Daniel Bernoulli and his Fluid Dynamics

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Bernouilli and his fluid mechanics

Daniel Bernouilli (1700-1782)

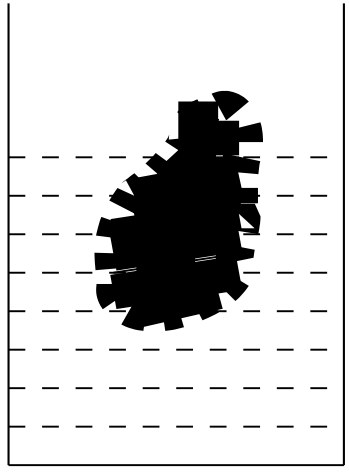
- The fluids : liquids and gases, are fascinating.
- Archimedes (287 B. C. - 212 B.C.) - mathematician, physicist, engineer, astronomer, and philosopher - of Syracuse, Sicily
- Archimedes did work on static fluids - **Hydrostatics**
- Eureka Story
- Hiero II - monarch of Syracuse - doubted the purity of metal used in making his crown - Archimedes principle - laws of floatation
- Why and when does a body float?
- an object weighs less when in water - due to buoyancy; the loss of weight equals boyancy force;

It all started with Archemedes

Figure 1: Archimedes (287 BC - 212 BC)

- mass is an intrinsic property of matter;
- a body has the same mass above ground; inside water; when taken to moon or to distant planets;
- But weight is relative;
- weight equals mass times gravity;
- we use the terms mass and weight interchangeably. Why ?
- It is because we all live on earth and
- do not have plans, at least as of now, to move to other planets here in our solar system or elsewhere in the universe!

- consider an object which weighs W;
- when in water let us say it weighs W';
- Archimedes principle:
- weight loss ($W - W'$) equals weight of water displaced:
- *i.e.* buoyancy force = weight of water displaced;
- if the body floats in water then W equals the weight of water displaced;



Self Portrait

Leonardo da Vinci and his principle of continuity - the very first step in fluid dynamics

Leonardo-da-Vinci (1452-1519)

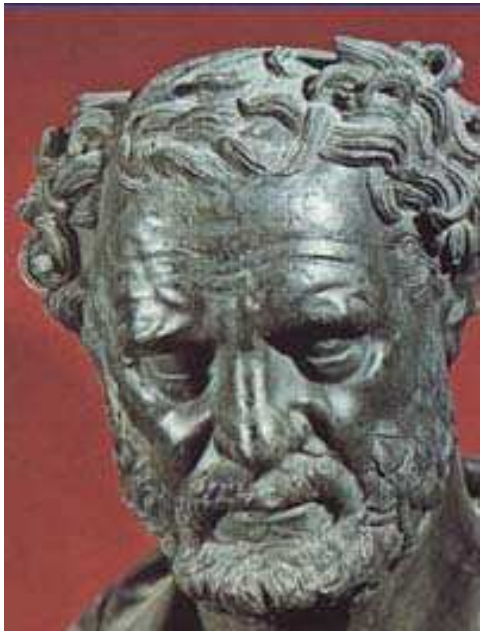
- architect, anatomist, sculptor, engineer, inventor, geometer, scientist, mathematician, musician, and painter - remember *the last supper*
- First to study **hydrodynamics**:
- Principle of Continuity: Fluid flows faster through narrower sections: *Still waters run deep*
- Imagine students moving in a file four in a row at some speed.
- There comes a passage that lets only one student at a time.
- What happens then ?

- To maintain continuity of flow, the students once they enter the passage, must move four times faster.
- Leonardo da Vinci used to sit near fountains and water falls, observe patterns of water flow by tossing grass seeds and sketched them.
- Atomistic View:
- Ancient man - Indian or Greek, Mohist or Mayan - must have definitely speculated on the possibility of tiny, invisible and indivisible particles assembling in very large numbers into a visible continuum of solids and liquids and an invisible continuum of air that surrounds us.

- Indians called it ANU and the Greek called it ATOM.
- The two arrived at the notion in opposite ways : Indians by "assembling - mountain versus mole hill" and the Greeks by "disassembling - cutting into half"
- According to
  Leucippus (440 B.C.),
  Democritus (370 B.C.) and
  Epicurus (341 - 271 BC)
  an atom moves in void unceasingly and changing course upon collision with another atom.
- Titus Lucretius Carus (94 BC - 55 BC) wrote on nature of things - De Rerum Natura - a Six book long poem

Leucippus, Epicurus and Democritus

Leucippus (440 BC)



Democritus (370 BC)

Epicurus (341 - 271 BC)

.... raiment, hung by surf-beat shore, grows moist.
The same, spread out before the sun, will dry.
Yet no one said how sank the moisture in,
nor how by heat off-driven. Thus we know
that moisture is dispersed about in bits
too small for eyes to see .....

According to his verses all the phenomena we see around

are caused by invisible atoms moving hither and thither

It was certainly not by design that the particles fell into order.
They did not work out what they were going to do,
but because many of them by many chances
struck one another in the course of infinite time
and encountered every possible form and movement,
that they found at last the disposition they have,
and that is how the universe was created.
Particles, kept together for so many years,
when by a chance they had found harmonious movements,
brought it about that rivers flow into the sea
to keep it going, while earth by the heat of the sun
renews its products, and living creatures breed on
and the gliding lights in the sky are never put out.

- Galileo Galilei (1564 - 1642)
- wrote of air surrounding earth and its ability to stand 34 feet of water; he knew of air expanding upon heating;
- Evangelista Torricelli (1608 - 1647)
- correctly concluded of air pressure; surmised that mercury fourteen times denser would rise only thirty inches against atmospheric pressure;
- Blaise Pascal (1623 - 1662)
- was quick to point out that pressure of air on mountain top would be less than it is on the ground if Torricelli is right;
- his statement was later verified experimentally;

- Robert Boyle (1627 - 1691) discovered through careful experimentation that $\boldsymbol{P \times V = \mathrm{constant}}$, for air: called Boyle's law (strictly valid for a given quantity of ideal gas under constant temperature):

$$\begin{aligned} \boldsymbol{PV} &= \boldsymbol{N\, k_B\, T} \\ &= \boldsymbol{n\, R\, T} \end{aligned}$$

- Boyle modeled air as a collection of springs: resists compression; expands into available space.
R. Boyle, *The spring of air* (1660); reprinted in S. G. Brush, *Kinetic Theory* vol. 1; Pergamon (1965)p.43

Galileo, Torricelli, Pascal and Boyle

Galileo Galilei

(1564 - 1642)

Evangelista Torricelli

(1608 - 1647)

Blaise Pascal

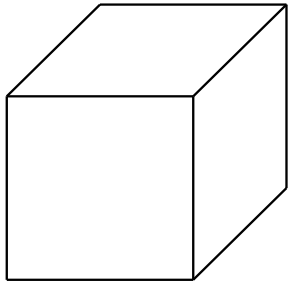
(1623 - 1662)

Robert Boyle

(1627 - 1691)

Daniel Bernoulli's model of gas and his derivation of ideal gas law:

- Gas consists of atoms/molecules: like billiard balls moving unceasingly and colliding with each other and with the walls of the container
- when a molecule bounces off the wall it exerts a tiny force; several molecules bouncing off the wall exert a tangible force;
- Pressure is force per unit area
- consider an ideal gas in a cube; double the cube length without changing the quantity of gas inside and without altering the molecular speeds.



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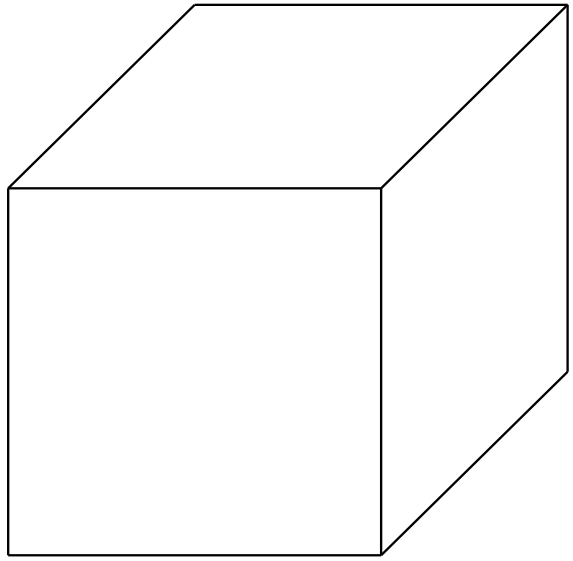


2L

- force exerted per impact remains the same; but a molecule has to travel twice the distance between successive collisions; hence the force is halved.
- since each edge is doubled, the area of a side is quadrupled;
- half the force is exerted on a four-fold increased area; therefore the pressure is eight times less;
- however the volume of the cube is now eight times more;
- Hence, Bernoulli concludes: pressure times volume of a given quantity of gas remains the same at constant temperature.

The simplicity of the argument and

the elegance with which it captures

an important experimental truth

remain unmatched to date.

Story of the Bernoullis:

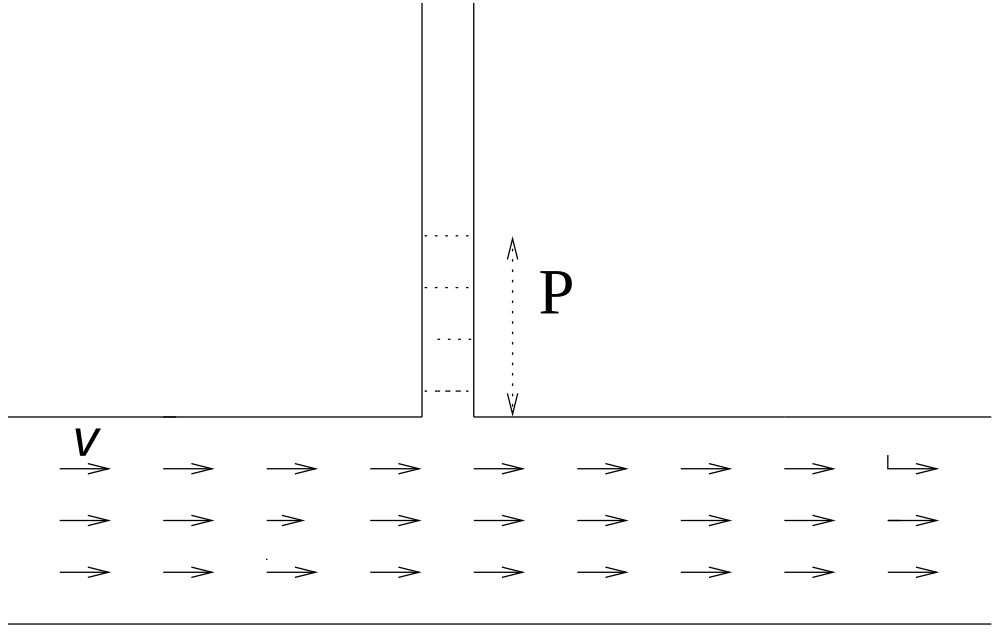
- The Bernoullis dominated Mathematics for over a century:
- Jacob I (1645 - 1705):
  Nicolous I (1662 - 1716):
  Johannes I (1667 - 1748): father of Daniel Bernoulli; quarreled with his brother Jacob; threw his son (Daniel) out of his house - Reason:
- French Academy of Sciences gives award who provides the best solution to the challenging technical problem it poses every year;
- started by King Lois XIV in 1666;
- prestigious; carries loads of money;

- the prize for the year 1734 was awarded jointly to the father, Johannes and the son, Daniel;
- Next generation:
- Nicolous II (1667-1759) - - son of Nicolous I
- Nicolous III (1695-1726);
  Daniel (1700 - 1782);
  Johannes II (1710 - 1790)
  - - sons of Johannes I
- Next generation:
  Johannes III (1746- 1807);
  Jacob II (1759-1789)
  - - sons of Johannes II

- Daniel was in Russia under invitation to the Russian Academy of Sciences from Catherine - the queen: Daniel spent 8 years in Russia; made astonishing and historic discoveries on hydrodynamics:
- Daniel wanted to become as famous as Newton :
- Like Newton is known for his work on solids, Daniel wanted to become famous for work on fluids.
- Through careful experimentation Daniel confirmed the law of continuity discovered by Leonardo da Vinci;
- When the pipe becomes narrower water flows faster;
- when it becomes wider, water slows down
- Daniel has a hunch that pressure should play a role in the phenomenon;

- How does one measure the pressure of a flowing water ?
- Galileo has shown how to measure static pressure
- take a vertical tube sealed at one end
- immerse the open end in a tub of water;
- water would raise in the tube and stabilize at some level;
- the outside atmospheric pressure stands 34 feet of water;

- Make a small hole in the pipe and stick a vertical capillary tube to it;
- water would raise in the capillary tube;
- its height measures the pressure;



- Slow moving water: it rose higher in the vertical tube;
- faster the flow the lower is the pressure;
- WHY ?
- At microscopic level Pressure is momentum transferred in molecular collision;
- Imagine a room full of children running a muck jostling with each other and with the walls;
- a teacher stationed amidst would be bombarded by the children from all sides; each child would transfer a certain momentum; the teacher will feel the pressure; each of the colliding child would also feel the pressure.

- now ask the children to run along a passage with the teacher also running along keeping pace with them;
- the collision amongst the children and with the teacher will be gentler; less jostling; less pressure.
- Why?
- each child remains equally energetic;
- but most of the energy is used up in keeping pace with the moving group;
- hence there is less jostling; less pressure;

- the jostling felt by the teacher moving with the children corresponds to STATIC pressure - the one measured by the vertical capillary tube attached to the pipe.
- however if the teacher chooses to stand still, the children would ram into her;
- the body blows she receives shall be higher now than earlier;
- this would correspond to the so-called ram pressure
- hold your hand in a stream;
- when the current is fast moving your hand shal feel a higher ram pressure;
- with increase of speed static pressure decreases but the ram pressure increases;

- Bernoulli asked :
- Is there a trade off - between jostling and current ?
- Is there a conservation:
- jostling plus current remains constant -
- when one increases the other should decrease ?
- When the fluid flows faster does it do so at the expense of jostling ?

Bernoulli's Equation

Fluid flowing in a pipe:

consider two points 1 and 2 along the pipe;

P - static pressure;
ρ - density of the fluid;
v - the flow velocity;
h - the height;
g - the gravitational constant;

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Derivation of Bernoulli equation

- Consider a vertical slice of water flowing in a pipe;
- A denote the cross section area;
- Δx its width;
- ΔP is the pressure difference across the slice
- m mass of the fluid in the slice
- Force equals mass times acceleration:

$$
\begin{aligned}
-A\Delta P &= m\frac{\Delta v}{\Delta t} \\
\Delta P &= -\frac{m}{A\Delta x}\Delta x\frac{\Delta v}{\Delta t} \\
\Delta P &= -\rho\Delta x\frac{\Delta v}{\Delta t} \\
\Delta P &= -\rho v\Delta v \\
dP &= -\rho\, v\, dv
\end{aligned}
$$

$$\begin{aligned}
\int_{P_1}^{P_2} dP &= -\rho \int_{v_1}^{v_2} v dv \\
P_2 - P_1 &= \frac{1}{2}\rho(v_1^2 - v_2^2) \\
P_2 + \frac{1}{2}\rho v_2^2 &= P_1 + \frac{1}{2}\rho(v_1^2) \\
P + \frac{1}{2}v^2 &= \text{a constant}
\end{aligned}$$

Alternate derivation

- A solid object has potential energy (mgh) and kinetic energy ($mv^2/2$).
- Only the location of the centre of mass is considered for calculating potential energy;
- similarly only the speed of the centre of mass is considered for calculating translational kinetic energy.
- This is because the object is rigid.
- with fluids there is an additional energy - represented by the (static) pressure;
- remember the jostling of the molecules which can absorb energy (by jostling more), store energy and release energy (by jostling less);

- Fluid pressure can be viewed as energy per unit volume:

$$
\begin{aligned}
P &= \frac{F}{A} \\
&= \frac{F \times d}{A \times d} \\
&= \frac{W}{V} \\
&= \frac{E}{V} \\
PV &= E
\end{aligned}
$$

$$
\begin{aligned}
E &= \int_0^E dE \\
&= \int_0^L F dx \\
&= \int_0^L P A dx \\
&= \int_0^V P dV \\
&= PV
\end{aligned}
$$

- Bernoulli equation is simply an equation for conservation of energy:

$$P_1 V + \frac{1}{2} m v_1^2 + m g h_1 = P_2 V + \frac{1}{2} m v_2^2 + m g h_2$$

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 V + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

- Daniel writes up all his findings in a book : Hydrodynamics by Daniel Bernoulli son of Johannes
- Bernoulli dispatches several copies of his book to his trusted friend Euler - who got his job in Russian academy because of recommendations from Daniel - for release.

- Euler cheats Daniel: helps the father plagiarize son's work;
- Johannes writes a book called Hydraulics;
- Euler waits for an year to release **Hydraulics** first and later releases **Hydrodynamics**;
- Daniel, shattered, never recovered from the shock.

- Man, jealous of birds, has always wanted to fly
- Our Gods anyway fly all the time on their own;
- often they ride on birds, flying horses, or fancy flying machines;
- One of the earliest kidnapping event in the history of mankind happened with the help of a flying machine - Ravana kidnapping Sita
- it took another flying Hanuman to get Sita back
- Fly like birds - so went the saying
- stick wings to your shoulders
- flap and jump from a tall tower;

- the lucky ones died;
- the unlucky survived with broken bones to live out the rest of their lives in agony;
- moral: man is not made to fly - with wings are otherwise
- When you attach a heavy object to a cork and put it in water the heavy object floats
- Catch a hot-air balloon and float with it; was tried first 1783
- Called Aerostatics:
- Buoyancy of still air supports the weight:
- Archimedes principle: Buoyancy force equals weight of fluid displaced;

- what about aerodynamics:
- Moving air supporting weight
- 1903 : Wright brothers — flew for 12 seconds
- 1905 : Nikolai Yegorovich Zhukovsky (1847 - 1921) announced after careful experimentation:
- air plane flies because of Bernoulli principle
- the wing, moving fast, splits the air into an upper stream and a lower stream
- air in the upper stream moves faster than that in the lower stream
- pressure above wing is lower than that below wing;

The Wright brothers

Orville Wright (1871 - 1948)

Wilbur Wright (1867 - 1912)

Nikolai Yegorovich Zhukovsky (1847 - 1921)

- the pressure difference lifts the wing which in turn lifts all of us sitting inside the body of the plane
- once we knew mathematics of flying - we could design better air planes;
- we could design safer air planes with better and sophisticated control mechanisms;
- it took us 1900 years for mankind to learn to fly;
- but once we knew the mathematics of it it took just 50 years to soar into the sky ....to the moon ..... to the planets .... and in the coming years beyond the solar system ... and to the distant galaxies !

- this is precisely the power of mathematics - the beauty of mathematics - the poetry of mathematics

Age can not wither her
nor custom stale her
infinite variety

- THANKS