Quantum Mechanics 3, Spring 2012 CMI

Problem set 5

Due by beginning of class on Monday Feb 6, 2012

Frequency-Time relation, charged particle in an electromagnetic field

1. Consider an ensemble of atoms in an unstable and unfamiliar state i, whose energy we do not know. Such atoms are found to decay via the emission of radiation to a more familiar state f, whose energy E_f is known to us. Suppose we wait a time $\Delta t = \tau$ equal to the life-time of the unstable state i.

   (a) $\langle 2 \rangle$ On average, what remains after a time τ?

   (b) $\langle 6 \rangle$ Suppose we are able to find the energy carried away by radiation in each decay. Discuss the implications of the frequency-time relation for this situation. What does it predict for the distribution of frequencies of the emitted radiation? To what extent can we predict the energy of the mysterious state i?

2. $\langle 3 \rangle$ Consider the transformation of electromagnetic potentials by a scalar function $\chi(\vec{r}, t)$[1]

$$\vec{A} \to \vec{A}' = \vec{A} + \nabla\chi \quad \text{and} \quad \phi \to \phi' = \phi - \frac{\partial \chi}{\partial t} \tag{1}$$

   Find how the fields (a) $\vec{E}$, (b) $\vec{B}$ and (c) $\oint_C \vec{A} \cdot d\vec{l}$ change under this transformation for a closed curve C.

3. Consider the Schrödinger equation for a charge e mass m particle in an electromagnetic field in three dimensions

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \quad \text{where} \quad H = \frac{1}{2m}\left(\vec{p} - e\vec{A}\right)^2 + e\phi. \tag{2}$$

   (a) $\langle 3 \rangle$ Find the dimensions of the quantities (a) $\frac{e}{\hbar}$ (b) χ and (c) $\frac{e\chi}{\hbar}$ with χ as in the previous problem.

   (b) $\langle 6 \rangle$ Find the equation satisfied by the new wave function $\psi' = e^{\frac{ie\chi}{\hbar}}\psi$ and new potentials $\vec{A}', \phi'$ in as simple a form as possible.

[1] Note that primes do not denote differentiation in this problem set.