Quantum Mechanics 3, Spring 2012 CMI
Problem set 2
Due by beginning of class on Monday Jan 16, 2012
Adiabatic approximation & Spin

1. $\langle 4 \rangle$ Show that the geometric phase angle occurring in the adiabatic theorem

$$\theta_n^G(t) = \gamma = \int_0^t \langle \psi_n(t') \mid i \, \frac{\partial \psi_n(t')}{\partial t'} \rangle \; dt'. \tag{1}$$

   is real, so that we are justified in calling it a phase angle. Here $\psi_n(t)$ are orthonormal eigenstates of the hamiltonians $H(t)$ for each t with eigenvalues $E_n(t)$.

2. $\langle 2 \rangle$ With the same notation as above, show that

$$\dot{E}_n = \langle \psi_n | \dot{H} | \psi_n \rangle. \tag{2}$$

3. $\langle 2 \rangle$ Find a matrix representation of the component of spin $\vec{S} \cdot \hat{n}$ in the direction of the unit vector $\hat{n} = (n_x, n_y, n_z)$, for a spin half particle.

4. $\langle 3 \rangle$ Find the eigenvalues of the component of spin $\vec{S} \cdot \hat{n}$ in any direction $\hat{n}$ for a spin-half particle by evaluating the square of this operator and its trace.

5. $\langle 4 \rangle$ Find the corresponding normalized eigenvectors of the component of spin $\vec{S} \cdot \hat{n}$ where $\hat{n}$ is a unit vector with spherical polar coordinates $(1, \theta, \phi)$.