Quantum Mechanics 3, Spring 2012 CMI

Problem set 8

Due by beginning of class on Monday Mar 12, 2012

Charged particle spin half particle, SHO

1. ⟨11⟩ A hamiltonian for a spin half particle (charge e, mass m) in an electromagnetic field is

$$H = \frac{1}{2m}\left[\vec{\sigma} \dot{\otimes} (\vec{p} - e\vec{A})\right]^2 + e\, I \otimes \phi \tag{1}$$

   (a) ⟨8⟩ Expand out the hamiltonian using properties of the Pauli matrices and identify its three terms: (i) the usual coupling to a vector potential as for a spin zero particle, (ii) a spin magnetic dipole energy and (iii) an electric potential energy as for a spin zero particle.

   (b) ⟨3⟩ Extract the value of the gyromagnetic ratio and g-factor for the spin magnetic moment predicted by this hamiltonian and compare with the experimental value for an electron.

2. ⟨9⟩ Infinite product formula for sine function.

   (a) ⟨1⟩ Find all the zeros of the analytic function of a complex variable $f(z) = \sin \pi z$. Hint: write in terms of exponentials.

   (b) ⟨2⟩ Based on the location of zeros of $f(z)$ it is possible to write

$$\sin \pi z = e^{g(z)}\, z \prod_{n \neq 0} \left(1 - \frac{z}{n}\right) e^{z/n} \tag{2}$$

   where $g(z)$ is a function analytic everywhere, so that $e^{g(z)}$ is analytic and non-zero everywhere on the complex plane. *Evaluate the logarithmic derivative of both sides.*

   (c) ⟨1⟩ Determine $g'(z)$ by comparing with the infinite series

$$\pi \cot \pi z = \frac{1}{z} + \sum_{n \neq 0} \left(\frac{1}{z-n} + \frac{1}{n}\right). \tag{3}$$

   (d) ⟨1⟩ Determine $g(z)$ by taking the limit of $f(z)/z$ as $z \to 0$.

   (e) ⟨1⟩ Use the above results to write an infinite product formula for $\sin \pi z$ grouping together the n and $-n$ factors.

   (f) ⟨3⟩ Use the infinite product for $\sin(\pi z)$ to find its first two non-zero Taylor coefficients. Compare with their known values.