

Quiz for Intro to Math Finance

330 pm - 530pm

duration = **2 hours**; total = **35 marks**

April 11, 2025

1. **3 marks** Given standard/canonical Brownian Motion $B(t)$, find the probability $P(B(1) + B(2) > 1)$.
2. **3 marks** Find the distribution of $\int_0^T f(t) dB_t$.
3. **4 marks** Find the distribution of $\int_0^T B_t dt$.
4. **5 marks** You have access to a random number generator $rand()$ that generates real numbers between 0 and 1 uniformly randomly. Using this, describe an algorithm to estimate the value of π by a Monte Carlo method. [hint: generate numbers in the unit square and estimate the area of the circle.] Estimate the error in this estimation of π .
5. **5 marks** Given that Stock Prices $S(t) \equiv S_t$ follow a geometric Brownian Motion process,

$$\frac{dS_t}{S_t} = \mu dt + \sigma dt \quad (1)$$

find the distribution $S(T)$ at time $t = T$.

6. **15 marks** Given the OU process

$$dr = a(\theta - r)dt + \sigma dB(t),$$

compute (a) $E[r(t)]$ and (b) $\text{Var}[r(t)]$.