Mid Sem Exam for Intro to Math Finance

1 pm - 4pm

duration = **3 hours**; total = **50 marks**

April 3, 2025

1. Draw the graphs of the payoffs of the following combinations of options, as a function of the asset price:
   (a) **1 marks** buying one call plus one put option with the same strike price K (this is known as a straddle)
   (b) **3 marks** buying one call option and selling one put option with the same strike price K
   (c) **3 marks** buying one call option with strike price $K1$, and selling another call option with strike price $K2$ (this is known as a bull spread; you will want $K2 > K1$)
2. Consider a two period binomial model, with spot price of $S = 100$ at $t = 0$ and the stock price is either $S \times u$ or $S \times d$ after each $1year$ period. Take $u = 2$ and $d = 1/2$. We are thus only concerned with $t = 0, 1$ and 2 years. Also, assume an interest rate of $10\%/year$ compounded every 6 month.
   (a) **3 marks** Compute the risk neutral probability of the stock price at 400 at $t = 2y$.
   (b) **10 marks** Take a call option with strike $K = 110$ and maturity $T = 2years$. Compute its price at $t = 0$ using (a) replication portfolio method and (b) risk neutral probabilities.
3. **10 marks** Consider a two-period binomial model for the stock price with both periods of length one year. Let the initial stock price be S(0) = 100 and assume that the stock pays no dividends. Let the up and down factors be $u = 1.25$ and $d = 0.75$ respectively. Let the continuously compounded interest rate be $r = 5\%$ per annum.

   Roger is interested in purchasing a chooser option with the provision that he can choose if the option is a put or a call after one year. The strike for this option is \$100 and the expiry date is two years. Using the binomial tree mentioned in the first para, find the price of the chooser option.
4. Consider the knight's tour on a chess board: A knight selects one of the next positions at random independently of the past.
   (a) **5 marks** Set up the process as a Markov chain. Is it irreducible? Is it aperiodic?
   (b) **10 marks** Find the stationary distribution. Give an interpretation of it: what does it mean, physically?
   (c) **5 marks** Which are the most likely states in steady-state? Which are the least likely ones?