Mid Sem Exam for Intro to Math Finance

10 am - 1pm

duration = **3 hours**; total = **50 marks**

March 9, 2025

1. Consider a one period binomial model, i.e., (a) spot price of $S = 100$ at $t = 0$ and (b) the stock price is either $S \times u$ or $S \times d$ at maturity $t = 1 year$ depending on if stock price moved up or moved down. Let $u = 2$ and $d = 1/2$. Also, assume a continuous interest rate of $10\%/year$.

   (a) **5 marks** Compute the risk neutral probability of the stock going up.

   (b) **10 marks** Take a call option with strike $K = 110$ and maturity $T = 1 year$. Compute its price at $t = 0$ using (a) replication portfolio method and (b) risk neutral probabilities.

2. Consider a two period binomial model, with spot price of $S = 100$ at $t = 0$, $u = 2$ and $d = 1/2$. Also, take each period to be one year, i.e, we are only concerned with $t = 0, 1$ and 2 years. Also, assume a continuous interest rate of $10\%/year$.

   (a) **5 marks** Compute the risk neutral probability of the stock price at 400 at $t = 2y$.

   (b) **10 marks** Take a call option with strike $K = 110$ and maturity $T = 2 years$. Compute its price at $t = 0$.

3. I have 4 umbrellas, some at home, some in the office. I keep moving between home and office. I take an umbrella with me only if it rains. If it does not rain I leave the umbrella behind (at home or in the office). It may happen that all umbrellas are in one place, I am at the other, it starts raining and must leave, so I get wet.

   (a) **10 marks** If the probability of rain is p, what is the probability that I get wet, over a long time?

   (b) **10 marks** Current estimates show that p = 0.6 in Edinburgh. How many umbrellas should I have so that, if I follow the strategy above, the probability I get wet is less than 0.1?