Thermal Physics, Autumn 2016 CMI
Problem set 1
Due by 5pm Thursday Aug 18, 2016
Hydrostatic pressure, Exact differential

1. $\langle \mathbf{13} \rangle$ Recall that the hydrostatic pressure on a small flat membrane (with unit normal $\hat{n}$) in a fluid is the normal component of the force per unit area acting on the membrane due to the fluid on one side of it. Let us illustrate the fact that the pressure does not depend on the orientation of the membrane. Consider a small fluid element in the shape of a tetrahedron with three right isosceles faces perpendicular to each other. It may be visualized as filling a tetrahedral region in the first quadrant with vertices $(0,0,0), (a,0,0), (0,a,0)$ and $(0,0,a)$.

   (a) $\langle \mathbf{2} \rangle$ Draw the fluid element labelling the axes and vertices.

   (b) $\langle \mathbf{2} \rangle$ What are the lengths of the sides of the four faces of the tertahedron?

   (c) $\langle \mathbf{3} \rangle$ What are the areas of the four faces?

   (d) $\langle \mathbf{4} \rangle$ Suppose p_i is the compressional (inward) pressure acting on the small face perpendicular to the $\hat{x}_i$ direction and p_4 the pressure on the small oblique face. Find the forces $\vec{f}_{1,2,3,4}$ acting on the four faces.

   (e) $\langle \mathbf{2} \rangle$ If the tetrahedral element is in hydrostatic equilibrium, what is the sum of the four forces? What does this imply for the pressures $p_1, \cdots, p_4$?

2. $\langle \mathbf{6} \rangle$ Consider the Pfaffian differential expression in two variables $\omega(x,y) = y\,dx - x\,dy$. Find whether it satisfies the integrability condition to be exact. If it does, find a function $\sigma(x,y)$ such that $\omega = d\sigma$. If not, find an integrating denominator $\tau(x,y)$ and function $\sigma(x,y)$ such that $\omega/\tau = d\sigma$ is exact.