Topology, Problem Set 12

(1) Show that a covering map is open.
(2) Show that if $p : E \to B$ and $p' : E' \to B'$ are covering maps, then $p \times p' : E \times E' \to B \times B'$ is a covering map.
(3) Suppose that Y is a discrete topological space. Then show that the first projection map $X \times Y \to X$ is a covering map.
(4) Let $p : E \to B$ be a covering map with B connected. Show that if $p^{-1}(b_0$ has k elements for some $b_0 \in B$, then $p^{-1}(b)$ has k elements for every $b \in B$. In such a case, E is called a k**-fold covering of** B.
(5) Let $p : E \to B$ be a continuous and surjective map. Let $U \subset B$ be an evenly covered open set. Show that if U is connected, then the partition of $p^{-1}(U)$ into open slices is unique.
(6) Suppose that $p : X \to Y$ and $q : Y \to Z$ are covering maps. Show that if $q^{-1}(z)$ is finite for each $z \in Z$, then $q \circ p : X \to Z$ is also a covering map. This is not true when $q^{-1}(z)$ is not finite for some $z \in Z$.
(7) Let $p : E \to B$ be a covering map. Show that $p^{-1}(b) \subset E$ has discrete topology for any $b \in B$.
(8) Let $p : E \to B$ be a covering map. Assume that $p^{-1}(b)$ is finite and nonempty for all $b \in B$. Show that E is compact Hausdorff if and only if B is compact Hausdorff.
(9) Show that a retract of a contractible space is contractible.