Strong field tests of Gravity using Gravitational Wave observations

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1. Introduction

2. Tests of Gravity using GW observations of Inspiralling Compact Binaries

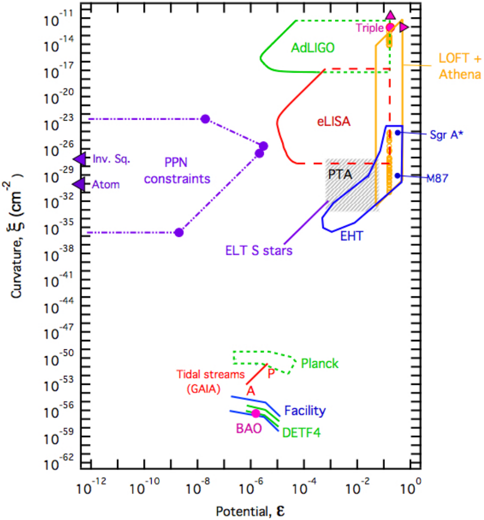
3. Scalar Tensor Theories of gravity

4. Massive Graviton Theories

5. Parametrized waveforms
   - Parametrized tests of post-Newtonian theory.
   - Parametrized post-Einsteinian Framework

6. Systematic effects affecting the Tests of Gravity

7. Conclusions



[Baker et al, 2014]

Parameters

$$\epsilon = \frac{GM}{rc^2},$$
$$\xi = R_{abcd} R^{abcd} .$$

ξ is called Kretschmann scalar and scales as $\sim \frac{GM}{r^3 c^2}$ for Schwarzchild metric.

Why test GR?

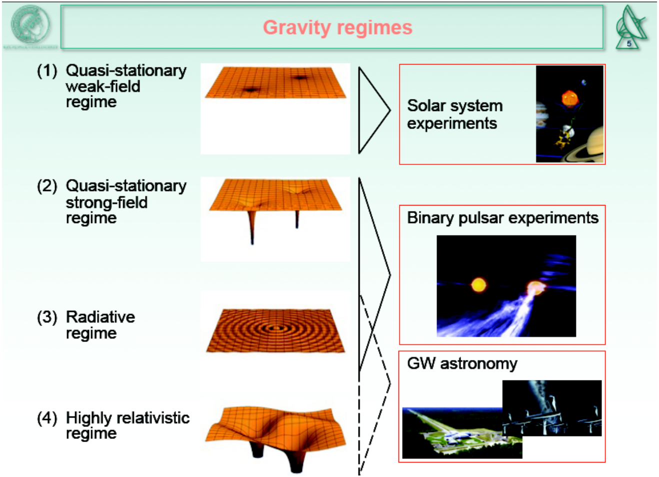
Successes of GR

GR has passed all the tests of gravity till date in flying colors.

Still..

- No fundamental reason why GR is the correct theory of gravity at all scales.
- Even if its 'correct', always good to quantify the correctness of GR.
- Weak-field tests put very stringent bounds, but these parameters may grow very rapidly as a function of field strength.
- Singularities in the theory.
- Early universe and quantum gravity.

Regimes of Gravity



[Figure Courtesy: N Wex]

Classifying Tests of Gravity

- Laboratory, Astrophysical, Cosmological.
- Static, Kinematic, Dynamical.
- Weak field, Moderate field, Strong-field.
- Model dependent, Model Independent.

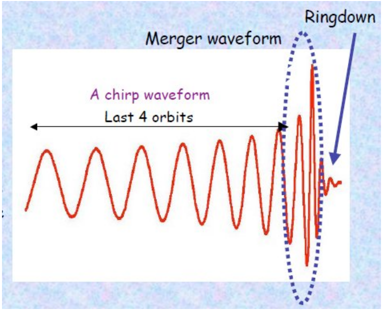
Tests of GR at a glance

[Living Review Articles by Clifford Will, Psaltis, Stairs]

- Fundamental principles of GR:
  - Strong & Weak equivalence principle.
  - Gravitational Redshift.
- GR predictions in *weak* fields:
  - Solar system bounds $\epsilon \sim 10^{-6}$
  - Parametrized post-Newtonian (PPN) formalism is used very efficiently. [Clifford Will & Collaborators.]
- GR predictions in the *strong* field regime:
  - Binary Pulsar Tests $\epsilon \sim 10^{-3}$
  - Parametrized post-Keplerian (PPK) parametrization used. [Damour & Collaborators.]
- Other tests:
  * Event Horizon.
  * Gravitational Lensing
  * No Hair Theorem.

GWs: natural way to probe strong-field gravity.

GWs from inspiralling compact binaries



Inspiral-Merger-Ringdown

- Inspiral $\Rightarrow$ PN theory
- Merger + Ringdown $\Rightarrow$ Numerical Relativity

- Inspiralling compact binaries composed of Black Holes (BHs) or neutron stars (NS) are the most promising sources for GW detection.
- Prior predictability using General Relativity $\Rightarrow$ Use of matched filtering for detection and parameter estimation.
- One can verify various strong-field predictions of GR, from GW observations each of which constitute a test of GR.

Tests of GR using GWs

Philosophy

- If the underlying theory of gravity is not GR but something else, the gravitational waveforms will be different in that theory.
- Estimating the additional parameters of the alternative theory will give us an estimate or bound on the parameters. (Parameter Estimation Problem)

Method

- Give the expected sensitivity (noise Power Spectral Density) of advanced detectors such as advanced LIGO (aLIGO), we can assess the ability of aLIGO to constrain the parameters of the alternative theories.
- We need to have at least the leading order correction to the GR waveforms from the alternative theory that we are interested to constrain.

Crucial: Use of Matched filtering to analyse the GW data

What are we after?

Signatures of of Possible deviations from GR

- Monopole, Dipole GW radiation.
- Additional modes of polarization of GWs (beyond $h_+, h_\times$).
- Non-null propagation of GWs.
- Correction to the GR phasing formula.

Once GWs are detected, we are interested if the detected GWs have any of these properties different from the GR predictions.

Important

- Many alternative theories predict one or more of these qualitative deviations (e.g., detection of dipole radiation will not give us any clue about the theory of gravity)
- Not all of them are independent effects (e.g. most of the theories which predict dipolar radiation also predicts other polarization modes)

Different alternative theories

- Scalar Tensor Theories.
- Tensor-Vector-Scalar (TeVeS) Theories.
- Massive Graviton Theories.
- $f(R)$ Theories.
- Higher Dimensional Gravity.

For any theory of gravity that we want to test, we want to know if there are any features in the theory which are different from GR and can those be observed with the sensitivity of GW detectors.

Scalar-Tensor Theories

Overview

Action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \left(\partial_\mu \phi\right) \left(\partial_\nu \phi\right) - U(\phi) \right] + S_M[\Psi, g_{\mu\nu}] \,,$$

- One more more scalar degrees of freedom appears in the gravitational sector of the theory, non-minimally coupled. Let the coupling parameter be $\omega(\phi)$.
- Jordan-Friez-Brans-Dicke theory is a special case of this where $\omega(\phi) = \omega_{\rm BD}$ and $U(\phi) = 0$.
- A generalization of this is a theory where $\omega(\phi) = \alpha_0 + \beta_0 (\varphi - \varphi_0)$. [Damour & Esposito-Farése]
- Another variant is massive scalar tensor theories, where $\omega(\phi) = \omega(\varphi_0)$ but $U(\phi) \neq 0$ (scalar field has is massive).

Gravitational Waveforms in ST theory

Dipolar GWs

- The important distinguishing feature of ST gravity is that it predicts dipolar Gravitational radiation, unlike GR whose leading order GW emission is quadrupolar.
- This is because ST gravity does not respect equivalence principle.
- The leading dipolar term in the energy flux & phasing is "pre-Newtonian" (one order earlier than the leading PN term).

Additional modes of GW polarization

- ST theories predict a third transverse mode ('breathing mode') of GW polarization (in addition to the two transverse modes of GR).

Phasing in JFBD theory

[Will 94, Krolak Kokkotas, Schäfer 1994.]

- This introduces a new term in GW phasing formula which is proportional to a parameter defined as $\omega_{\rm BD}$:

$$\Psi(f) = 2\pi f\, t_c - \phi_c + \frac{3}{128\eta}\, v^{-5}\left[1 + \frac{3\,\mathcal{S}^2}{84\,\omega_{\rm BD}} v^{-2} + \text{GR terms}\right]$$

where $v = (\pi m f)^{1/3}$ (characteristic velocity), $\mathcal{S} = s_2 - s_1$ (difference in 'sensitivities' of the two compact objects.)

- For binary neutron stars $\mathcal{S} \sim 0.05 - 0.1$ and for NS-BH binaries $\mathcal{S} \sim 0.3$ and for a binary BH $\mathcal{S} = 0$.
- **Hence one of the binary constituents should be a NS.**

Goal

The aim here is to bound $\omega_{\rm BD}$ which is ∞ in GR.

Higher order PN modelling of scalar-tensor theories?

Equations of motion (Mirshekari & Will 2013)

- 2.5PN accurate equations of motion for general scalar tensor theories.
- Motion of two BHs is same in both GR and ST theory.
- Motion of NS-BH binary is same as in GR through 1PN order. But from 1.5PN order, motion is different from GR and till 2.5PN order the difference is governed by single parameter (which depends on the coupling and the internal structure).

Tensor Gravitational Waveform (Lang 2014)

- 2PN $\tilde{h}_{ij}$ including hereditary and memory effects.
- BBH waveforms are same in both GR and ST theory, but NS-BH waveforms differ at 1PN order depending on a single parameter.

GW Energy flux (Lang 2014)

1PN (relative to the quadrupolar flux) correction to the GW energy flux is available. This includes flux due to scalar waves.

More general scalar-tensor theories

[KGA, 2012]

One can introduce a new *amplitude* parameter (α) in addition to the phase parameter β into the waveform which describes a generic dipolar GW emission.

$$\tilde{h}(f) \simeq \tilde{A}\left[\frac{v_2^2}{\sqrt{2\dot{F}_{\rm GR}^{\rm Newt}(v_2)}}\left(1-\frac{\beta}{2}v_2^{-2}\right)e^{2i\,\tilde{\Psi}(v_2)}+\frac{\alpha\, v_1}{\sqrt{\dot{F}_{\rm GR}^{\rm Newt}(v_1)}}\,e^{i\,\tilde{\Psi}(v_1)}\right] \quad (1)$$

In the above expression, the Fourier Domain phasing is given by

$$\tilde{\Psi}(v_k) = \tilde{\Psi}_{\rm GR}^{\rm Newt}(v_k)\left(1+\overline{\beta}v_k^{-2}+\cdots\right), \quad (2)$$

where it is straightforward to show that $\overline{\beta} = -4\beta/7$.

Bounds

One can then obtain the expected bounds α & β for various detector configurations.

'Light' scalar-tensor theories

Phasing formula for Massive variant of JFBD theory is computed in (Alsing et al 2012 & Berti et al, 2012).

$$
\begin{aligned}
\psi(f) = {} & 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128(\pi\mathcal{M}f)^{5/3}} \times \\
& \times \Big\{ 1 + \zeta \\
& + \frac{20}{9} A \eta^{-2/5} (\pi\mathcal{M}f)^{2/3} - 16\pi\eta^{-3/5}(\pi\mathcal{M}f) + \ldots \\
& + \xi\Gamma^2\nu \left[\frac{5}{462}\eta^{6/5}(\pi\mathcal{M}f)^{-2} - \frac{\nu}{1632}\eta^{12/5}(\pi\mathcal{M}f)^{-4} \right] \\
& \times \Theta(2\pi f - m_s) \\
& + \xi\mathcal{S}^2 \left[\frac{25\nu}{1248}\eta^{8/5}(\pi\mathcal{M}f)^{-8/3} - \frac{5}{84}\eta^{2/5}(\pi\mathcal{M}f)^{-2/3} \right] \\
& \times \Theta(\pi f - m_s) \Big\}, \qquad (16)
\end{aligned}
$$

where we defined

$$
\zeta = \frac{2}{3}\xi\left(s_1 + s_2 - 2 s_1 s_2\right) + \frac{\xi}{2} - \frac{\xi\Gamma^2}{12}\Theta(2\pi f - m_s). \quad (17)
$$

Summary: ST theory

- Distinguishing signature of gravitational waveforms in ST gravity are the dipolar GW emission and the corresponding modifications to the phase and a new transverse mode of polarization (breathing mode).
- Progresses have been made in modelling the phase more accurately for general scalar tensor theories as well as massive variants of ST theories.
- *Efficient* implementation of these in the data analysis pipelines may be an interesting thing.

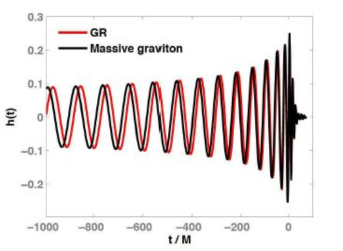
Massive Graviton Theories

Basic Idea: Will, 1998

- If gravitation is propagated by a massive field, then the velocity of gravitational waves (gravitons) will depend upon their frequency as $\left(\frac{v_g}{c}\right)^2 = 1 - \left(\frac{c}{f\,\lambda_g}\right)^2$.
- For compact binary inspiral, low frequency GWs would travel slightly slower compared to high frequency components, hence distorting the waveform w.r.t the GR waveforms.

Such a distortion can be parametrized in terms of an additional term in the phasing formula at 1PN order in terms of compton wavelength of the graviton λ_g which can be bounded from GW observations.

$$\psi_{\rm MG}(f) = \psi_{\rm GR}(f) + \delta\psi(\lambda_g)$$



[Figure courtesy: P Ajith]

Model Independent Bounds I: Parametrized tests of PN theory.

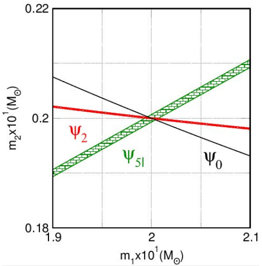
[KGA, Iyer, Qusailah, Sathyaprakash (2006)]

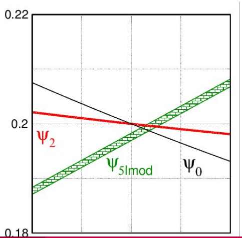
Basic idea: Parametrized phasing formula

[Blanchet & Sathyaprakash 1994, 1995, KGA, Iyer, Qusailah & Sathyaprakash, 2006a,b; Mishra, KGA, Iyer& Sathyaprakash, 2011.]

$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \sum_{k=0}^{7} (\psi_k + \psi_{kl} \ln f)\, f^{\frac{k-5}{3}},$$

- For nonspinning binaries, ψ_k & ψ_{kl} are functions of the masses of the constituent binaries.
- Measure at least 3 of these coefficients and require their consistency in the Mass plane.
- Similar in spirit to binary pulsar tests!





Problem of high correlation & solution

The problem

- The best case scenario is where you treat all the 8 PN coefficients as independent parameters and estimate them from the data.
- Since they are all functions of just two variables (m_1 & m_2), there is very high correlation among these eight parameters which results in a very poor estimation of these coefficients.

Solution-I

[KGA, Iyer, Qusailah, Sathyaprakash, 2006b]

- One simple way out is to keep only 3 of these coefficients as independent and express all others in terms of two of them.
- For example, one can treat $\{\psi_0, \psi_2, \psi_3\}$ as the set of parameters and express all higher order coefficients in terms of ψ_0 & ψ_2.
- This way, the full test is now being split into many (${}^8C_2$ in total) tests.

Solution 2: Singular Value Decomposition of Fisher matrix

[Pai & KGA, 2012]

- A more systematic way to remove degeneracies is to use the SVD of the Fisher matrix and obtain a *reduced* set of parameters which are combinations of the phasing coefficients which can be best estimated.
- These new set of parameters can be estimated with much better accuracy (compared to the original ones) (see figures).
- One can reformulate the parametrized tests in terms of these new combinations.

Parametrized post-Einsteinian Framework

[Yunes, Pretorius (2009)]

Basic Idea

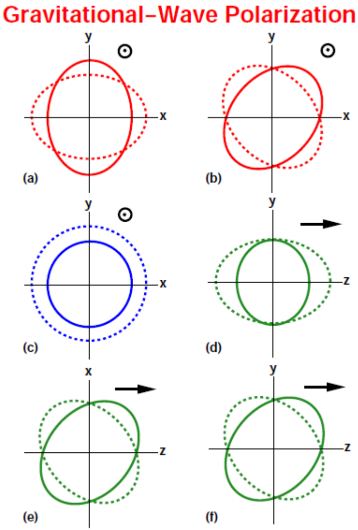
Understand and quantify the theoretical bias due to the use of GR waveforms by writing down a parametrized waveform which characterizes the departure from GR by the presence of additional parameters (called as ppE parameters).

- Kind of a generalization of PTPN framework.
- Guiding assumptions:
  - ⇒ Metric theories of gravity.
  - ⇒ Weak-field consistency with GR.
  - ⇒ Strong field inconsistency.
- Inclusion of merger & ringdown waveforms in addition to the inspiral.

ppE waveforms

$$\tilde{h}(f) = \begin{cases} \tilde{h}_{\mathrm{I}}^{(\mathrm{GR})}(f) \cdot (1 + \alpha u^a) e^{i\beta u^b} & f < f_{\mathrm{IM}}, \\ \gamma u^c e^{i(\delta + \epsilon u)} & f_{\mathrm{IM}} < f < f_{\mathrm{MRD}}, \\ \zeta \frac{\tau}{1 + 4\pi^2 \tau^2 \kappa (f - f_{\mathrm{RD}})^d} & f > f_{\mathrm{MRD}}, \end{cases}$$

Generic Metric theory of gravity and GW polarizations



- Three transverse modes + three longitudinal modes.
- Scalar-Tensor theories predict 3 modes of polarization, while TeVeS theories predict all 6 modes.

Extended ppE with complete polarization content

[Chatziioannou, Yunes & Cornish (2012)]

Summary

- A general metric theory of gravity can have 6 states of polarizations.
- A parametrized waveform, in the Fourier domain, incorporating the contributions from all 6 modes of polarization was written down for the inspiral phase of the binary evolution.
- These new polarization parameters can be constrained by using Null streams from the data of multiple detectors.

Systematic effects which affect Tests of Gravity

There may be physical effects within GR, which can mimic a deviation from GR and hence affect our ability to accurately test GR.

- Spin effects is among the most important effect which can affect our ability to test GR accurately.
- Using analytical waveform (based on PN expansion, say) will also affect our ability to test GR.

Conclusion

- Lots of proposals have been made to test Gravity in the strong field regime using GW observations.
- Careful implementation of some of these interesting ideas will be very valuable by the time we have first detection.
- It will be important to refine some of these proposals and study how they will be affected by systematic errors.
- What may be more important will be to develop efficient parameter estimation pipelines which implement these ideas at the algorithm level.