Complex Algebraic Surfaces, Take-Home Midterm Exam
Due: Friday, March 17, 2017, 3:30 pm

Notation: All surfaces are complex, projective and smooth.

(1) Let D be a nonzero effective divisor on a surface S. Show that D is not linearly equivalent to the zero divisor.

(2) Let C be an irreducible and reduced curve of arithmetic genus 0 on a surface S. Show that C is smooth rational. Give an example of a reduced and irreducible **singular** curve C of arithmetic genus 1 on a surface S.

(3) Let S be a minimal surface with $q(S) = P_2(S) = 0$. Show that for every divisor D on S, the linear system $|D + nK_S|$ is empty for $n \gg 0$. (Hint: consider the two cases $K_S^2 \geq 0$ and $K_S^2 < 0$ separately.)

(4) Let S be a surface and let $\hat{S} \to S$ be the blow up of a point $p \in S$. Show **directly** that there are isomorphisms $H^0(\hat{S}, mK_{\hat{S}}) \cong H^0(S, mK_S)$ for every $m \geq 0$. Conclude that the Kodaira dimension is a birational invariant.

You may use (without proof) the following fact: Let $f : X \to Y$ be a surjective morphism of projective varieties with Y normal and connected fibres. Then for any line bundle L on Y there is a canonical isomorphism $H^0(Y, L) \cong H^0(X, f^\star L)$.

(5) Let S be a surface. Suppose that there exists an irreducible and reduced curve C on S such that $C^2 \geq 0$ and $K_S \cdot C < 0$. Then show that $\kappa(S) = -\infty$.

(6) Exercise II.20 (2) in Beauville's book *Complex Algebraic Surfaces*.

(7) Exercise II.20 (3) in Beauville's book *Complex Algebraic Surfaces*.