

Quiz for Intro to Math Finance, 2026

330 pm - 530pm

duration = **2 hours**; total = **35 marks**

April 11, 2024

1. **4 marks** Find the distribution of the random variable $X = \int_0^T W_t dt$, where W_t is a standard Brownian motion. (*Note: Clearly state the mean and variance*).
2. **6 marks** Let W_t be a standard Brownian motion. First, demonstrate that the process W_t^2 is not a martingale. Next, find a deterministic function $f(t)$ such that the modified process $M_t = W_t^2 + f(t)$ is a martingale.
3. **5 marks** Given a standard Brownian motion $W(t)$, compute the correlation between $W(5)$ and $W(10)$.
4. **5 marks** Assume an asset price S_t follows the geometric Brownian motion $dS_t = \mu S_t dt + \sigma S_t dW_t$. Consider a new process defined by $X_t = S_t^2$. Use Itô's Lemma to derive the stochastic differential equation (SDE) satisfied by X_t . (*Note: Express your final SDE entirely in terms of X_t , μ , σ , and dt , dW_t*).
5. **7 marks** Given the OU / Vasicek process

$$dr = a(\theta - r)dt + \sigma dB(t),$$

compute the expectation $E[r(t)]$.

6. **10 marks** Derive the Black-Scholes Partial Differential Equation (PDE) for the price of an arbitrary derivative $V(S, t)$. For your derivation, assume the underlying asset price S_t follows the geometric Brownian motion $dS_t = \mu S_t dt + \sigma S_t dW_t$.