times B^2$ with A its boundary torus $S^1 \times S^1$.

(5) Show that if a path connected, locally path connected space X has finite fundamental group, then every map $X \to S^1$ is null homotopic (hint: use the covering map $\mathbb{R} \to S^1$).